



Equilibrium pricing in a (partial) search market: The shopbot paradox[☆]

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Received 15 February 2006; received in revised form 13 June 2006; accepted 18 August 2006
Available online 13 November 2006

Abstract

Price competition is explored when consumers costlessly learn product prices but not ancillary fees and have greater disutility to paying these fees. There is a unique symmetric Nash equilibrium price between the competitive (Bertrand) and monopoly (Diamond) prices.

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Keywords: Search; Oligopoly; Diamond paradox; Shopbot

JEL classification: L11; L13

1. Introduction

One of the most significant attributes of online markets is the low cost of consumer search. In particular, the advent of shopbots has made it almost costless for prospective buyers to see the prices of many online sellers. Though impressive, the price information provided at a shopbot can be incomplete. For example, early versions of shopbots did not report shipping fees. To learn the total price, a consumer had to go through much of the purchasing process at a retailer's web site. Furthermore, the total price at

[☆] The first author conducted this research while visiting the Department of Economics at Harvard University. He expresses his appreciation for their hospitality and the quality of their students. The second author conducted this research while an undergraduate in the Faculty of Arts and Sciences at Harvard University.

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the end of that search could be vastly higher than the product price found at the shopbot. Ellison and Ellison (2004) provide some striking anecdotes with respect to the online market for computer hardware.

This paper explores price competition by online retailers when consumers can costlessly learn firms' product prices — by going to a shopbot — but can only learn the shipping charge at a cost — by initiating the process of buying the product at the firm's web site. The model then lies between the classical set-ups of Bertrand (1883) — where all price information is costlessly available — and Diamond (1971) — where all price information is costly to learn.

Some other recent theoretical work on pricing in online markets includes Bakos (1997) and Baye and Morgan (2001). Bakos (1997) considers the implications for price competition when there are distinct search costs associated with price and product traits though the analysis is seriously flawed (for details, see Harrington, 2001). Baye and Morgan (2001) focus on the behavior of closed shopbots (which charge a fee for online retailers to be listed) and derive implications for product price competition. Neither of these papers — nor the more general literature on pricing in search markets — encompasses the partial information setting that we analyze.

Though the analysis is motivated by online markets and product prices and shipping fees, it is applicable to other market settings in which only part of the total price of purchasing a product is observed costlessly. A common offline situation is when a firm advertises the price of a product but then a consumer learns about “hidden” fees when she shows up at the store ready to purchase. This partial information setting is then one that is common in conventional markets as well as online markets.

2. Model

There are $n \geq 2$ online firms selling a homogeneous product. Each firm chooses two distinct prices — a product price and a shipping price. Let p_i and s_i denote the product price and shipping price, respectively, of firm $i \in \{1, 2, \dots, n\}$ where $p_i, s_i \geq 0$. Each firm has a common constant marginal of $c \geq 0$.

Consumers are assumed to have a stronger disutility for paying for shipping than for the product. Let the perceived price to a consumer from firm i be $p_i + \theta s_i$ where $\theta \geq 1$. If $\theta = 1$ then we have the standard assumption that a consumer is indifferent as to how expenditure is allocated between the product and shipping. The empirical findings of Brynjolfsson and Smith (2000) support the hypothesis that $\theta > 1$, which is the assumption we make.

As this preference assumption is unorthodox and crucial to our analysis, let us briefly review Brynjolfsson and Smith (2000). They have data on clickstream behavior at a shopbot for books (which happens to provide shipping fees). A logit model was estimated that included product price, brand name, shipping charge, and delivery time as explanatory variables. Estimates showed that both a higher product price and a higher shipping charge results in a lower probability of buying from an online retailer. However, quite strikingly, the marginal effect of the shipping price on the probability of a customer purchase was twice as large as that for the product price. This is the basis for our assumption that $\theta > 1$.

There is a unit mass of consumers, each with a demand function $D(P)$ where $P = p + \theta s$ is the perceived price. $D : \mathfrak{R}_+ \rightarrow \mathfrak{R}_+$ is assumed to be twice continuously differentiable and if $D(P) > 0$ then $D'(P) < 0$. Further assume $D'(c) > 0$ and that we have the necessary concavity to ensure the existence of a unique optimal price; a sufficient condition is $D''(P) \leq 0$.

Firms simultaneously choose a product price and a shipping charge. Consumers use a shopbot to costlessly learn the n product prices but have to go to an online retailer's web site to learn its shipping

charge. In the spirit of [Diamond \(1971\)](#), there is a search cost $x > 0$ for each shipping charge that a consumer learns. Upon observing the n product prices, consumers form expectations of the associated n shipping charges and then visit the web site of the firm with the lowest expected perceived price.¹

Industry profit at a common price pair is then $(p + s - c)D(p + \theta s)$ and, by our assumptions, the monopoly solution exists and is unique:

$$(p^m, 0) = \arg \max_{p,s} (p + s - c)D(p + \theta s).$$

To understand why the optimal shipping charge is zero, suppose that a monopolist instead had a price pair (p', s') where $s' > 0$. It could then raise the product price to $p' + \theta s'$ and lower the shipping charge to zero and consumers would demand the same amount since the perceived price is unchanged. However, firm profit has risen since per unit profit has grown by $(\theta - 1)s'$. Hence, the monopoly shipping price must be zero. The monopolist then avoids charging for services that consumers find particularly distasteful.

