The Social Selection of Flexible and Rigid Agents

By JOSEPH E. HARRINGTON, JR.*

People differ in how they respond to their environment. Some individuals treat each situation as unique and tailor their behavior accordingly while others respond in the same manner regardless of the situation. My objective is to explore how social systems select from such a heterogeneous population. A class of simple hierarchical systems is considered which encompasses some features of corporations and electoral systems. A selection process operates on this population which results in successful agents going on to compete against equally successful agents for further advancement. I characterize the population dynamics and the type of agent that ultimately dominates. (*JEL* D00, D23, D72)

Who will cure the nation's ills?
A leader with a selfless will.
But how will you find this leader of yours?
By a process of natural selection of course.

W. H. Auden (1940)

Across diverse circumstances, there is often a regularity in the manner in which a person responds to his or her environment. This observed unity to a person's behavior is part of what defines that person. In the political arena, some politicians routinely support the popular positions on key issues while others, often referred to as ideologues, appear to embody a particular world view and support positions consistent with that view. The former are driven by the "here and now" of getting elected while the behavior of ideologues is rooted in those past events which determined their ideology and their commitment to that ideology.[1] Representative of such a contrast are Winston Churchill and Prime Minister Stanley Baldwin during the 1930's (William Manchester, 1988 p. 219):

> Winston was guided by a built-in gyroscope which would carry him toward his objective through tumult, while the prime minister relied on a kind of sociological radar—signals from voters—to determine his course.

A related form of heterogeneity is manifested in organizations. There are those individuals who strive to please their superior—commonly referred to as "yes men"—while others (let us call them mavericks) appear to possess a definite opinion as to what is the proper action, independent of their superior's opinions, and are committed to acting on it. "Yes men" are driven by the expedient

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[1] See Aaron Wildavsky (1965) for a comparative discussion of ideologues and pragmatists and Gary M. Maranell (1970) for a ranking of U.S. presidents in terms of flexibility.

dictum "to get along, you go along," while mavericks "march to the tune of a different drummer." Turning to the ranks of businessmen, one finds that some are quick to adjust their business strategy to new market conditions while others dogmatically cling to their original plan.[2] A classic example is Henry Ford, whose stubborn commitment to the same strategy arguably caused Ford Motor Company to squander its leadership. Alfred P. Sloan, CEO for General Motors, commented (1965 pp. 163, 437):

> Mr. Ford, who had so many brilliant insights in earlier years, seemed never to understand how completely the market had changed ... [He] stayed too long with his old and once dominantly successful concept of the business.

An entrepreneur who was no stranger to change is Sam Walton (1992 p. 169):

> When folks buy into a way of doing things, and really believe it's the best way, they develop a tendency to think that's exactly the way things should always be done. So I've made it my own personal mission to ensure that constant change is a vital part of the Wal-Mart culture itself.

The commonalty to these diverse settings is that they all encompass people who differ in how they respond to a change in circumstances. Politicians who follow the dictates of their constituents, members of an organization who "go along," and entrepreneurs who routinely modify their business strategy are *flexible* in that their behavior is driven by the conditions of the environment—whether it is to win an election, get promoted, or increase profit. In contrast, political ideologues, organizational mavericks, and dogmatic entrepreneurs are *rigid* in that they appear to be endowed with some fundamental approach and rarely veer from it. Their behavior is relatively unresponsive to the current shape of the environment and is instead determined by past events that established the properties of their code of conduct.[3]

The central premise of this study is that a substantive source of heterogeneity among people is the degree to which they modify their behavior in response to a change in their environment. Indeed, there is a considerable psychology literature that documents and characterizes heterogeneity along a rigidity–flexibility dimension.[4] Let me now place such a diverse population within the confines of a government, corporation, or army. This leads to a central research question: given a population of flexible and rigid agents and a hierarchical social system within which they interact and compete, which type of agent will rise to the top? Are ideologues or office-seekers more likely to occupy high-level offices like governor, senator, and president? Are apparatchiks or mavericks more proficient at climbing the corporate ladder and becoming senior vice presidents and chief executive officers? Are successful entrepreneurs more likely to have a business strategy built on persistence and dogma or flexibility and change? Are the leaders of our society more likely to be pragmatists who "blow with the wind" or visionaries with a "selfless will" who would sacrifice themselves as martyrs before compromising their beliefs?

Toward addressing these questions, I develop a simple class of hierarchical societies in which the population of agents differ in the flexibility of their behavior. My research objective is to understand how the properties of

[2] William D. Guth and Renato Tagiuri (1965) discuss how personal values might influence corporate strategy. At the level of business strategy, the trade-off between committing to an approach and committing to being flexible is the focus in Robert H. Hayes and Gary P. Pisano (1994).

[3] This type of heterogeneity manifests itself in the judicial realm as well, as was pointed out to me by Keith Scharfman. A rigid judge may be interpreted as one who acts in "good faith" in that he administers his position based "on the reasons provided by the law" even though he may personally disagree with the decision or know that it makes society worse off (Steven J. Burton, 1992 pp. 36–37). In opposition is the expedient judge who fails to exercise self-restraint and lets personal interests and preferences, rather than overriding principles, influence his decisions.

[4] See, for example, Abraham S. Luchins and Edith Hirsch Luchins (1959), William A. Scott (1966), and David H. Jonassen and Barbara L. Grabowski (1993).

a social system influence the characteristics of those agents who do well within that system. This research fits into a growing line of work which uses an evolutionary-style framework to explore the types of preferences or behavioral rules that survive in a competitive environment. This work is unique in modelling the social environment as hierarchical and in having agents differ in terms of their flexibility. A similar form of heterogeneity is considered in John Haltiwanger and Michael Waldman (1991), Robert W. Rosenthal (1993), and Dale O. Stahl (1993) and, at a population level, in Elchanan Ben-Porath et al. (1993). Encompassing distinct forces, Ronald A. Heiner (1983) investigates the appropriate amount of flexibility. The flexibility of behavioral rules is also central to the time-consistency problem in macroeconomics and is the root of the debate on rules versus discretion; see Finn E. Kydland and Edward C. Prescott (1977). From this perspective, this paper questions whether agents endowed with a rule survive at a higher or lower rate than those endowed with discretion.

I. A Class of Hierarchical Competitive Systems

I start with a hierarchical system with a lowest level and no upper bound on the highest level. At each level there is a large population of agents, specifically, a continuum of agents.[5] Whether it is a politician trying to advance from the state legislature to the House of Representatives or a regional manager striving to become a vice president in a corporation, advancement typically requires performing relatively better than some subset of peers who are also eligible for promotion. This process is modelled by assuming that, at each level, agents are randomly matched into pairs and compete for "promotion" to the next level.[6] The terms promotion, advancement, and survival interchangeably are used. The assumption that an agent is compared to only one other agent is a concession to tractability but would seem to be reasonable for electoral systems where general elections typically involve two candidates. Each of these matchings is faced with a stochastic environment. Once the environment is determined and revealed to the agents, they choose actions. The agent with greater performance is promoted to the next level while the other agent is assumed to drop out of the system or, more to the point, no longer be eligible for promotion. While this "up-or-out" structure is extreme, it is not without merit. Casual observation suggests that a large percentage of candidates who lose do not run again and corporate employees who are "passed over" when their time has come may no longer be on the "fast track," which makes them less likely to be considered for promotion.[7]

There are two possible environments which I denote type 0 and type 1. At each level, a proportion b of all matchings have a type 1 environment. This is assumed to be i.i.d. across levels so that the probability an agent faces a type 1 environment is b and this is independent of his personal history. While each agent faces an uncertain future environment, the absence of aggregate uncertainty simplifies matters. Without loss of generality, I make a type 1 environment more common: $b \in (1/2, 1)$.[8]

In responding to one's environment, there are two generic approaches. Depending on the context, they could correspond to a political ideology, a corporate ethic, a business strategy, or yet some other concept. In this simple setup, an approach or a strategy corresponds

[5] Assuming a continuum of agents simplifies the analysis by making the process deterministic. Richard T. Boylan (1992) shows that a deterministic system can be a valid approximation for a stochastic system when there are many agents.

[6] It would be interesting to allow for correlated matching as it might simulate self-selection among agent types. This would seem applicable to electoral systems where the decision to run for higher office depends on one's prospective opponent.

[7] Joseph A. Schlesinger (1966, 1991) documents the progressive paths taken to higher office. The tournament-style structure of organizations is examined in Sherwin Rosen (1986), while Raaj K. Sah and Joseph E. Stiglitz (1991) also explore the determinants of upper-level management.

[8] The case of $b = 1/2$ turns out to be knife-edge. When $b \neq 1/2$, there is a finite number of rest points. When $b = 1/2$, there is a continuum of rest points and at most one of them is locally stable. Details are in Harrington (1994).

to a particular action to play in all environments. By definition, action 0 (1) is the action associated with strategy 0 (1).[9] In a manner to be described momentarily, action 0 (1) is the best action for a type 0 (1) environment. Since $b > 1/2$, action 1 is more frequently the appropriate response to the environment.

Selection is determined as follows. If the two matched agents choose distinct actions, then the agent whose action matches the environment survives and is promoted to the next level. If both agents select the action which matches the environment, the agent who has chosen that particular action more frequently in the preceding h rounds advances with probability $p \in [1/2, 1]$. If they have chosen that action equally frequently, then an agent is randomly selected to survive. There is no need to specify whom is promoted if both agents choose the less appropriate action as the set of equations which describe the population dynamics is independent of it.

The idea is that survival depends on current performance which itself is determined by one's action and one's proficiency with that action where proficiency comes from experience. If $p > 1/2$, then experience yields an advantage which could be due to learning-by-doing or, as in the electoral context, credibility that comes from being relatively consistent in one's positions over time. If $p = 1/2$, then there is no experiential advantage. p is a measure of how much experience matters.[10] Note that survival depends lexicographically on one's current action and one's experience with that action. This means that the incremental effect from choosing a better action exceeds the incremental effect from more experience. h is a parameter which determines how much of an agent's history is relevant for proficiency. I initially consider the case of $h = \infty$ so that an agent is more effective in using a particular action if he has chosen that action more often over the entire history of play. I then examine the case when $h = 1$ so that only the most recent past matters. The value of h could be determined by the rate of depreciation of knowledge regarding the proper use of an action or, in the electoral context, by memories of voters. Having advancement depend only on current performance is like the old Hollywood adage, "You're only as good as your last picture," or voters' lament of "What have you done for me lately?" and obviously implies a certain myopia or forgetfulness. This might be a reasonable assumption for the electoral context but is admittedly uncompelling for the corporate setting. In Section III, past performance is allowed to play a limited role.

