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Some implications of antitrust laws for cartel pricing

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Abstract

Price dynamics are characterized for a price-fixing cartel. Antitrust laws reduce cartel prices even though cartel detection occurs with probability zero. In response to cheating, the non-collusive price gradually moves from the collusive price to the static Nash equilibrium price.

Keywords: Collusion; Cartel detection; Antitrust

JEL classification: L1; L4

1. Introduction

As evidenced by recent cases in lysine, graphite electrodes, vitamins, and auction houses, price-fixing remains a perennial problem. In light of the illegality of price-fixing, a critical goal faced by a cartel is to avoid the appearance that there is a cartel. If high prices or rapidly increasing prices may make customers suspicious that a cartel is operating, this should have implications for how a cartel prices. My initial research on this topic characterized the joint profit maximizing price path (Harrington, 2002a). The desire to avoid detection in addition to the evolving size of penalties led to a rich set of pricing dynamics. That analysis presumed the incentive compatibility constraints ensuring the internal stability of the cartel were not binding. Research is currently in progress that analyzes cartel pricing when those constraints bite (Harrington, 2002b). The current note offers some early results on this research with an example that highlights some of the new forces at work when existing cartel models are augmented to allow for the prospect of detection.

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2. Model

Consider an industry with n symmetric firms and a sequence of markets represented by $\{D^h(\cdot)\}_{h=1}^{\infty}$, where $D^h:\Omega^2 \to \Re_+$ is a (symmetric) firm demand function and Ω is a compact and convex subset of $\Re_+$. $D^h(P', P'')$ is a firm's demand when it charges P' and all other firms charge P''. Assume the profit function is $\pi^h(P_i, P_{-i}) \equiv (P_i - c)\, D^h(P_i, P_{-i})$ so that the cost function is linear.

A1. $D^h(P_i, P_{-i})$ is continuous, non-increasing in P_i, and non-decreasing in P_{-i}. If $D^h(P_i, P_{-i}) > 0$ then $D^h(P_i, P_{-i})$ is continuously differentiable and increasing in P_i and decreasing in P_{-i}.

A2. $\pi^h(P_i, P_{-i})$ is quasi-concave in P_i and if $\pi^h(P_i, P_{-i}) > 0$ then it is strictly quasi-concave in P_i.

A3. $\exists \hat{P}^h > c$ such that $\psi^h(P) \lesseqgtr P$ as $P \gtreqless \hat{P}^h$ where $\psi^h(P_{-i}) \in \operatorname{argmax}\ \pi^h(P_i, P_{-i})$.

Define $D(P)$ and $\pi(P)$ as a firm's demand and profit, respectively, when all firms charge P. A4 eases the analysis by assuming that market demand at a common price is the same along the sequence of markets.

A4. $\exists D:\Omega \to \Re_+$ such that $D^h(P, P) = D(P) \forall P, \forall h$.

A5. $\pi(P) \equiv (P - c)\, D(P)$ is quasi-concave in P and if $\pi(P) > 0$ then it is strictly quasi-concave in P. $\exists P^m > \hat{P}^h$ such that $\pi(P^m) > \pi(P) \forall P \neq P^m$.

Firms engage in this price game for an infinite number of periods with perfect monitoring and a discount factor of $\delta \in (0,1)$. To make the analysis interesting, suppose, in the absence of antitrust policy, incentive compatibility constraints (ICCs) are binding. Specifically, A6 specifies that firms cannot support the monopoly price if they use a grim trigger strategy. $\tilde{P}^h$ represents the benchmark collusive price in the absence of antitrust laws.

A6. $\exists \tilde{P}^h \in (\hat{P}^h, P^m)$ such that

$$\pi(P)/(1-\delta) \gtreqless \pi^h(\psi^h(P), P) + \delta[\pi(\hat{P}^h)/(1-\delta)] \text{ as } P \lesseqgtr \tilde{P}^h, \forall P \in [\hat{P}^h, P^m].$$

Given A3, when all competitors price at P, the best reply is to price below P when it exceeds the static Nash equilibrium price. By A7, the extent of the optimal undercutting goes to zero as $h \to \infty$. By A8, a firm's demand curve is increasingly elastic as $h \to \infty$. Both of these properties are associated with firms' products becoming homogeneous. An example satisfying these assumptions is provided in Shubik (1984, Section 6.2).