3. Equilibrium

Equivalent to having each firm choose both prices simultaneously is to have each firm first choose a product price — which is revealed to consumers — and then choose a shipping price when consumers show up at its online store. With that interpretation, define $\phi(p)$ as the profit-maximizing shipping price when a firm receives a fraction $\alpha \in (0, 1]$ of demand (that is, a fraction α of consumers visit this retailer's web site) after it has committed to a product price p :

$$\phi(p) = \arg \max_s (p + s - c)\alpha D(p + \theta s).$$

First note that if a symmetric equilibrium product price is p' then the shipping price must be $\phi(p')$. For suppose instead that the shipping price is $s'' \neq \phi(p')$. The firm could then marginally change its shipping price in the direction of $\phi(p')$ and, if α is unchanged, raise profit. As long as the change in the shipping price is sufficiently small, α will indeed be unchanged because a consumer will prefer to buy than to incur the search cost of x from learning another retailer's shipping fee. Hence, if the symmetric equilibrium product price is p' then the shipping price must be $\phi(p')$.

To derive some properties of $\phi(p)$, take the derivative of the profit function with respect to s ,

$$\alpha[D(p + \theta s) + \theta(p + s - c)D'(p + \theta s)], \tag{1}$$

and evaluate it at the simple monopoly price,

$$\alpha[D(p^m + \theta s) + \theta(p^m + s - c)D'(p^m + \theta s)]. \tag{2}$$

¹ As is standard in the Diamond model, it is assumed that a consumer's expected surplus from incurring the search cost x exceeds x . An alternative demand specification is that each consumer buys at most one unit and $D(P)$ is the mass of consumers with a maximum willingness to pay exceeding P . In that case, the standard assumption is that a consumer's first search is free.

Since

$$D(p^m) + (p^m - c)D'(p^m) = 0$$

then $\theta > 1$ implies

$$D(p^m) + \theta(p^m - c)D'(p^m) < 0$$

Thus, Eq. (2) is negative when evaluated at $s=0$ and, given the concavity in s , it follows that profit is decreasing in s for all $s \geq 0$. Therefore, $\phi(p^m)=0$; a firm provides free shipping when its product price is the simple monopoly price.

Next consider a product price equal to marginal cost. Evaluating (1) at $p=c$, marginal profit is $\alpha [D(c+\theta s) + \theta s D'(c+\theta s)]$ which is positive for $s=0$. Hence, $\phi(c) > 0$.

Finally, let us characterize the relationship between the optimal shipping price and the product price. If $\phi(p) > 0$ then $\phi(p)$ is defined by the first-order condition (where we've divided through by α):

$$D(p + \theta\phi(p)) + \theta(p + \phi(p) - c)D'(p + \theta\phi(p)) = 0.$$

Taking the total derivative with respect to p yields

$$\phi'(p) = -\left(\frac{1}{\theta}\right) \left[\frac{(1 + \theta)D' + \theta(p + \theta\phi(p) - c)D''}{2D' + \theta(p + \theta\phi(p) - c)D''} \right]. \quad (3)$$

By our concavity assumptions, it follows that: if $\phi(p) > 0$ then $\phi'(p) < 0$. Charging a higher product price implies a lower fee for shipping.

An important element in our analysis is the relationship between the product price and the perceived price, $p + \theta s$. Under equilibrium expectations, a consumer that sees a firm charging a product price of p' will expect to find a shipping price of $\phi(p')$ at the firm's web site and thus expect a perceived price of $p' + \theta\phi(p')$. To evaluate how the perceived price changes with respect to the announced product price, take the derivative of $p + \theta\phi(p)$ with respect to p and, using Eq. (3), we derive:

$$\frac{\partial(p + \theta\phi(p))}{\partial p} = \frac{(1 - \theta)D'}{2D' + \theta(p + \theta\phi(p) - c)D''} < 0.$$

Interestingly, a higher product price is more attractive to a consumer; a higher product price signals a sufficiently smaller shipping price that the perceived price is actually lower. That a firm will set such a lower shipping fee is because consumer demand is more sensitive to the shipping fee (when $\theta > 1$). Consumers are then better off if the firm announces a higher product price.

Define p^* as the lowest product price for which the optimal shipping price is zero. It is that product price such that Eq. (1) is zero when evaluated at $s=0$:

$$D(p^*) + \theta(p^* - c)D'(p^*) = 0. \quad (4)$$

p^* is depicted in Fig. 1.

We now claim that there is a unique symmetric Nash equilibrium and it has firms charging a product price of p^* and providing free shipping.