Each agent is endowed with a behavioral rule. The space of agent types is then associated with the space of feasible behavioral rules. Until Section III, attention is limited to behavioral rules that condition only on the current environment. It is then the set of functions which map the set of environments, {0, 1}, into the set of actions, {0, 1}, with the exception that the pathological case of always choosing an action inappropriate for the current environment is excluded. I believe this simplifies the analysis without any loss of generality. A flexible agent is defined to be one who always selects the action best suited for the environment: he chooses action 0 (1) when the environment is type 0 (1). A rigid agent chooses the same action irrespective of the environment. A type 0 rigid agent always uses action 0 and a type 1 rigid agent always uses action 1.

In concluding, there are a variety of sources of heterogeneity in rigidity. Faced with a complex meta-environment and computational constraints, people may simply differ in their opinion as to which behavioral rule is best. Alternatively, some agents may be endowed with a greater ability to modify their behavior. Perhaps more flexible agents can more finely discriminate among different environments. An agent who cannot tell the difference between two environments cannot condition his behavior on which environment occurs.[11] The

[9] Since there is a one-to-one mapping between strategy sets and action sets, there is no formal distinction between the two. Conceptually, there is a distinction in that a strategy is a rule which maps from the space of environments into the space of actions. It just so happens these strategies call for the same action for all environments.

[10] The results also extend to $p < 1/2$, but that range is inconsistent with the notion of experience contributing to proficiency.

[11] For the relation between an organism's representational system and his action set, see Derek Bickerton (1990).

following argument for constrained choice sets is also relevant (Heiner, 1983 p. 585):

> ... agents cannot decipher all of the complexity of the decision problems they face, which literally prevents them from selecting most preferred alternatives. Consequently, the flexibility of behavior to react to information is constrained to smaller behavioral repertoires that can be reliably administered.

It is then quite plausible that people with different powers of reasoning, recognition, and memory would have different behavioral repertoires.

II. The Unbounded Memory System

The initial population resides at the lowest level of the system and is comprised of three types: rigid agents endowed with action 1, rigid agents endowed with action 0, and flexible agents. The objective is to understand how the characteristics of this cohort of agents evolves as it migrates up through the hierarchy.[12] Though the system begins with only three distinct types, there are potentially many types after the population has a chance to interact with the environment because an agent's performance depends not only on his endowed behavioral rule but also his personal history. For present purposes, it is sufficient to partition the population as follows.

Definition: Let $z_s^t \in \{0, 1\}$ denote the action of agent s at level t. For an h-memory system, agent s is **proficient in action i** at level t if $z_s^\tau = i$ for all $\tau \in \{\max\{1, t-h\}, \dots, t-1\}$.

$r_i^t \equiv$ proportion of the level t population that are rigid agents endowed with action i (Ri); $i \in \{0, 1\}$.

$f_i^t \equiv$ proportion of the level t population that are flexible agents who are proficient in action i (Fi); $i \in \{0, 1\}$.

$x^t \equiv$ proportion of the level t population that are flexible agents who are not proficient in any action (FN).

The level t state of the system is defined by $(r_1^t, f_1^t, r_0^t, f_0^t, x^t)$. It is assumed that $r_1^1 > 0$, $r_0^1 \geq 0$, and $1 - r_1^1 - r_0^1 > 0$.

In this section, I consider the case of unbounded memory ($h = \infty$) so that an agent's entire history matters. For $t \geq 2$, the state at level $t + 1$ is described by the following system of equations:[13]

$$(1)\quad r_1^{t+1} = 2r_1^t[(1/2)r_1^t + (b/2)f_1^t + br_0^t + bp(1 - r_1^t - f_1^t - r_0^t)],$$

$$(2)\quad f_1^{t+1} = 2f_1^t[(b/2)r_1^t + (b/2)f_1^t + br_0^t + bp(1 - r_1^t - f_1^t - r_0^t)],$$

$$(3)\quad r_0^{t+1} = 2r_0^t[(1/2)r_0^t + ((1-b)/2)f_0^t + (1-b)r_1^t + (1-b)p(1 - r_1^t - r_0^t - f_0^t)],$$

$$(4)\quad f_0^{t+1} = 2f_0^t[((1-b)/2)r_0^t + ((1-b)/2)f_0^t + (1-b)r_1^t + (1-b)p(1 - r_1^t - r_0^t - f_0^t)],$$

where I have substituted $1 - r_1^t - f_1^t - r_0^t - f_0^t$ for x^t. In each of these equations, the bracketed term is the probability that an agent of that type survives. Multiplying this by the level t proportion of that type and doubling it (because an agent could be the first or second draw in a matching) gives a type's proportion at the next level. Consider randomly matching two agents and suppose the first agent drawn is an $R1$. With probability r_1^t he is matched with the same type, in which case his probability of surviving is $1/2$. With probability f_1^t he is matched with an $F1$, in which case he survives with probability $1/2$ when the environment is type 1, which occurs with probability b, and does not survive when the environment is type 0 as an $F1$ selects action 0. With probability r_0^t, he is matched with an $R0$ and survives when the environment is type 1. Finally, if he is matched with either an $F0$ or an FN,

[12] If the hierarchy is to be kept "full" then, at the end of each round, a fresh cohort of agents must enter the lowest level to replace those who moved on. This is not relevant to the current analysis in that I am interested in what happens to a single cohort. In Harrington (1996), the flow of new agents is modelled where a new agent's behavioral rule is assumed to be partly endowed and partly the result of imitation of those at higher levels.

[13] The system of equations which determines $(r_1^2, f_1^2, r_0^2, f_0^2, x^2)$ is different from that in (1)–(4) because agents do not have a history at level 1.

which occurs with probability $(1 - r_1^t - f_1^t - r_0^t)$, and the environment is type 1, he survives with probability p because of his greater experience with action 1. In a similar fashion, the other three equations can be explained.

Theorem 1 establishes that a population with all $R1$s is globally stable when the experiential advantage is sufficiently great and/or the type 1 environment is sufficiently common. All proofs are in the Appendix.

THEOREM 1: *With unbounded memory* $(h = \infty)$, *if* $pb > 1/2$ *then* $\lim_{t\to\infty} r_1^t = 1$.

Regardless of their initial presence in the population, systems with sufficiently many levels find their highest levels dominated by rigid agents who use the action that is more frequently the appropriate response to the environment. Rigid agents endowed with the less effective action and flexible agents are eventually eliminated.[14]

Theorem 1 is best understood by examining the system's dynamics. It is straightforward to show that the presence of agents who are proficient in action 0 is always diminishing: $\Delta r_0^t < 0$ and $\Delta f_0^t < 0$, where $\Delta r_i^t \equiv r_i^{t+1} - r_i^t$ and $\Delta f_i^t \equiv f_i^{t+1} - f_i^t$. Turning to those agents who are proficient in action 1, one can use (1)–(2) to derive:

$$(5)\quad \text{If } r_1^t > 0 \text{ then: } \Delta r_1^t \gtreqless 0 \text{ as } f_1^t \lesseqgtr [(2pb - 1)(1 - r_1^t) + 2b(1 - p)r_0^t]/b(2p - 1).$$

$$(6)\quad \text{If } f_1^t > 0 \text{ then: } \Delta f_1^t \gtreqless 0 \text{ as } r_1^t \lesseqgtr [((2pb - 1) + 2b(1 - p)r_0^t)/b(2p - 1)] - f_1^t.$$

Though $F1$s are eventually driven to extinction, their presence is increasing when the proportion of $R1$s is sufficiently small. Though $R1$s are eventually driven to domination, they are decreasing when the proportion of $F1$s is sufficiently great. Note that when $r_0^t = 0$ (or $p = 1$), (5)–(6) depend only on (r_1^t, f_1^t) and Figure 1 depicts the associated phase diagram. The use of Figure 1 to explore the dynamics is reasonable since $R0$s are monotonically decreasing and simulations reveal that they rapidly converge to zero.

Suppose the system currently has many $R1$s and $F1$s so that it is initially in region I. For example, when $p \cong 1$ and $r_1^1 \cong r_0^1$, it can be shown that the level 2 state of the system is in region I. Let me describe the resulting dynamical path, an example of which is depicted in Figure 2. The large presence of $F1$s means that $R1$s are frequently meeting agents who are equally experienced in action 1 but are superior by means of their flexibility. As a result, the proportion of $R1$s is shrinking since they survive at a rate of only $b/2$ $(<1/2)$ in their encounters with $F1$s. Due to the large presence of $R1$s in the population, the relative advantage of an $F1$ is not his proficiency in action 1 (since that is matched by $R1$s) but rather his flexibility. Hence, $F1$s tend to do best when the environment is type 0 as then, by choosing action 0, they capitalize on $R1$s being locked into action 1. Though $F1$s are surviving at a high rate, they are losing proficiency in action 1. This results in the proportion of $F1$s shrinking and the proportion of FNs growing (see Figure 2). Since the presence of $R1$s is diminishing in region I and the presence of $R0$s is always diminishing, the presence of flexible agents is growing in region I. When the proportion of $F1$s becomes sufficiently small, the system moves into region II. At that point the population is favorable to $R1$s in that they are frequently meeting agents who are not proficient in action 1, in which case their survival rate is at least pb $(>1/2)$. From then on the proportion of $R1$s grows. For this path, the population dynamics are nonmonotonic in that the presence of flexible agents is higher at intermediate levels than at low and high levels.

Alternatively, consider when the system begins in region III. Due to their small presence, $R1$s and $F1$s are largely meeting agents who are not proficient in action 1. Hence, $R1$s and $F1$s are thriving. Note, in particular, that the

[14] It is shown in Harrington (1994) that (r_1, f_1, r_0, f_0, x) is a rest point if and only if $(r_1, f_1, r_0, f_0, x) \in \{(1, 0, 0, 0, 0), (0, 0, 1, 0, 0), (0, 0, 0, 0, 1), (0, (2b - 1)/b, 0, 0, (1 - b)/b)\}$.