A7. $\lim_{h \to \infty} \psi^h(P) = P, \forall P \in [\hat{P}^h, P^m]$.

A8. $\forall \varepsilon < 0, \exists h_\varepsilon$ such that if $P' \geq P''$ and $D^h(P', P'') > 0$ then $\partial D^h(P', P'')/\partial P_i < \varepsilon, \forall h > h_\varepsilon$.

If firms form a cartel, it is detected with some probability in each period. For simplicity, suppose detection results in the discontinuance of collusion forever so that it generates a terminal payoff of

$[\hat{\pi}/(1-\delta)]-F$ where $F>0$ is a fixed antitrust penalty. If not detected, collusion continues on to the next period. In modelling detection, I do not model buyers (or the antitrust authority) as strategic agents but instead assume that their suspicions are triggered by large price movements. In the context of a presumed stationary environment, such movements should seem anomalous to buyers which may cause them to consider the possibility of firms having cartelized. $\phi:\Omega^{2n}\rightarrow[0,1]$ denotes the probability of detection and is allowed to depend on the current and previous periods' price vectors. More specifically, detection occurs for sure when the absolute value of the change in a summary statistic, f^h, is sufficiently large and doesn't occur otherwise.

A9. $\phi^h(\underline{P}^t, \underline{P}^{t-1})=0$ (1) if $|f^h(\underline{P}^t)-f^h(\underline{P}^{t-1})|\leq(>)\eta$ where $\eta\in(0, (\tilde{P}^h-\hat{P}^h)/3)$ and: (i) f^h: $\Omega^n\rightarrow\Omega$ is continuously differentiable; ii) if $D^h(P_i, P_{-i})>0$ then $\partial f^h/\partial P_i>0$ and is bounded from above when $P_i\geq P_{-i}$; and (iii) if all firms with positive demand charge the same price of P then $f^h=P$.

All of the properties on the summary statistic hold if it is the average transaction price. After presenting the main result, a discussion of the sensitivity of the analysis to A9 is provided. Finally, it is assumed $\Omega=[c, P^m]$ which is not essential for results though it does simplify some of the proofs.

3. Optimal symmetric subgame perfect equilibrium

The cartel's problem is to choose an infinite price path so as to maximize the expected sum of discounted income subject to the price path being incentive compatible (IC). In determining the set of IC price paths, the assumption is made that deviation from the collusive path results in the cartel being dissolved and firms pricing according to a symmetric Markov Perfect Equilibrium (MPE) where $W^h:\Omega\rightarrow\Re$ denotes the associated payoff. The state variable is the lagged summary statistic which is of relevance because it enters the probability of detection function.

Assume that firms inherit the non-collusive price: $P^0=\hat{P}$. The firms' problem is either to not form a cartel—and price at $\hat{P}$ in every period—or form a cartel and choose a price path so as to:

$$\begin{aligned}\max_{\{P^t\}_{t=1}^{\infty}\in\Gamma} V(\{P^t\}_{t=1}^{\infty}) \equiv &\sum_{t=1}^{\infty}\delta^{t-1}\prod_{j=1}^{t-1}\left[1-\phi^h\left((P^j,\ldots,P^j),(P^{j-1},\ldots,P^{j-1})\right)\right]\pi(P^t)\\ &+\sum_{t=1}^{\infty}\delta^t\phi^h\left((P^t,\ldots,P^t),(P^{t-1},\ldots,P^{t-1})\right)\prod_{j=1}^{t-1}\left[1-\phi^h\left((P^j,\ldots,P^j),(P^{j-1},\ldots,P^{j-1})\right)\right]\\ &\times\left[(\hat{\pi}^h/(1-\delta))-F\right] \end{aligned}\tag{1}$$