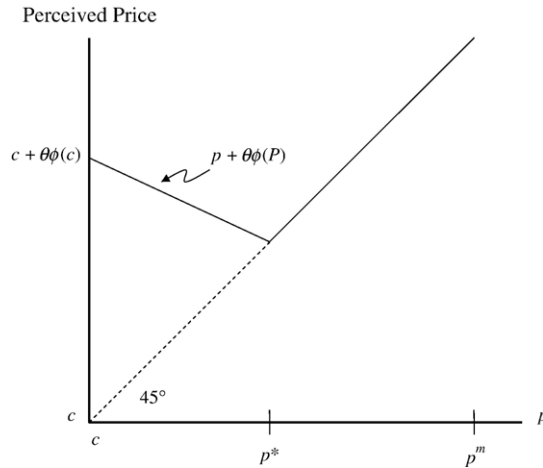


Fig. 1. Perceived price as a function of the product price.

Consider p' as a candidate symmetric equilibrium product price and suppose $p' > p^*$. Note that the optimal shipping price is zero in a small neighborhood of p' (recall that p^* is the lowest price for which firms offer free shipping). Given all firms charge a product price of p' , consumers are indifferent among firms. Making the standard assumption that each firm gets an equal share of market demand, a firm's profit from pricing at p' is then $(p' - c)(1/n)D(p')$ where we've presumed the firm sets the optimal shipping charge of $\phi(p') = 0$. Now consider a firm instead charging a lower product price of $p' - \varepsilon$ where $\varepsilon > 0$ and ε is sufficiently close to zero so that $\phi(p' - \varepsilon) = 0$. As the new perceived price is $p' - \varepsilon$, which is less than the perceived price of p' of the other firms, all consumers go to this firm. Its profit is then $(p' - \varepsilon - c)D(p' - \varepsilon)$ which exceeds $(p' - c)(1/n)D(p')$ when ε is sufficiently small. Hence, p' is not a symmetric equilibrium product price when $p' > p^*$.

Now suppose $p' < p^*$. Analogous to the above, the profit from charging a product price of p' , given every other firm does so, is

$$(p' + \phi(p') - c)(1/n)D(p' + \theta\phi(p'))$$

though now $\phi(p') > 0$. Consider a firm raising its product price to $p' + \varepsilon$. Since $p + \theta\phi(p)$ is decreasing in p when $p < p^*$, all consumers will prefer to buy from this firm. Its profit is

$$(p' + \varepsilon + \phi(p' + \varepsilon) - c)D(p' + \varepsilon + \theta\phi(p' + \varepsilon))$$

which is higher than that earned from pricing at p' when ε is sufficiently small. We conclude that p' is not a symmetric equilibrium product price when $p' < p^*$.

This leaves us with the one remaining candidate for a symmetric equilibrium which is that every firm charges a product price of p^* . In that situation, a firm's profit is $(p^* - c)(1/n)D(p^*)$. By charging a lower product price, its perceived price increases which makes its demand and profit zero (see Fig. 1). By charging a higher product price, its perceived price increases as well since the product price increases and the shipping price remains at zero; again, its demand and profit is zero. Therefore, it is the unique symmetric Nash equilibrium for all firms to charge a product price of p^* and offer free shipping.

In our partial information setting with a consumer distaste for paying for shipping (relative to paying for the product), the equilibrium price is less than the simple monopoly price. Taking the total differential of Eq. (4), the product (and total) price can be shown to be lower when the distutility from shipping fees is more severe:

$$\frac{\partial p^*}{\partial \theta} = -\frac{(p^*-c)D'(p^*)}{(1+\theta)D'(p^*) + \theta(p^*-c)D''(p^*)} < 0.$$

The intuition as to why consumer welfare is higher when there is a greater dislike for shipping charges is as follows. For any product price, the profit-maximizing shipping price is (weakly) reduced when θ is higher because consumer demand is more sensitive to the shipping price. If the equilibrium price was $p^*(\theta')$ when $\theta = \theta'$ then, when θ is raised from θ' to θ'' , the optimal shipping fee is zero in a neighborhood around $p^*(\theta')$.² This induces firms to lower their product price from $p^*(\theta')$ as doing so lowers the perceived price (as the shipping fee remains zero) and thereby serves to attract consumers to their site. We then have that when consumer demand is more sensitive to shipping charges, firms reduce their product prices and continue to avoid charging for shipping.

Another way in which to see this result is that as consumers experience more disutility from shipping fees, firms have a greater incentive to avoid such fees since demand is more sensitive to them. This serves to shift competition to the product price which, since it is costlessly observed, intensifies competition.

4. The shopbot paradox

The main result from Diamond (1971) is that, as long as search costs are positive (no matter how small), competition completely vanishes as all firms charge the simple monopoly price. This is referred to as the Diamond Paradox because it runs so strikingly counter to what we observe in practice. The main result of the current paper is also a bit paradoxical. When consumers costlessly learn product prices but not some ancillary fees and consumers find such fees especially distasteful, it is predicted that ancillary fees are zero which runs counter to a lot of anecdotal evidence. Possible reasons for this disparity between theory and reality are that consumers have incorrect expectations about shipping fees, firms do not believe consumers dislike paying for shipping more than paying for the product, and, contrary to the initial evidence, consumers do not particularly dislike shipping fees.

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² In comparison, when $\theta = \theta'$, a firm would charge a positive shipping fee when the product price is less than $p^*(\theta')$.

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