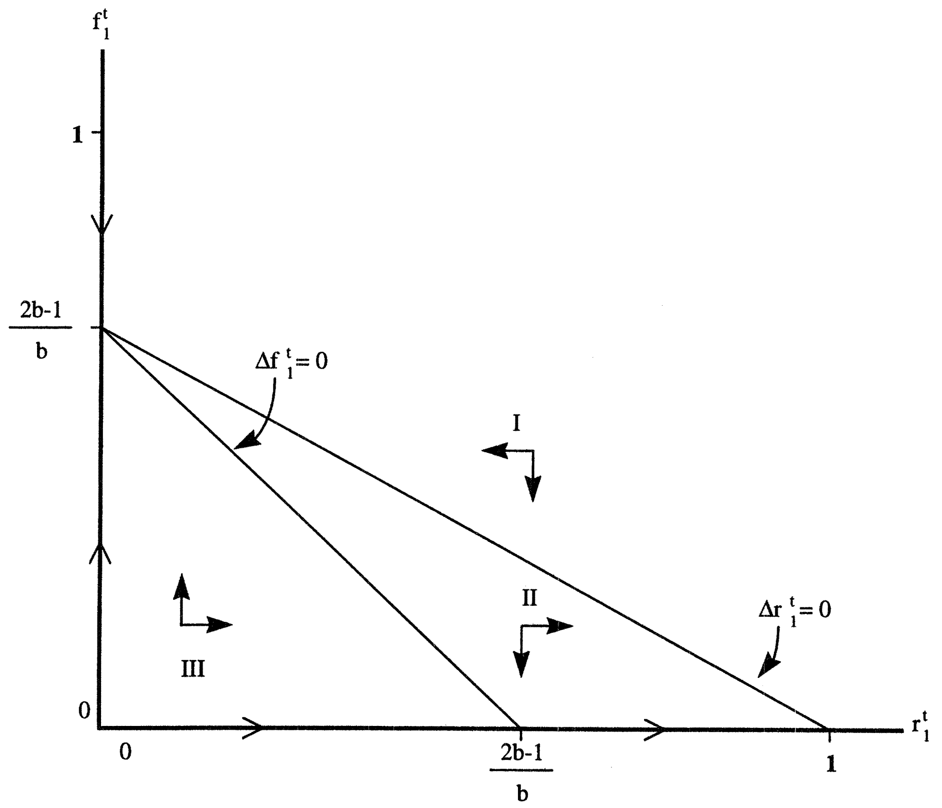

FIGURE 1. UNBOUNDED MEMORY SYSTEM: PHASE DIAGRAM (ASSUMPTION: $r_0^t = 0$ or $p = 1$)

proportion of $F1$s is growing because their differential advantage against an $R0$, $F0$, or FN is being proficient in action 1 rather than being flexible. Thus, their survival rate is highest when the environment is type 1, in which case they choose action 1, survive with probability at least p, and maintain their proficiency in action 1. With $F1$s and $R1$s growing, eventually they start frequently meeting each other. Given their frequent encounters with $R1$s, the differential advantage of an $F1$ shifts to being flexible, in which case their presence shrinks as they convert to FNs. This is represented by the system moving into region II. At that point, the presence of $R1$s is sufficiently great that they continue to grow and that growth accelerates as the presence of $F1$s shrinks. An interesting property of this path is that the proportion of $F1$s is growing at low levels even though they eventually become extinct.

While $R1$s can initially founder, they start thriving once the proportion of $F1$s is sufficiently small. What is surprising is how long this can take. Since a flexible agent must have faced $T - 1$ consecutive type 1 environments to be proficient in action 1 at level T, one would expect there to be very few proficient flexible agents when T is not small and b is not close to one since the likelihood of $T - 1$ consecutive type 1 environments is b^{T-1}. It would then seem that $R1$s would quickly overwhelm $F1$s. What this ignores, however, is that those flexible agents who are lucky enough to be proficient in action 1 have a much higher chance of surviving by virtue of their proficiency. For example, suppose (b, p, r_1^1, r_0^1) = (0.6, 1, 0.25, 0.25), as in Figure 2. Of the proficient agents, those who are flexible are on the order of 1 in every 2 agents at level 10, 1 in every 3 at level 15, 1 in every 4

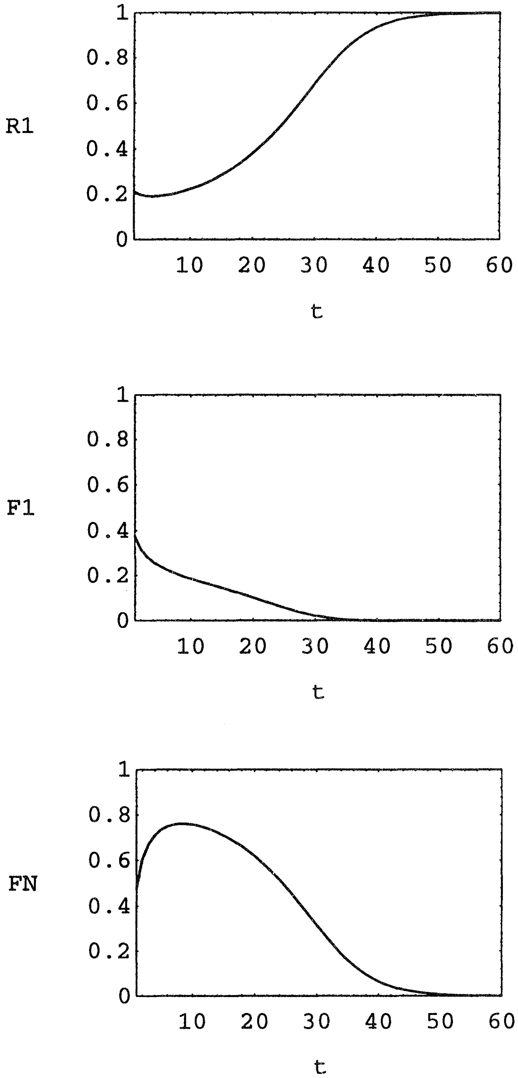

FIGURE 2. UNBOUNDED MEMORY SYSTEM: EXAMPLE ($b = 0.6, p = 1, r_1^1 = 0.25, r_0^1 = 0.25$)

at level 20, and 1 in every 8 at level 25. By comparison, when selection is inoperative so that all agents survive, of all proficient agents those who are flexible are around 1 in every 100 at level 10, 1 in every 1,000 at level 15, 1 in every 15,000 at level 20, and 1 in every 200,000 at level 25. Thus, selection allows proficient flexible agents to survive for many more rounds. By maintaining proficiency, flexible agents may be able to limit the dominance of rigid agents even in systems with relatively many levels.

Now suppose experience with an action is sufficiently unproductive in improving survival. Theorem 2 establishes that flexible agents will dominate.

THEOREM 2: *With unbounded memory* ($h = \infty$), *if* $pb \leq 1/2$ *then* $\lim_{t\to\infty} (r_1^t + r_0^t) = 0$.

Since, when the system is run long enough, most flexible agents are not proficient in any action, the long-run dominance of $R1$s is contingent upon them doing well against such agents. Given that when an $R1$ meets such an agent his probability of survival is pb, then long-run survival requires $pb > 1/2$. Otherwise, as shown in Theorem 2, flexible agents will dominate.

III. Extensions of the Unbounded Memory System

The objective of this research is not to determine whether, in general, rigid behavior is prevalent but rather to understand how the properties of a social system relate to the type of behavior that thrives within that system. Contrary to a common view that flexibility is a superior trait, the analysis of the previous section identified one class of social systems in which rigid agents do quite well. I now want to consider related systems so as to both assess the robustness of this result and to understand what properties promote rigid behavior. To simplify proofs, I assume $p = 1$ for the remainder of the analysis.

A. *More Sophisticated Flexible Agents*

Whenever one engages in an analysis that assumes agents are endowed with behavioral rules, it is important to ask whether there is a behavioral rule not in the feasible set that would have thrived. Along these lines, I go beyond strategies that condition only on the current environment and consider making flexible agents a bit more sophisticated. The preceding analysis suggests that a flexible agent, when faced with a rigid agent, might want to mimic a rigid agent. This is embodied in the following behavioral rule.

(7) At level 1, choose the action that matches the environment. At level t (≥ 2), choose the action that matches the environment except when:

(i) one has always chosen action i;
(ii) the current environment is type j ($\neq i$); and
(iii) one is matched with an agent who is type Ri; $i \in \{0, 1\}$.

Compared to the rule of always choosing the best action for the current environment, this rule enhances proficiency while maintaining a chance of surviving today though the probability of that event is lower.

THEOREM 3: *With unbounded memory* ($h = \infty$), *if flexible agents use the rule in* (7) *then* $\lim_{t\to\infty} r_1^t = 1$.

While this more sophisticated rule may delay the time at which an flexible agent loses proficiency, such remains inevitable. Interestingly, since $F1$s no longer take advantage of their flexibility when facing $R1$s, the proportion of $R1$s is now monotonically increasing for all initial conditions (see the proof of Theorem 3 in the Appendix).[15]

A related possibility to (7) is for a proficient flexible agent to act "rigid" when facing any identically proficient agent. If all flexible agents use such a rule then one maintains a chance of surviving today and lengthens the time for which one is maximally proficient.[16]

(8) At level 1, choose the action that matches the environment. At level t (≥ 2), choose the action that matches the environment except when:

(i) one has always chosen action i;
(ii) the current environment is type j ($\neq i$); and
(iii) one is matched with an agent who has always chosen action i; $i \in \{0, 1\}$.

THEOREM 4: *With unbounded memory* ($h = \infty$), *if flexible agents use the rule in* (8) *then* $\lim_{t\to\infty} (r_1^t + f_1^t) = 1$.

While flexible agents can now survive in the long run, this is only achieved by perfectly mimicking rigid agents! Since, in the limit, $F1$s face only other $F1$s and $R1$s, it follows from (8) that they choose action 1 regardless of the environment. Furthermore, if they ever consider deviating from this rule and choosing action 0 when the environment is type 0, it is known by the argument in the preceding section that they will eventually be driven out.

B. *Survival Depends on Past Performance*

Thus far, survival depends only on current performance which itself depends on an agent's action and his experience with that action. In many settings, past performance would seem quite relevant. For example, promotion in a corporation is apt to depend on one's performance throughout one's tenure, though perhaps with disproportionate weight placed on recent results.[17] To encompass past performance, consider the following selection rule.

(9) The agent who chose the best action for the current environment survives.
If both agents chose the best action then the one who more frequently in the past chose the best action for the environment at that time survives.
If both agents chose the best action and equally frequently in the past chose the best action for the environment at that

[15] An agent who only knows the history of environments and actions of the other agents may not be able to implement the rule in (7). For example, if another agent has always chosen action 1 and always faced a type 1 environment then he could be either rigid or flexible. The rule in (7) is then a best-case scenario for flexible agents in that it presumes more information than they realistically would have.

[16] This rule can be implemented with information only on another agent's past actions.

[17] A corporation might choose to focus on current performance under the presumption (perhaps false) that one's past performance must have been reasonably good for the agent to have advanced to the current level.

time, then the one who chose the current action more frequently in the past survives.