where

$$\begin{aligned}\Gamma\equiv\Big\{&\{P^t\}_{t=1}^{\infty}\in\Omega^{\infty}: V(\{P^t\}_{t=\tau}^{\infty})\geq\max_{P_i\in\Omega}\pi^h(P_i,P^{\tau})\\ &+\delta\phi^h\left((P^{\tau},\ldots,P_i,\ldots,P^{\tau}),(P^{\tau-1},\ldots,P^{\tau-1})\right)\left[(\hat{\pi}^h/(1-\delta))-F\right]\\ &+\delta\left[1-\phi^h\left((P^{\tau},\ldots,P_i,\ldots,P^{\tau}),(P^{\tau-1},\ldots,P^{\tau-1})\right)\right]W^h\left(f^h(P^{\tau},\ldots,P_i,\ldots,P^{\tau})\right),\forall\tau\geq1\Big\}\end{aligned}$$

Γ is the set of price paths that are IC. A solution to (1) is referred to as an Optimal Symmetric Subgame Perfect Equilibrium (OSSPE) price path.

The first result shows that if products are sufficiently similar and the penalty is sufficiently severe then there is a symmetric MPE in which firms move their prices closer to the static Nash equilibrium price by an amount η until it is reached. The prospect of detection after a deviation then induces a gradual unwinding of the cartel.

Lemma 1. *$\exists h'$ such that if $h > h'$, the following is true for a market with D^h. $\exists \bar{F}^h > 0$ such that if $F > \bar{F}^h$ then $\bar{\rho}^h : \Omega \rightarrow \Omega$ is a MPE where*:

$$\bar{\rho}^h\left(f^h\left(\underline{P}^{t-1}\right)\right) = \begin{cases} f^h\left(\underline{P}^{t-1}\right) - \eta & \text{if } f^h\left(\underline{P}^{t-1}\right) \in \left(\hat{P}^h + \eta, P^m\right] \\ \hat{P}^h & \text{if } f^h\left(\underline{P}^{t-1}\right) \in \left[\hat{P}^h - \eta, \hat{P}^h + \eta\right] \\ f^h\left(\underline{P}^{t-1}\right) + \eta & \text{if } f^h\left(\underline{P}^{t-1}\right) \in \left[c, \hat{P}^h - \eta\right) \end{cases}$$

Proof. For any h, if F is large enough then it's not optimal for a firm to trigger detection when it is under its control not to do so. Suppose all other firms are using the strategy specified in the lemma and define $\mu^{t-1} \equiv f^h\left(\underline{P}^{t-1}\right)$ as the summary statistic in period $t-1$. Suppose $\mu^{t-1} > \hat{P}^h + \eta$. Given other firms are charging $\mu^{t-1} - \eta$, an individual firm triggers detection if its price is less than $\mu^{t-1} - \eta$ so a firm then prefers pricing at $\mu^{t-1} - \eta$ to pricing below $\mu^{t-1} - \eta$. Next consider firm i pricing at $P_i > \mu^{t-1} - \eta$ (but not so high that detection occurs) and make the standard presumption that all firms, including this one, acts according to $\bar{\rho}^h$ in the future. The future price path then involves all firms pricing at $\max\left\{f^h\left(\mu^{t-1} - \eta | P_i\right) - \tau\eta,\ \hat{P}^h\right\}$ in period $t + \tau$ where $f^h(P|P_i) \equiv f^h(P, \ldots, P_i, \ldots, P)$. Firm i's payoff is then:

$$(P_i - c)\, D^h\left(P_i, \mu^{t-1} - \eta\right) + \sum_{\tau=1}^{\infty} \delta^\tau \pi\left(\max\left\{f^h\left(\mu^{t-1} - \eta | P_i\right) - \tau\eta, \hat{P}^h\right\}\right)$$