Otherwise, let me retain the specification in Section II. Survival depends lexicographically on the current action, past performance, and experience. Given that flexible agents always choose the best action and always have maximal past performance, encompassing past performance results in flexible agents thriving.

THEOREM 5: *With unbounded memory* ($h = \infty$) *and the selection rule in* (9), $\lim_{t\to\infty}(r_1^t + r_0^t) \in ((2b-1)r_1^1/b(1-r_0^1),\ r_1^1 + r_0^1)$.

Since $\lim_{t\to\infty}(r_1^t + r_0^t) < r_1^1 + r_0^1$, then this social system favors flexible agents. However, rigid agents are not driven out. For example, when $b = 0.75$ and the initial population is equally divided between $R1$s, $R0$s, and flexible agents, rigid agents make up around 36 percent of the highest levels in systems with sufficiently many levels.

To explain how rigid agents survive, let $R1^*$ denote a rigid agent endowed with action 1 who has always faced a type 1 environment. The notable property of an $R1^*$ is that he has always chosen the action which is best for the environment, just like a flexible agent. By the selection criterion in (9), survival of rigid agents rests upon $R1^*$s doing well. This may not seem very promising because it would appear unlikely for any agent to continually face the same environment. Indeed, for the selection criterion in Section II, one can show that the proportion of agents who are $R1^*$s goes to zero as $t \to \infty$. However, that selection criterion provided no pressure for $R1$s to accumulate such a history. In contrast, given the selection criterion in (9), $R1$s who just happen to face a type 1 environment in, say, the first two rounds, have a higher chance of surviving then $R1$s who have not. In particular, when faced with any type other than an $F1$, the survival rate of an $R1^*$ (as an $R1^*$) is b (which exceeds $1/2$) in that he survives for sure when the environment is type 1. For example, against a flexible agent who has not always chosen the same action, an $R1^*$ survives when the environment is type 1, as he has the advantage of having always chosen the same action [the third part in (9)] and, like any flexible agent, has always chosen the action right for the environment including the current one. Against an $F1$, however, an $R1^*$'s survival rate is only $b/2$ ($<1/2$) since, when the environment is type 1, the two agent types have identical histories and choose identical actions. Given that long-run survival requires a survival rate of at least $1/2$, the long-run survival of $R1^*$s (and thus rigid agents) is then contingent upon there not being too many $F1$s. What assures that there are not too many $F1$s is that the dynamic affecting $F1$s is exactly the same as that for $R1^*$ since the two agent types are identical (and, as a result, the ratio of $R1^*$s to $F1$s is constant; see the proof of Theorem 5 in the Appendix). Therefore, $F1$s and $R1^*$s shrink together and eventually stabilize at levels above zero. In particular, one can show that the proportion of agents who have always faced a type 1 environment—$R1^*$s and $F1$s—converges to $(2b-1)/b$ as $t \to \infty$ with the split between them depending on the level 1 population.[18] In short, while it is relatively unlikely for any specific agent to exclusively face a type 1 environment, some agents will have done so and selection results in them being disproportionately represented in the next generation. In this manner, the unlikely—a rigid agent having always chosen the action best for the environment—is made likely by selection pressures and results in rigid agents surviving.

I find that allowing selection to depend on past performance is conducive to flexible agents dominating though rigid agents still manage to survive. Finally, let me note that if the third part of (9) were replaced with a random selection of agents, there would be no virtue to being rigid. In that case, rigid agents would be driven out and only flexible agents would be present.[19]

[18] Since, the proportion of $R1^*$s is positive as $t \to \infty$, this follows immediately from (A10).

[19] If the third part in (9) was replaced with the flip of a fair coin then $\Delta\hat{r}_1^t = \hat{r}_1^t[(2b-1) - (\hat{r}_1^t + f_1^t)]$, where $\hat{r}_1^t$ denotes the proportion of $R1^*$ agents. Since $\hat{r}_1^t > 0$ implies $\Delta\hat{r}_1^t < 0$, it follows that $\lim_{t\to\infty} \hat{r}_1^t = 0$.

IV. The Bounded Memory System

An implication of assuming the system has unbounded memory is that a flexible agent who becomes less experienced than a rigid agent is always less experienced. Since such an assumption is clearly extreme and could be crucial for the domination of rigid agents, I now consider a bounded memory system while otherwise maintaining the structure of Section II (except for assuming $p = 1$). As I am particularly interested in assessing the robustness of my earlier results, the extreme case of a one-period memory is assumed. Proficiency is then determined solely by an agent's behavior in the preceding round and, in principle, is relatively easy to achieve.

With a one-period memory, all agents are proficient so that there are four types: $\{R1, F1, R0, F0\}$. The dynamical system for $t \geq 2$ is:

$$(10)\quad r_1^{t+1} = 2r_1^t[(1/2)r_1^t + (b/2)f_1^t + br_0^t + bf_0^t],$$

$$(11)\quad f_1^{t+1} = 2f_1^t[(b/2)r_1^t + (b/2)f_1^t + br_0^t + bf_0^t] + 2f_0^t[br_0^t + (b/2)f_0^t],$$

$$(12)\quad r_0^{t+1} = 2r_0^t[(1-b)r_1^t + (1-b)f_1^t + (1/2)r_0^t + ((1-b)/2)f_0^t],$$

$$(13)\quad f_0^{t+1} = 2f_0^t[(1-b)r_1^t + (1-b)f_1^t + ((1-b)/2)r_0^t + ((1-b)/2)f_0^t] + 2f_1^t[(1-b)r_1^t + ((1-b)/2)f_1^t].$$

In (11), $2f_0^t[(b/2)f_0^t + br_0^t]$ represents those agents who switch from being an $F0$ to being an $F1$, while in (13) $2f_1^t[(1-b)r_1^t + ((1-b)/2)f_1^t]$ captures the flow of $F0$s to $F1$s. In contrast, with unbounded memory, the only flow was from $F1$s and $F0$s to FNs.

Though the bounded memory system does not have a global attractor, it does have asymptotic attractors. Let $d((r_1', f_1', r_0', f_0'), (r_1'', f_1'', r_0'', f_0''))$ be the Euclidean distance between states (r_1', f_1', r_0', f_0') and $(r_1'', f_1'', r_0'', f_0'')$.

Definition: (r_1, f_1, r_0, f_0) is an **asymptotic attractor** if there exists $\varepsilon > 0$ such that if $d((r_1^1, f_1^1, r_0^1, f_0^1), (r_1, f_1, r_0, f_0)) < \varepsilon$ then $\lim_{t\to\infty}(r_1^t, f_1^t, r_0^t, f_0^t) = (r_1, f_1, r_0, f_0)$.

A state is an asymptotic attractor if the system converges to it when it is in a sufficiently small neighborhood of it. This is then a kind of local stability.

It is shown below that the bounded memory system has two asymptotic attractors. Given the ease with which proficiency is achieved, one has all flexible agents. More interestingly is that the other has all rigid agents (endowed with action 1).[20]

THEOREM 6: *With bounded memory* ($h = 1$), (r_1, f_1, r_0, f_0) *is an asymptotic attractor if and only if* $(r_1, f_1, r_0, f_0) \in \{(1, 0, 0, 0), (0, b, 0, 1-b)\}$.

To explore the system's dynamics, I construct a phase diagram under the assumption that $r_0^t = 0$. As with unbounded memory, the proportion of $R0$s monotonically converges to

[20] A qualification to Theorem 6 is in order. By definition, a rest point is not an asymptotic attractor if within any neighborhood around the rest point there are states such that the system does not converge to it. This leaves open the possibility that there could be other states within the neighborhood for which the system does converge. While this is not true for the rest point with all $R0$s, I have been unable to prove that there do not exist convergent paths for the rest point with a mix of flexible and rigid agents though, after an extensive numerical search, no convergent paths were found. Note, however, that if the limit is taken so as to make this a continuous time system, this rest point would be a saddlepoint which would imply that almost all paths would lead away from it.

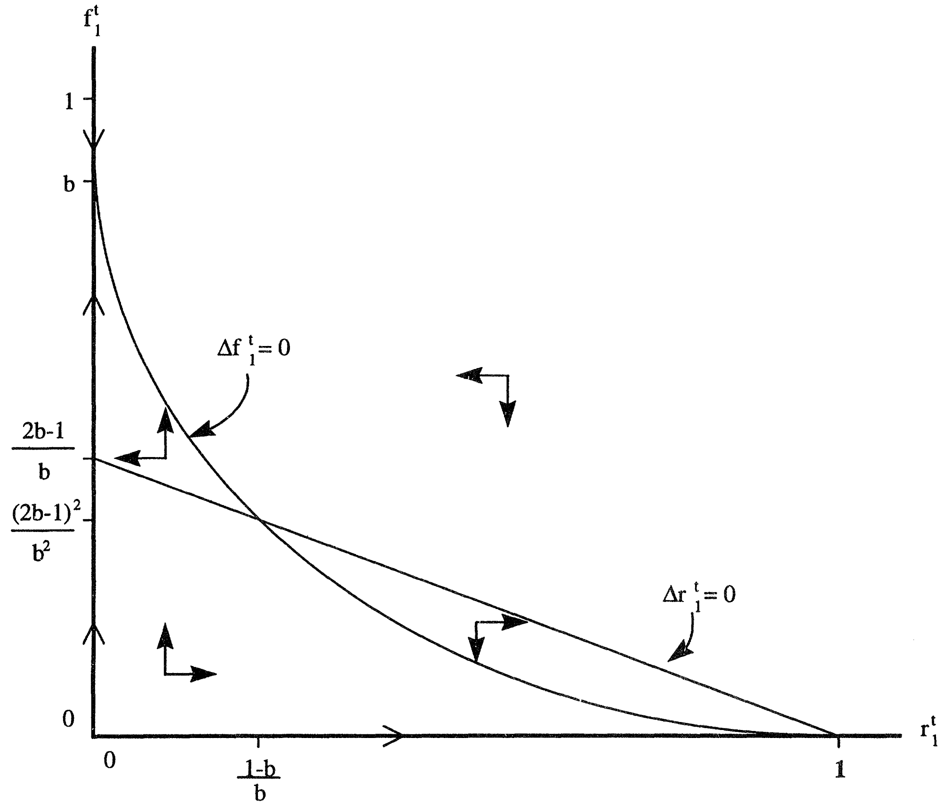

FIGURE 3. BOUNDED MEMORY SYSTEM: PHASE DIAGRAM (ASSUMPTION: $r_0^t = 0$)

zero and simulations show that the rate of convergence is rapid. Given $r_0^t = 0$ and substituting $1 - r_1^t - f_1^t$ for f_0^t in (10)–(11), we derive:

$$\text{(14)} \quad \text{If } r_1^t > 0 \text{ then: } \Delta r_1^t \gtreqless 0 \text{ as } f_1^t \lesseqgtr [(2b-1)/b](1 - r_1^t) \equiv \Gamma(r_1^t),$$

$$\text{(15)} \quad \text{If } f_1^t > 0 \text{ then: } \Delta f_1^t \gtreqless 0 \text{ as } f_1^t \lesseqgtr [b(1 - r_1^t)^2/(1 - br_1^t)] \equiv \Omega(r_1^t).$$

$\Gamma(\cdot)$ is the same as when the system has unbounded memory (assuming $p = 1$) while $\Omega(\cdot)$ is distinct.

$$\text{(16)} \quad \Omega(0) = b > (2b-1)/b = \Gamma(0);$$

$$\Omega(1) = 1 = \Gamma(1).$$

$$\Omega'(r_1^t) = -[b(1 - r_1^t)/(1 - br_1^t)^2] \times [(1-b) + (1 - b(r_1^t)^2)] < 0 \;\; \forall r_1^t \in [0, 1); \; \Omega'(1) = 0.$$

$$\Gamma'(r_1^t) = -[(2b-1)/b] < 0 \quad \forall r_1^t \in [0, 1].$$

Using the properties in (16), the phase diagram is constructed in Figure 3. The two asymptotic attractors are represented by $(r_1, f_1) = (1, 0)$ and $(r_1, f_1) = (b, 0)$. $(r_1, f_1) = ((1-b)/b, (2b-1)^2/b^2)$ represents a rest point that is not asymptotically stable.

First note that when the proportion of $R1$s is sufficiently great, the population dynamics are qualitatively similar to when memory is unbounded (see Figure 1). This is interesting in light of the qualitative distinction between unbounded and bounded memory systems. When memory is unbounded, proficiency in the better action requires a flexible agent to have always faced a type 1 environment (and played action 1). Flexible agents are then almost surely less experienced than rigid agents when the system has sufficiently many levels. In the bounded memory system, a flexible agent who chooses action 1 becomes proficient in action 1 *regardless* of his preceding history. In spite of this difference, bounding the memory does not qualitatively affect the population dynamics as long as the initial presence of $R1$s is sufficiently great.

To explain this finding, recall that flexible agents can continuously gain and lose proficiency in action 1 when memory is bounded. An $F1$ faced with a type 0 environment at level t will become an $F0$ (due to having chosen action 0) but switches back to being an $F1$ at level $t + 1$ if the environment at that time is type 1. The difficulty, however, lies in that an agent is unlikely to survive long enough to regain proficiency in action 1. With a strong presence of $R1$s, a flexible agent is very likely to meet an $R1$ who will take advantage of this flexible agent's "temporary" lack of proficiency in action 1. Rigid agents can then dominate even when it only takes one period for a flexible agent to achieve comparable proficiency to that of a rigid agent.

When instead the initial proportion of rigid agents is small, Figure 3 shows that the population dynamics are very different from when the system's memory is unbounded. Even though $R1$s can grow (when there are sufficiently few $F1$s), eventually flexible agents dominate as the system converges to $(r_1, f_1, r_0, f_0) = (0, b, 0, 1 - b)$. To understand the long-run domination of flexible agents, one must understand how $R1$s are driven out. As described above, the key to $R1$s surviving is that flexible agents who lose their proficiency in action 1 have a difficult time regaining it. However, when there are sufficiently few $R1$s in the population, a flexible agent who loses proficiency in action 1 has a reasonable chance

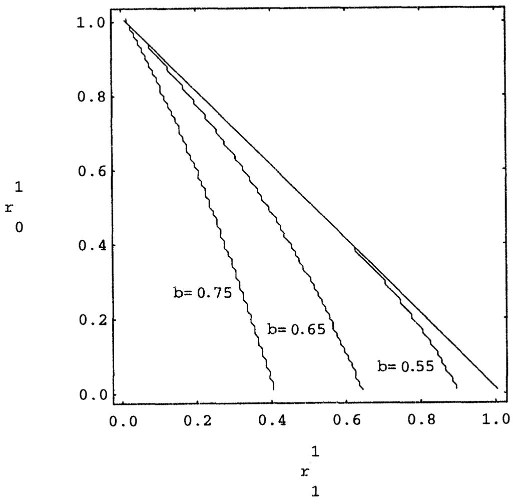

FIGURE 4. BOUNDED MEMORY SYSTEM: BASINS OF ATTRACTION

of avoiding an $R1$. This gives a flexible agent the opportunity to successfully regain proficiency in action 1. In particular, this is achieved by an $F0$ meeting his own kind. With this steady replenishment, the proportion of $F1$s is consistently high when there are initially few $R1$s. By facing a perpetually inhospitable population, $R1$s are driven out so that flexible agents flourish.

To characterize the basins of attraction of the two asymptotic attractors, numerical methods were used. In contrast to Figure 3, I allow $r_0^1 \neq 0$. The basins are defined over the space of initial populations where an initial population is represented by (r_1^1, r_0^1). Simulations showed that the basins partitioned the initial state space. These results are shown in Figure 4 where all points between a jagged line (where the associated value for b is noted) and the diagonal comprise the basin for the asymptotic attractor with all rigid agents, and all points to the left of a jagged line comprise the basin for the asymptotic attractor with all flexible agents. Note that if the initial population is biased towards flexible agents then this bias is magnified so that eventually nearly every surviving agent is flexible. When instead the initial population is biased towards rigid agents then the system converges to the asymptotic attractor with all rigid agents. As one increases the frequency with which action 1 is the appropriate response to the environment, the basin for the rest point with all rigid agents expands. If action 1 is made more effective then it is more likely that agents who exclusively use that action will dominate. This result can also be interpreted as follows. As b gets closer to 1/2, the environment becomes more volatile in which case being flexible is relatively more advantageous. Hence, flexible agents are more likely to dominate in that case.

V. Concluding Remarks

Environments routinely change and with them changes what is required to be well adapted. The extent of behavioral response to an altered set of surroundings is a crucial form of variation among organisms in the animal kingdom and would seem central to the determination of which organisms thrive and which perish. In spite of the importance of behavioral plasticity in biological systems, its implications for social systems have generally gone unexplored.

In investigating this issue, I assumed flexible agents—by choosing the action best suited for the current environment—maximize the probability of surviving today, while rigid agents—by always choosing the same action—become relatively proficient with a particular action. A key assumption is that an agent's current performance (where current survival depends on relative performance) is determined primarily by his current action and secondarily by his proficiency with that action. A central finding is that rigid agents can thrive even when experience and the proficiency it generates counts much less in determining current survival than doing what is right for the moment. It was also found, unsurprisingly, that flexible agents do better when proficiency requires less experience. More interestingly, if there are initially enough rigid agents present, rigid agents will dominate even when proficiency is achieved with a trivial amount of experience. While having current performance be more heavily influenced by one's action than one's proficiency biases the model against rigid agents, a feature of the model that favors rigid agents is having an agent's current survival depend only on current and not past performance because, over all types of environments, flexible agents generate higher average performance in that they always choose the best action—and that is what primarily determines current performance. Relatedly, another feature of the model that would seem to favor rigid agents is the "up-or-out" structure in which agents who perform relatively better advance and the remainder drop out and no longer compete. If an agent who fails to be promoted is not immediately kicked out of the system, the higher average performance of flexible agents may result in them doing better. Indulging in some speculation, this suggests that rigid agents might do relatively better in electoral systems than in corporations in that the former are characterized by an up-or-out structure and arguably put more weight on current than past performance. It is this type of insight that I am striving for and which will help identify how the properties of a social system relate to the behavioral types that thrive in such systems.

In light of the simplicity of our model, it is natural to wonder about the robustness of our results. To begin, one needs to properly pose this issue. If the model were to be generalized to allow for many environments, many actions, and many behavioral types, I certainly would not expect purely rigid agents to thrive. They are an extreme type that perhaps only does well in the extreme model. The pertinent issue is instead whether *relatively* rigid agents do well. Robustness then pertains to whether the identified forces would still be operative in a more general setting and thereby contribute to the determination of which type prevails. I suspect that consistency of action would continue to contribute to rigid agents surviving, the process by which flexible agents lose proficiency would continue to result in nonmonotonic dynamics, and a quicker learning curve for achieving proficiency would continue to promote flexible agents.

While having more environments and actions might seem opportune for flexible agents, let me put forth some speculative arguments as to why this might not necessarily be so. Suppose there are n environments, n actions, and each action is best for some environment. In that they always use the same action, there are n types of fully rigid agents. With a fully flexible agent using a different action for each environment, there are $n!$ types of fully flexible agents. Let me further suppose that m actions are highly detrimental in any environment other than the one for which they are best while the other $n-m$ actions do reasonably well in all environments. If each possible rule is initially equally represented then a proportion $(n-m)/n$ of purely rigid agents avoids those actions that are highly detrimental in most environments while only a proportion $1/n(n-1)\cdots(n-m+1)$ of fully flexible agents do so. Even if the best rule is one that is fully flexible, one could imagine the subpopulation of purely rigid agents outperforming, at least for a time, the subpopulation of fully flexible agents. Another factor arises if I generalize a step further and suppose that an agent observes the environment with noise. A rigid agent who avoids actions that are detrimental in certain situations may perform better then a flexible agent who includes them in his repertoire. As Heiner (1983) argues, for a bigger behavioral repertoire to be advantageous, an agent must be more skillful at recognizing the environment for which an action is appropriate. This is a consideration that has thus far been ignored in that I have presumed that a flexible agent is able to perfectly deploy his strategy. It is then not so obvious that more environments and more actions are necessarily conducive to flexible agents thriving.

The objective of this particular piece of research was a modest one—to begin to accumulate insight into how a class of competitive selection processes selects among a population of agents who differ in terms of their rigidity. The ultimate goal of this line of research is to generate predictive statements about real-world systems. One class of statements would describe how rigidity varies across levels within a hierarchical social system. For example, should more "yes men" be expected to be found in middle or upper management? In contrast, statements across social systems concern how the overall rigidity of a social system varies with such characteristics as the number of levels and the volatility of the environment. For example, do "yes men" have a smaller presence in flatter organizations? While the model of this paper is far too simple to be a reasonable representation of such a complex system as an electoral system or a corporation, the hope is that later generations of models will be.