Note that by pricing above $\mu^{t-1} - \eta$, firm i may lower its current profit but it could raise future profit for at least a finite number of periods as $f^h(P|P_i)$ is increasing in P_i. Taking the derivative of this expression with respect to P_i:

$$\begin{aligned} &(P_i - c)\left(\partial D^h\left(P_i, \mu^{t-1} - \eta\right) / \partial P_i\right) + D^h\left(P_i, \mu^{t-1} - \eta\right) \\ &+ \left(\partial f^h\left(\mu^{t-1} - \eta | P_i\right) / \partial P_i\right) \\ &\times \sum_{\tau=1}^{\infty} \delta^\tau \pi'\left(\max\left\{f^h\left(\mu^{t-1} - \eta | P_i\right) - \tau\eta, \hat{P}^h\right\}\right) \kappa\left(f^h\left(\mu^{t-1} - \eta | P_i\right) - \tau\eta\right) \end{aligned} \tag{2}$$

where $\kappa\left(f^h\left(\mu^{t-1} - \eta | P_i\right) - \tau\eta\right) = 1$ if $f^h\left(\mu^{t-1} - \eta | P_i\right) - \tau\eta \geq \hat{P}^h$ and 0 otherwise. If P_i is set so high that $D^h\left(P_i, \mu^{t-1} - \eta\right) = 0$ then the firm's payoff from that price is lower than if it prices at $\mu^{t-1} - \eta$ because current profit is lower and future profit is unaffected as the summary statistic is the same whether it prices at P_i or P: $f^h(P|P_i) = P = f^h(P, \ldots, P)$. Now consider $P_i > \mu^{t-1} - \eta$ but where $D^h\left(P_i, \mu^{t-1} - \eta\right) > 0$. The second term in (2) is bounded from above as is the third term since $\partial f^h\left(\mu^{t-1} - \eta | P_i\right) / \partial P_i$ and π' are bounded and $\kappa = 1$ for only a finite number of periods. By A8, a demand system can be selected to make $|\partial D(P_i,\ \mu^{t-1}-\eta)/\partial P_i|$ arbitrarily large. Since $P_i > \mu^{t-1}-\eta > \hat{P}^h \geq c$ then P_i is bounded above c. Hence, if h is sufficiently high then, in absolute terms, the first term exceeds the sum of the second and third terms. It follows that a firm's payoff is strictly decreasing in P_i for $P_i \geq \mu^{t-1}-\eta$ when $\mu^{t-1} > \hat{P}^h + \eta$ and h is sufficiently high. As it has already been shown that pricing below $\mu^{t-1}-\eta$ is non-optimal, pricing at $\mu^{t-1}-\eta$ is then optimal. This concludes the proof that $\overline{\rho}^h$ is optimal when the state is such that $f^h(\underline{P}^{t-1}) > \hat{P}^h + \eta$.

An analogous argument applies when $f^h(\underline{P}^{t-1}) < \hat{P}^h - \eta$. When $f^h(\underline{P}^{t-1}) \in [\hat{P}^h - \eta,\ \hat{P}^h + \eta]$, pricing at $\hat{P}^h$ maximizes current profit, while pricing below it reduces current profit and can only lower the future price path. By the argument given above, pricing above $\hat{P}^h$ lowers the firm's payoff when h is sufficiently high. □

The main result is that if the punishment for deviation is $\overline{\rho}^h$ then the collusive price path is rising and is bounded below the price that would have occurred in the absence of antitrust laws. Intuitively, when there is a deviation, firms do not immediately drop price to $\hat{P}^h$ because such a large price change would create suspicions about non-collusive pricing. Firms instead lower price in sufficiently small increments so as to avoid detection and, in finite time, reach the static Nash equilibrium. This more gradual price response to a deviation weakens the punishment and thereby reduces the level of collusive prices that can be supported relative to when there are no antitrust laws. It is worth noting that antitrust laws lower prices even though the cartel is never detected.