APPENDIX

PROOF OF THEOREM 1:

The proof is comprised of three steps. First, as $t \to \infty$ the proportion of $F1$s is shown to go to zero (Lemma A1). Lemma A1 is then used to show that the proportion of $R1$s is not bounded below 1 (Lemma A2). It follows from Lemmas A1 and A2 that T can be chosen so as to make f_1^t arbitrarily close to zero $\forall t \geq T$ and r_1^T arbitrarily close to one. From this result, it is argued that r_1^t remains arbitrarily close to one $\forall t > T$.

LEMMA A1: $\lim_{t\to\infty} f_1^t = 0$.

PROOF:

Using (1)–(2), it is straightforward to show that $b < 1$ implies that f_1^t/r_1^t is strictly decreasing in t. Since f_1^t/r_1^t is strictly decreasing and f_1^t/r_1^t is bounded below by 0, $\lim_{t\to\infty} f_1^t/r_1^t$ exists and is nonnegative. It follows that:

$$\text{(A1)} \qquad \lim_{t\to\infty}[(f_1^{t+1}/r_1^{t+1}) - (f_1^t/r_1^t)] = 0.$$

Using (1)–(2), it is straightforward to show that (A1) is equivalent to:

$$\text{(A2)} \qquad \lim_{t\to\infty} \frac{-f_1^t(1-b)}{r_1^t + bf_1^t + 2br_0^t + 2pb(1-r_1^t-f_1^t-r_0^t)} = 0.$$

Given the denominator is bounded below one and $b < 1$, it follows from (A2) that $\lim_{t\to\infty} f_1^t = 0$.

LEMMA A2: *If $pb > 1/2$ then there does not exist $\bar{r} < 1$ such that $r_1^t < \bar{r}$ for all t.*

PROOF:

Suppose not so that $\exists \bar{r} < 1$ such that $r_1^t < \bar{r}\,\forall t$. Using (1), it is derived that:

$$\text{(A3)}\quad \Delta r_1^t \equiv r_1^{t+1} - r_1^t = r_1^t[(2pb-1)(1-r_1^t) - b(2p-1)f_1^t + 2b(1-p)r_0^t].$$

By Lemma A1, $\lim_{t\to\infty} - b(2p-1)f_1^t = 0$. By supposition, $r_1^t < \bar{r}\,\forall t$ which implies, with $pb > 1/2$, that $(2pb-1)(1-r_1^t) > (2pb-1)(1-\bar{r}) > 0\ \forall t$. From these two properties, it follows that $\exists$ finite T such that $\Delta r_1^t > (2pb-1)(1-\bar{r})\ \forall t \geq T$. But if r_1^t is increasing at a rate bounded above zero for all $t \geq T$ then $\lim_{t\to\infty} r_1^t = +\infty$ which is not possible since, by construction, $r_1^t \in [0, 1]\ \forall t$. Given that this contradiction was derived under the supposition that Lemma A2 is false, it is concluded that Lemma A2 is true.

With the aid of Lemmas A1 and A2, I will now prove Theorem 1 by showing that: $\forall \varepsilon > 0\ \exists$ finite T such that $r_1^t \in (1-\varepsilon, 1)\ \forall t \geq T$. By Lemma A1, $\exists$ finite t' such that $f_1^t < [(2pb-1)/b(2p-1)](\varepsilon/2)\ \forall t \geq t'$, where $\varepsilon > 0$ (note that $pb > 1/2$ implies $[(2pb-1)/b(2p-1)] > 0$). It is an implication of Lemma A2 that $\forall$ finite t, $\exists$ finite $T \geq t$ such that $r_1^T > 1 - (\varepsilon/2)$. Setting $t = t'$, I conclude that $\exists$ finite T such that $f_1^t < [(2pb-1)/b(2p-1)](\varepsilon/2)\ \forall t \geq T$ and $r_1^T > 1-(\varepsilon/2)$. The next step is to derive sufficient conditions for $\Delta r_1^t > 0$. Since $r_1^t \in (0, 1-[b(2p-1)/(2pb-1)]f_1^t)$ implies $\Delta r_1^t > 0$ and given $f_1^t < [(2pb-1)/b(2p-1)](\varepsilon/2)\ \forall t \geq T$, I conclude that if $r_1^t \in (0, 1-(\varepsilon/2))$ then $\Delta r_1^t > 0\ \forall t \geq T$. Next, a lower bound on Δr_1^t is derived. From (A3) and $p \geq 1/2b > 1/2$, $\Delta r_1^t > -b(2p-1)r_1^t f_1^t > -bf_1^t$. If $t \geq T$ then $-bf_1^t > -b[(2pb-1)/b(2p-1)](\varepsilon/2) = -(2b-1)(\varepsilon/2) > -\varepsilon/2$. Therefore, $\Delta r_1^t > -\varepsilon/2\ \forall t \geq T$.

To summarize, I have shown that $\forall \varepsilon > 0\ \exists$ finite T such that: (i) $r_1^T > 1-(\varepsilon/2)$; (ii) if $r_1^t \in (0, 1-(\varepsilon/2))$ then $\Delta r_1^t > 0\ \forall t \geq T$; and (iii) $\Delta r_1^t > -\varepsilon/2\ \forall t \geq T$. By (i) and (iii), $r_1^{T+1} > 1-\varepsilon$. Now suppose that $t > T$ and $r_1^t > 1-\varepsilon$. If $r_1^t \in (1-\varepsilon, 1-(\varepsilon/2))$ then, by (ii), $\Delta r_1^t > 0$, which implies $r_1^{t+1} > r_1^t$ and therefore $r_1^{t+1} > 1-\varepsilon$. If $r_1^t > 1-(\varepsilon/2)$ then, by (iii), $r_1^{t+1} > r_1^t - (\varepsilon/2)$ and therefore $r_1^{t+1} > 1-\varepsilon$. I have then shown that $r_1^{T+1} > 1-\varepsilon$ and if $r_1^t > 1-\varepsilon$ then $r_1^{t+1} > 1-\varepsilon$, $t \geq T+1$. By induction, it is concluded that $r_1^t > 1-\varepsilon\ \forall t > T$. Since ε was arbitrary, this proves Theorem 1.

PROOF OF THEOREM 2:

Using (1) and (3), after a few steps one can derive:

$$\text{(A4)}\quad \Delta r_1^t + \Delta r_0^t = (1-r_1^t-r_0^t)[(2pb-1)r_1^t + (2p(1-b)-1)r_0^t].$$

Since $p \leq 1/2b$ then $\Delta r_1^t + \Delta r_0^t < 0$. Given that $r_1^t + r_0^t$ is monotonically decreasing and has a lower bound of zero, $\lim_{t\to\infty} \Delta r_1^t + \Delta r_0^t = 0$. Since $(1-r_1^t-r_0^t)$ is bounded above zero $\forall t$, it follows from (A4) that $\lim_{t\to\infty} \Delta r_1^t + \Delta r_0^t = 0$ iff $\lim_{t\to\infty} r_1^t = 0$ and $\lim_{t\to\infty} r_0^t = 0$.

PROOF OF THEOREM 3:

It is straightforward to show that:

$$\text{(A5)}\quad \Delta r_1^t = r_1^t(2b-1)(1-r_1^t-f_1^t),$$

$$\text{(A6)}\quad \Delta f_1^t = f_1^t[(2b-1)(1-r_1^t) - bf_1^t].$$

By the same method used to prove Lemma A1, one can establish that $\lim_{t\to\infty} f_1^t = 0$. By (A5), r_1^t is monotonically increasing. Since r_1^t is bounded above by one, it then has a limit which implies $\lim_{t\to\infty} \Delta r_1^t = 0$. Since $\lim_{t\to\infty} f_1^t = 0$ and r_1^t is increasing then $\lim_{t\to\infty} r_1^t = 1$.

PROOF OF THEOREM 4:

It is straightforward to show that:

$$\text{(A7)}\quad \Delta r_1^t = r_1^t(2b-1)(1-r_1^t-f_1^t),$$

$$\text{(A8)}\quad \Delta f_1^t = f_1^t(2b-1)(1-r_1^t-f_1^t).$$

Since $r_1^t + f_1^t$ are monotonically increasing and bounded above by one then $r_1^t + f_1^t$ has a limit so that $\lim_{t\to\infty}(\Delta r_1^t + \Delta f_1^t) = 0$. Since $\Delta r_1^t + \Delta f_1^t = (2b-1)(r_1^t + f_1^t)(1 - r_1^t - f_1^t)$ then $\lim_{t\to\infty}(\Delta r_1^t + \Delta f_1^t) = 0$ implies $\lim_{t\to\infty}(r_1^t + f_1^t) = 1$.

PROOF OF THEOREM 5:

The first step is to partition rigid agents into those who have always chosen the action best for the environment and those who have not.

$\hat{r}_i^t \equiv$ proportion of the level t population that are rigid agents endowed with action i and who have always faced a type i environment.

$\bar{r}_i^t \equiv$ proportion of the level t population that are rigid agents endowed with action i and who have previously faced a type j ($\neq i$) environment.

Note that $r_i^t = \bar{r}_i^t + \hat{r}_i^t$. It is straightforward to derive the equations of motion for $t \geq 2$:

$$\text{(A9)}\quad \bar{r}_1^{t+1} = \bar{r}_1^t[2b(\bar{r}_0^t + \hat{r}_0^t) + 2(1-b)\hat{r}_1^t + \bar{r}_1^t] + (1-b)(\hat{r}_1^t)^2,$$

$$\text{(A10)}\quad \hat{r}_1^{t+1} = \hat{r}_1^t[2b - b\hat{r}_1^t - bf_1^t],$$

$$\text{(A11)}\quad f_1^{t+1} = f_1^t[2b - b\hat{r}_1^t - bf_1^t],$$

$$\text{(A12)}\quad \bar{r}_0^{t+1} = \bar{r}_0^t[2(1-b)(\bar{r}_1^t + \hat{r}_1^t) + 2b\hat{r}_0^t + \bar{r}_0^t] + b(\hat{r}_0^t)^2,$$

$$\text{(A13)}\quad \hat{r}_0^{t+1} = \hat{r}_0^t[2(1-b) - (1-b)\hat{r}_0^t - (1-b)f_0^t],$$

$$\text{(A14)}\quad f_0^{t+1} = f_0^t[2(1-b) - (1-b)\hat{r}_0^t - (1-b)f_0^t].$$

LEMMA A3: $\lim_{t\to\infty} r_0^t = 0$.

PROOF:

Since $\Delta\hat{r}_0^t = \hat{r}_0^t[-(2b-1) - (1-b)\hat{r}_0^t - (1-b)f_0^t] < 0$, it is easy to show that $\lim_{t\to\infty} \hat{r}_0^t = 0$. Using this fact then a little manipulation shows: $\lim_{t\to\infty} \Delta\bar{r}_0^t = \lim_{t\to\infty} -\bar{r}_0^t[1 - \bar{r}_0^t - 2(1-b)(\bar{r}_1^t + \hat{r}_1^t)] \leq 0$. From this it is concluded that $\lim_{t\to\infty} \bar{r}_0^t = 0$ and thus $\lim_{t\to\infty} r_0^t = 0$.

LEMMA A4: $\lim_{t\to\infty} \hat{r}_1^t = [(2b-1)/b][r_1^1/(1-r_0^1)] \equiv \hat{\rho}$.

PROOF:

Since $\hat{r}_1^{t+1} = \hat{r}_1^2 \prod_{\tau=2}^{t}[2b - b\hat{r}_1^\tau - bf_1^\tau]$ and $f_1^{t+1} = f_1^2 \prod_{\tau=2}^{t}[2b - b\hat{r}_1^\tau - bf_1^\tau]$ then $\hat{r}_1^{t+1}/f_1^{t+1} = \hat{r}_1^2/f_1^2$. It is straightforward to derive: $\hat{r}_1^2 = br_1^1(1 + r_0^1)$ and $f_1^2 = b(1 - r_1^1 - r_0^1)(1 + r_0^1)$. It is then obtained that $\hat{r}_1^t/f_1^t = r_1^1/(1 - r_1^1 - r_0^1)$ $\forall t \geq 3$. Substituting $[(1 - r_1^1 - r_0^1)/r_1^1]\hat{r}_1^t$ for f_1^t in (A10), the following is derived:

$$\text{(A15)}\quad \hat{r}_1^{t+1} = \hat{r}_1^t[2b - b((1-r_0^1)/r_1^1)\hat{r}_1^t].$$

Using (A15), it is easy to show that $\Delta\hat{r}_1^t \gtreqless 0$ as $\hat{r}_1^t \lesseqgtr \hat{\rho}$. If $\hat{r}_1^t$ has a limit, it must then be $\hat{\rho}$. $\hat{r}_1^t$ converges to $\hat{\rho}$ if $|\hat{r}_1^{t+1} - \hat{\rho}| < |\hat{r}_1^t - \hat{\rho}|$. This will only be shown for $\hat{r}_1^t < \hat{\rho}$, as the proof is analogous for $\hat{r}_1^t > \hat{\rho}$. Given $\hat{r}_1^t < \hat{\rho}$ and therefore $\hat{r}_1^{t+1} > \hat{r}_1^t$, if $\hat{r}_1^{t+1} < \hat{\rho}$ then I am done. Suppose instead that $\hat{r}_1^{t+1} > \hat{\rho}$. Using (A15) and substituting $((2b-1)/\hat{\rho})$ for $b((1-r_0^1)/r_1^1)$, one derives that $|\hat{r}_1^{t+1} - \hat{\rho}| < |\hat{r}_1^t - \hat{\rho}|$ if and only if:

$$\text{(A16)}\quad \varphi(\hat{r}_1^t) \equiv 2\hat{\rho} - (2b+1)\hat{r}_1^t + (2b-1)(1/\hat{\rho})(\hat{r}_1^t)^2 > 0.$$

Since $\varphi'(\hat{\rho}) < 0$, and $\varphi''(\cdot) > 0$ $\forall \hat{r}_1^t$ then $\varphi(\hat{r}_1^t) > \varphi(\hat{\rho})$ $\forall \hat{r}_1^t < \hat{\rho}$. Given $\varphi(\hat{\rho}) = 0$, it can be concluded that (A16) is true. Hence, $\lim_{t\to\infty} \hat{r}_1^t = \hat{\rho}$.

LEMMA A5: $\lim_{t\to\infty} \bar{r}_1^t = (1/2)\{1 - 2(1-b)\hat{\rho} - [1 - 4(1-b)\hat{\rho} - 4b(1-b)\hat{\rho}^2]^{1/2}\} \equiv \bar{\rho}$.

PROOF:

Since $\lim_{t\to\infty} r_0^t = 0$ and $\lim_{t\to\infty} \hat{r}_1^t = \hat{\rho}$, it follows that:

$$\text{(A17)}\quad \lim_{t\to\infty} \left| (\partial \bar{r}_1^{t+1}/\partial \bar{r}_1^t) - [2(1-b)\hat{\rho} + 2\bar{r}_1^t] \right| = 0.$$

Thus, as $t \to \infty$, $(\partial \bar{r}_1^{t+1}/\partial \bar{r}_1^t) < 1$ iff $\bar{r}_1^t < (1/2) - (1-b)\hat{\rho}$. It follows that if $\bar{r}_1^t < (1/2) - (1-b)\hat{\rho}$ as $t \to \infty$ then $\bar{r}_1^t$ converges. Furthermore, it can be shown that, as $t \to \infty$, $\Delta \bar{r}_1^t \gtreqless 0$ as $\lim_{t\to\infty} \bar{r}_1^t \lesseqgtr \bar{\rho}$. Hence, if $\bar{r}_1^t$ converges it must converge to $\bar{\rho}$. Given that $\bar{\rho} < [(1/2) - (1-b)\hat{\rho}]$ then, as $t \to \infty$, $\Delta \bar{r}_1^t < 0$ when $\bar{r}_1^t > (1/2)[1 - 2(1-b)\hat{\rho}]$. Therefore, $\exists\, T$ such that $\forall t \geq T$ if $\bar{r}_1^t > (1/2)[1 - 2(1-b)\hat{\rho}]$ then $\Delta \bar{r}_1^t < 0$. This implies $\exists\, T' > T$ such that $\bar{r}_1^{T'} < [(1/2) - (1-b)\hat{\rho}]$. Since then $|(\partial \bar{r}_1^{t+1}/\partial \bar{r}_1^t)| < 1$ $\forall t \geq T'$, it follows that $\bar{r}_1^t$ converges to $\bar{\rho}$.

I now want to show that $r_1^1 + r_0^1 > \hat{\rho} + \bar{\rho}$. Substituting $(1 - r_0^1)[b/(2b-1)]\hat{\rho}$ for r_1^1 and using the definition of $\bar{\rho}$, $r_1^1 + r_0^1 > \hat{\rho} + \bar{\rho}$ is equivalent to:

$$\text{(A18)}\quad (1 - r_0^1)[b/(2b-1)]\hat{\rho} + r_0^1 > \hat{\rho} + (1/2)\{1 - 2(1-b)\hat{\rho} - [1 - 4(1-b)\hat{\rho} - 4b(1-b)\hat{\rho}^2]^{1/2}\}.$$

The left-hand side can be rearranged to: $[b/(2b-1)]\hat{\rho} + [1 - (b/(2b-1))\hat{\rho}]r_0^1$; and it is straightforward to show that $[1 - (b/(2b-1))\hat{\rho}]r_0^1 > 0$. Hence, (A18) holds if:

$$\text{(A19)}\quad [b/(2b-1)]\hat{\rho} > \hat{\rho} + (1/2)\{1 - 2(1-b)\hat{\rho} - [1 - 4(1-b)\hat{\rho} - 4b(1-b)\hat{\rho}^2]^{1/2}\}.$$

Working through a few algebraic steps shows that (A19) is equivalent to: $(2b-1)/b > \hat{\rho}$, which is indeed true.

PROOF OF THEOREM 6:

Since an asymptotic attractor must be a rest point, the set of rest points is first derived and then their asymptotic stability is evaluated.

PROPOSITION A1: *(r_1, f_1, r_0, f_0) is a rest point if and only if* $(r_1, f_1, r_0, f_0) \in \{(1,0,0,0), (0,0,1,0), (0,b,0,1-b), ((1-b)/b, (2b-1)^2/b^2, 0, (1-b)(2b-1)/b^2)\}$.

PROOF:

The proof is by construction. Substituting $1 - r_1^t - f_1^t - f_0^t$ for r_0^t in (10), (11), and (13), the dynamical system can be represented by:

$$\text{(A20)}\quad \Delta r_1^t = r_1^t[(2b-1)(1-r_1^t) - bf_1^t],$$

$$\text{(A21)}\quad \Delta f_1^t = f_1^t[(2b-1) - br_1^t - bf_1^t] + f_0^t[2b(1-r_1^t) - 2bf_1^t - bf_0^t],$$

$$\text{(A22)}\quad \Delta f_0^t = f_0^t[-b + (1-b)r_1^t + (1-b)f_1^t] + f_1^t[2(1-b)r_1^t + (1-b)f_1^t].$$

$(r_1, f_1, 1 - r_1 - f_1 - f_0, f_0)$ is a rest point if and only if (A20)–(A22) equal zero. If $r_0 = 1$ then $r_1 = f_1 = f_0 = 0$. Since (A20)–(A22) equal zero, $(0,0,1,0)$ is a rest point. Given $\Delta r_0^t = -r_0^t[(2b-1)(r_1^t + f_1^t) + bf_0^t]$ then $\Delta r_0^t < 0$ when $r_1^t < 1$. Hence, if (r_1, f_1, r_0, f_0) is a rest point and $r_0 < 1$ then $r_0 = 0$.

Since $r_0 = 0$ at all other rest points, a rest point is defined by (r_1, f_1, f_0) such that $\Delta r_1^t = 0$, $\Delta f_1^t = 0$, $\Delta f_0^t = 0$, and $r_1 + f_1 + f_0 = 1$. Substitute $1 - r_1 - f_1$ for f_0 in (A20)–(A21). Simplifying, $(r_1, f_1, 0, 1 - r_1 - f_1)$ is a rest point if and only if (r_1, f_1) satisfies:

$$\text{(A23)}\quad r_1[(2b-1)(1-r_1) - bf_1] = 0,$$

$$\text{(A24)}\quad b - f_1(1 - br_1) - br_1(2 - r_1) = 0.$$

If $r_1 = 0$ then (A23) is satisfied while (A24) is satisfied by $f_1 = b$. Hence, $(0, b, 0, 1 - b)$ is a rest point. Now consider $r_1 = 1$. Since then $f_1 = 0$, (A23)–(A24) are satisfied so that (1, 0, 0, 0) is a rest point. The final case to consider are rest points for which $r_1 \in (0, 1)$. Solving (A23)–(A24), one derives $f_1 = ((2b - 1)/b)^2$ and $r_1 = (1 - b)/b$. The unique rest point for which $r_1 \in (0, 1)$ is then $((1 - b)/b, (2b - 1)^2/b^2, 0, (1 - b)(2b - 1)/b^2)$.