Theorem 2. *$\exists h'$ such that $\forall h > h'$, the following is true for a market with $D^h(\cdot)$. $\exists \overline{F}^h > 0$ such that if $F > \overline{F}^h$ then: if a cartel is formed and $\{\overline{P}^t\}_{t=1}^{\infty}$ is an OSSPE price path for which deviation results in a continuation equilibrium of $\overline{\rho}^h$ then: (i) $\{\overline{P}^t\}_{t=1}^{\infty}$ is non-decreasing over time; and (ii) sup$\{\overline{P}^1, \overline{P}^2, \ldots\} < \tilde{P}^h$.*

Proof. By Lemma 1, I can choose h and F sufficiently high such that $\overline{\rho}^h$ is a MPE. As an initial step, let us first show that an OSSPE price path is non-decreasing and does not trigger detection. The latter is immediate from setting F sufficiently high. To show that the price path is non-decreasing, suppose not so that $\exists t'$ such that $\overline{P}^{t'-1} > \overline{P}^{t'}$. Consider the alternative path of pricing at the level $\overline{P}^{t'-1}$ in all periods for which price is below $\overline{P}^{t'-1}$. This results in a higher payoff since profit is higher in those periods and the probability of detection is still zero. If, for the original price path, it was IC to price at $\overline{P}^{t'-1}$ in $t'-1$ then this new path will be IC as well since the future collusive payoff is higher and the punishment payoff is the same.

Given that an OSSPE price path is non-decreasing and bounded from above, $\lim_{t\to\infty} \overline{P}^t$ exists. Define $P^x \equiv \sup\{\overline{P}^1,\ \overline{P}^2, \ldots\}$ and suppose, contrary to the theorem, $P^x \geq \tilde{P}^h$. (Though it is not denoted, $\{\overline{P}^t\}_{t=1}^{\infty}$ and P^x also depend on h.) There are two cases: (i) $\exists t'$ such that $\overline{P}^{t'} = P^x$; and (ii) $\nexists t$ such that $\overline{P}^t = P^x$. Consider case (i). The payoff from optimally deviating at t' is $\max_{P_i} \pi^h(P_i, P^x) + \Sigma_{\tau=1}^{\infty} \delta^\tau \pi\left(\max\left\{f^h(P^x|P_i) - \tau\eta,\ \hat{P}^h\right\}\right)$. I want to argue that an optimal deviation results in $f^h(P^x|P_i) - \eta > \hat{P}^h$. I know that $\overline{P}^{t'-1} \in [P^x - \eta,\ P^x]$ as the price path is non-decreasing and does not trigger detection. If $f^h(P^x|P_i) \geq \overline{P}^{t'-1}$ then $f^h(P^x|P_i) \geq P^x - \eta$. $P^x \geq \tilde{P}^h$ then implies $\hat{f}^h(P^x|P_i) \geq \tilde{P}^h - \eta$. As $\tilde{P}^h > \hat{P}^h + 3\eta$ by A9, it follows that $\hat{f}^h(P^x|P_i) \geq \hat{P}^h + 2\eta$ and, therefore, $f^h(P^x|P_i) - \eta > \hat{P}^h$. If instead $f^h(P^x|P_i)<\overline{P}^{t'-1}$ then, as an optimal deviation does not trigger detection, it must be true that $\overline{P}^{t'-1}-f^h(P^x|P_i)\leq\eta$. As $\overline{P}^{t'-1}\geq P^x-\eta$, it follows that $f^h(P^x|P_i)+\eta\geq\overline{P}^{t'-1}\geq P^x-\eta$ which implies $f^h(P^x|P_i)\geq P^x-2\eta$. Since $P^x\geq\tilde{P}^h$ and $\tilde{P}^h>\hat{P}^h+3\eta$ then $f^h(P^x|P_i)\geq\tilde{P}^h-2\eta>\hat{P}^h+\eta$, and I have the desired result that $f^h(P^x|P_i)-\eta>\hat{P}^h$.