To prove that (0, 0, 1, 0) is not an asymptotic attractor, (12) is used to derive $\Delta r_0^t = -r_0^t[(2b - 1)(r_1^t + f_1^t) + bf_0^t]$. Since $\Delta r_0^t < 0\ \forall r_0^t \in (0, 1)$, it follows that $\lim_{t\to\infty} r_0^t = 0 < 1$.

To prove that $((1 - b)/b, (2b - 1)^2/b^2, 0, (1 - b)(2b - 1)/b^2)$ is not an asymptotic attractor, it will be shown that in any neighborhood of this rest point, there exists a set of points such that the system does not converge to this rest point. Consider an initial state such that: $r_1^1 > (1 - b)/b$, $r_0^1 = 0$, and

$$\text{(A25)}\quad [(2b - 1)/b](1 - r_1^1) > f_1^1 > b(1 - r_1^1)^2/(1 - br_1^1).$$

Note that the expressions on either side of f_1^1 equal $(2b - 1)^2/b^2$ when $r_1^1 = (1 - b)/b$. Therefore, by choosing r_1^1 sufficiently close to $(1 - b)/b$ and having f_1^1 satisfy (A25), this state can be made arbitrarily close to the rest point. I want to show that $\Delta r_1^t > 0\ \forall t \geq 1$. Since r_1^1 exceeds its value at the rest point, it would follow that the system diverges.

From (A20), one knows that $\Delta r_1^1 > 0$ iff $r_1^1 < 1 - [b/(2b - 1)]f_1^1$. Rearranging the left-hand side of (A25), one finds that $r_1^1 < 1 - [b/(2b - 1)]f_1^1$. Hence, $\Delta r_1^1 > 0$. When $r_0^1 = 0$, one knows that $\Delta f_1^1 > 0$ iff $f_1^1 > b(1 - r_1^1)^2/(1 - br_1^1)$. Thus, by the right-hand-side inequality in (A25), it is concluded that $\Delta f_1^1 < 0$. I next want to show: if $\Delta r_1^t > 0$ and $\Delta f_1^t < 0$ then $\Delta r_1^{t+1} > 0$ and $\Delta f_1^{t+1} < 0$. If $\Delta r_1^t > 0$ then $r_1^t < 1 - [b/(2b - 1)]f_1^t$ so that $r_1^t = 1 - [b/(2b - 1)]f_1^t - \varepsilon$ for some $\varepsilon > 0$. Using (10), it follows that: $r_1^{t+1} = 1 - [b/(2b - 1)]f_1^t - \varepsilon[2(1 - b) + bf_1^t + \varepsilon]$; and therefore $r_1^{t+1} < 1 - [b/(2b - 1)]f_1^t$. Since, by supposition, $\Delta f_1^t < 0$ then $f_1^{t+1} < f_1^t$. It follows from $r_1^{t+1} < 1 - [b/(2b - 1)]f_1^t$ that $r_1^{t+1} < 1 - [b/(2b - 1)]f_1^{t+1}$. It is concluded that $\Delta r_1^{t+1} > 0$. Now I turn to show that $\Delta f_1^{t+1} < 0$. Since $r_0^t = 0$ and $\Delta f_1^t < 0$ then $f_1^t = [b(1 - r_1^t)^2/(1 - br_1^t)] + \varepsilon$ for some $\varepsilon > 0$. From (11), $f_1^{t+1} = [b(1 - r_1^t)^2/(1 - br_1^t)] + br_1^t\varepsilon$ so that $f_1^{t+1} > b(1 - r_1^t)^2/(1 - br_1^t)$. Since $\Delta r_1^t > 0$ then $r_1^{t+1} > r_1^t$. Given that $b(1 - r_1^t)^2/(1 - br_1^t)$ is decreasing in r_1^t, it follows that $f_1^{t+1} > b(1 - r_1^{t+1})^2/(1 - br_1^{t+1})$ and thus $\Delta f_1^{t+1} < 0$.

It has been shown that: (i) $\Delta r_1^1 > 0$ and $\Delta f_1^1 < 0$; and (ii) if $\Delta r_1^t > 0$ and $\Delta f_1^t < 0$ then $\Delta r_1^{t+1} > 0$ and $\Delta f_1^{t+1} < 0$. By induction, $\Delta r_1^t > 0$ and $\Delta f_1^t < 0\ \forall t \geq 1$. It is inferred from $r_1^1 > (1 - b)/b$ and $\Delta r_1^t > 0\ \forall t \geq 1$ that $\lim_{t\to\infty} r_1^t > (1 - b)/b$. This establishes that $((1 - b)/b, (2b - 1)^2/b^2, 0, (1 - b)(2b - 1)/b^2)$ is not an asymptotic attractor.

The final step is to show that (1, 0, 0, 0) and $(0, b, 0, 1 - b)$ are asymptotic attractors. Substituting $1 - r_1^t - f_1^t - f_0^t$ for r_0^t in (10), (11), and (13) and defining $\Pi(f_1^t, r_0^t, f_0^t)$ to be the matrix of first derivatives, it is straightforward to derive (A26) below.

Let $\Lambda(r_1^t, f_1^t, f_0^t) \equiv \max\{\lambda_1, \ldots, \lambda_k\}$ where $\lambda_1, \ldots, \lambda_k$ are the eigenvalues to $\Pi(r_1^t, f_1^t, f_0^t)$. By Theorem 4.10 in Kelley and Peterson (1991), if $\Lambda(r_1, f_1, f_0) < 1$ then (r_1, f_1, f_0) is an asymptotic attractor. Using (A26), one can show that $\Lambda(1, 0, 0) = \max\{2(1 - b), b, 1 - b\} < 1$ and $\Lambda(0, b, 1 - b) = \max\{b(2 -$

$$\text{(A26)}\quad \Pi(r_1^t, f_1^t, f_0^t) = \begin{bmatrix} 2b - 2(2b-1)r_1^t - bf_1^t & -br_1^t & 0 \\ -b(f_1^t + 2f_0^t) & b(2 - r_1^t - 2f_1^t - 2f_0^t) & 2b(1 - r_1^t - f_1^t - f_0^t) \\ (1-b)(2f_1^t + f_0^t) & (1-b)(2r_1^t + 2f_1^t + f_0^t) & (1-b)(1 + r_1^t + f_1^t) \end{bmatrix}.$$

b), 0, $1 - b^2\} < 1$. Therefore, $(r_1, f_1, r_0, f_0) = (1, 0, 0, 0)$ and $(r_1, f_1, r_0, f_0) = (0, b, 0, 1 - b)$ are asymptotic attractors.

REFERENCES

Ben-Porath, Elchanan; Dekel, Eddie and Rustichini, Aldo. "On the Relationship between Mutation Rates and Growth Rates in a Changing Environment." *Games and Economic Behavior*, October 1993, *5*(4), pp. 576–603.

Bickerton, Derek. *Language and species*. Chicago: University of Chicago Press, 1990.

Boylan, Richard T. "Laws of Large Numbers for Dynamical Systems with Randomly Matched Individuals." *Journal of Economic Theory*, August 1992, *57*(2), pp. 473–504.

Burton, Steven J. *Judging in good faith*. Cambridge: Cambridge University Press, 1992.

Guth, William D. and Tagiuri, Renato. "Personal Values and Corporate Strategy." *Harvard Business Review*, September–October 1965, *43*(5), pp. 123–32.

Haltiwanger, John and Waldman, Michael. "Responders versus Non-Responders: A New Perspective on Heterogeneity." *Economic Journal*, September 1991, *101*(408), pp. 1085–102.

Harrington, Joseph E., Jr. "The Social Selection of Principled and Expedient Agents." Department of Economics Working Paper No. 324, Johns Hopkins University, March 1994.

______. "Social Learning and Rigid Behavior." Mimeo, Johns Hopkins University, February 1996.

Hayes, Robert H. and Pisano, Gary P. "Beyond World Class: The New Manufacturing Strategy." *Harvard Business Review*, January–February 1994, *72*(1), pp. 77–86.

Heiner, Ronald A. "The Origin of Predictable Behavior." *American Economic Review*, September 1983, *73*(4), pp. 560–95.

Jonassen, David H. and Grabowski, Barbara L. *Handbook of individual differences, learning and instruction*. Hillsdale, NJ: Lawrence Erlbaum Associates, 1993.

Kelley, Walter G. and Peterson, Allan C. *Difference equations*. Boston: Harcourt Brace Jovanovich, 1991.

Kydland, Finn E. and Prescott, Edward C. "Rules Rather than Discretion: The Inconsistency of Optimal Plans." *Journal of Political Economy*, June 1977, *85*(3), pp. 473–91.

Luchins, Abraham S. and Luchins, Edith Hirsch. *Rigidity of behavior*. Eugene, OR: University of Oregon Books, 1959.

Manchester, William. *Alone 1932–1940*. Boston: Little, Brown & Co., 1988.

Maranell, Gary M. "The Evaluation of Presidents: An Extension of the Schlesinger Polls." *Journal of American History*, June 1970, *57*(1), pp. 104–13.

Rosen, Sherwin. "Prizes and Incentives in Elimination Tournaments." *American Economic Review*, September 1986, *76*(4), pp. 701–15.

Rosenthal, Robert W. "Rules of Thumb in Games." *Journal of Economic Behavior and Organization*, September 1993, *22*(1), pp. 1–13.

Sah, Raaj K. and Stiglitz, Joseph E. "The Quality of Managers." *Quarterly Journal of Economics*, February 1991, *106*(1), pp. 289–95.

Schlesinger, Joseph A. *Ambition and politics: Political careers in the United States*. Chicago: Rand McNally, 1966.

______. *Political parties and the winning of office*. Ann Arbor, MI: University of Michigan Press, 1991.

Scott, William A. "Flexibility, Rigidity, and Adaptation: Toward Clarification of Concepts," in O. J. Harvey, ed., *Experience, structure, and adaptability*. New York: Springer, 1966, pp. 369–400.

Sloan, Alfred P. *My years with General Motors*. New York: Macfadden-Bartell, 1965.

Stahl, Dale O. "Evolution of Smart_n Players." *Games and Economic Behavior*, October 1993, *5*(4), pp. 604–17.

Walton, Sam and Huey, John. *Sam Walton: Made in America*. New York: Doubleday, 1992.

Wildavsky, Aaron. "The Goldwater Phenomenon: Purists, Politicians, and the Two-Party System." *Review of Politics*, July 1965, *27*(3), pp. 386–413.