Given an optimal deviation involves $f^h(P^x|P_i)-\eta>\hat{P}^h$, the future payoff associated with deviating is strictly higher than $\delta[\pi(\hat{P}^h)/(1-\delta)]$ as price exceeds $\hat{P}^h$ for at least one period and is never below it (and recall that detection doesn't occur). By choosing h sufficiently high, I can make $\psi^h(P^x)$ arbitrarily close to P^x by A7 and thereby f^h arbitrarily close to P^x by A9. This ensures that $P^x-f^h(P^x|\psi^h(P^x))\in(0,\eta)$. Therefore, the payoff to deviating strictly exceeds $\pi(\psi^h(P^x),\ P^x)+\delta[\pi(\hat{P}^h)/(1-\delta)]$. Since $\pi(P^x)\geq\pi(\overline{P}^t)\forall t$, an upper bound on a firm's equilibrium payoff is $\pi(P^x)/(1-\delta)$. A necessary condition for incentive compatibility is then

$$\pi(P^x)/(1-\delta)>\pi^h(\psi^h(P^x),P^x)+\delta[\pi(\hat{P}^h)/(1-\delta)]$$

But if $P^x\geq\tilde{P}^h$ then by A6 this cannot hold. Contrary to our original supposition, I conclude that $P^x<\tilde{P}^h$ and thus $\{\overline{P}^t\}_{t=1}^{\infty}$ is bounded below $\tilde{P}^h$.

For case (ii), $\overline{P}^t<P^x\ \forall t$ and $\lim_{t\to\infty}\overline{P}^t=P^x\geq\tilde{P}^h$. An upper bound on the future payoff from colluding is then $\pi(P^x)/(1-\delta)$ so that, in period t, an upper bound on the payoff from colluding is $\pi(\overline{P}^t)+\delta(\pi(P^x)/(1-\delta))$. Consider a firm deviating in period t by pricing at $\psi(\overline{P}^t)$. By the same argument as given above, if h is sufficiently high then the payoff from this deviation is strictly greater than $\pi^h(\psi^h(\overline{P}^t),\overline{P}^t)+\delta(\pi(\hat{P}^h)/(1-\delta))$. Hence, a necessary condition for equilibrium is:

$$\pi(\overline{P}^t)+\delta(\pi(P^x)/(1-\delta))>\pi^h(\psi^h(\overline{P}^t),\overline{P}^t)+\delta(\pi(\hat{P}^h)/(1-\delta))$$

As $t\to\infty$, this condition becomes:

$$\pi(P^x)/(1-\delta)>\pi^h(\psi^h(P^x),P^x)+\delta(\pi(\hat{P}^h)/(1-\delta))$$

However, since $P^x\geq\tilde{P}^h$ then this condition cannot hold by A6. □

Though results were derived for an extreme specification of the detection technology, what is critical is that the probability of detection is sensitive to an individual firm's price. If, by lowering its price on the punishment path, a firm raises the probability of detection then one would generally expect it to balance off the increase in profit from lowering price to that target and the increase in the expected penalty from doing so. The qualitative property of the MPE—firms gradually lower price after the cartel collapses—would then seem quite general. Thus, one can generally expect the punishment path, under antitrust laws, to involve higher prices and thus a higher payoff. Potentially offsetting this effect is that expected penalties may be positive which will tend to reduce the punishment payoff. That effect is absent here because there exist prices for which the probability of detection is zero. With a less extreme detection technology, there will be a positive probability of detection on the punishment path. What is key for the result is that, on net, the higher prices offset the prospect of penalties so that the punishment payoff is higher. This resulted in a greater incentive to deviate; thereby forcing the cartel to set lower prices than in the absence of antitrust laws.

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References

Harrington, Jr. J.E., 2002a. Optimal Cartel Pricing in the Presence of an Antitrust Authority. Working Paper No. 460. Johns Hopkins University, Baltimore, May 2001, Revised July 2002.

Harrington, Jr. J.E., 2002b. Cartel Pricing Dynamics in the Presence of an Antitrust Authority. Working Paper No. 486. Johns Hopkins University, Baltimore, December 2002.

Shubik, M., 1984. A Game−Theoretic Approach to Political Economy. MIT Press, Cambridge, MA.