Modelling Cartel Formation and Cartel Pricing Dynamics

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Introduction

- Research Goals:
  - Short-run: Develop a theory that is able to generate price paths consistent with actual cartel price paths.
  - Long-run: Develop a test that would be effective in screening markets for the presence of a cartel.
- Screening is the monitoring of markets for illegal activity.
- Increased interest in screening for cartels.
  - Currently under consideration by the European Commission.
  - Leniency programs enhance the efficacy of screening.

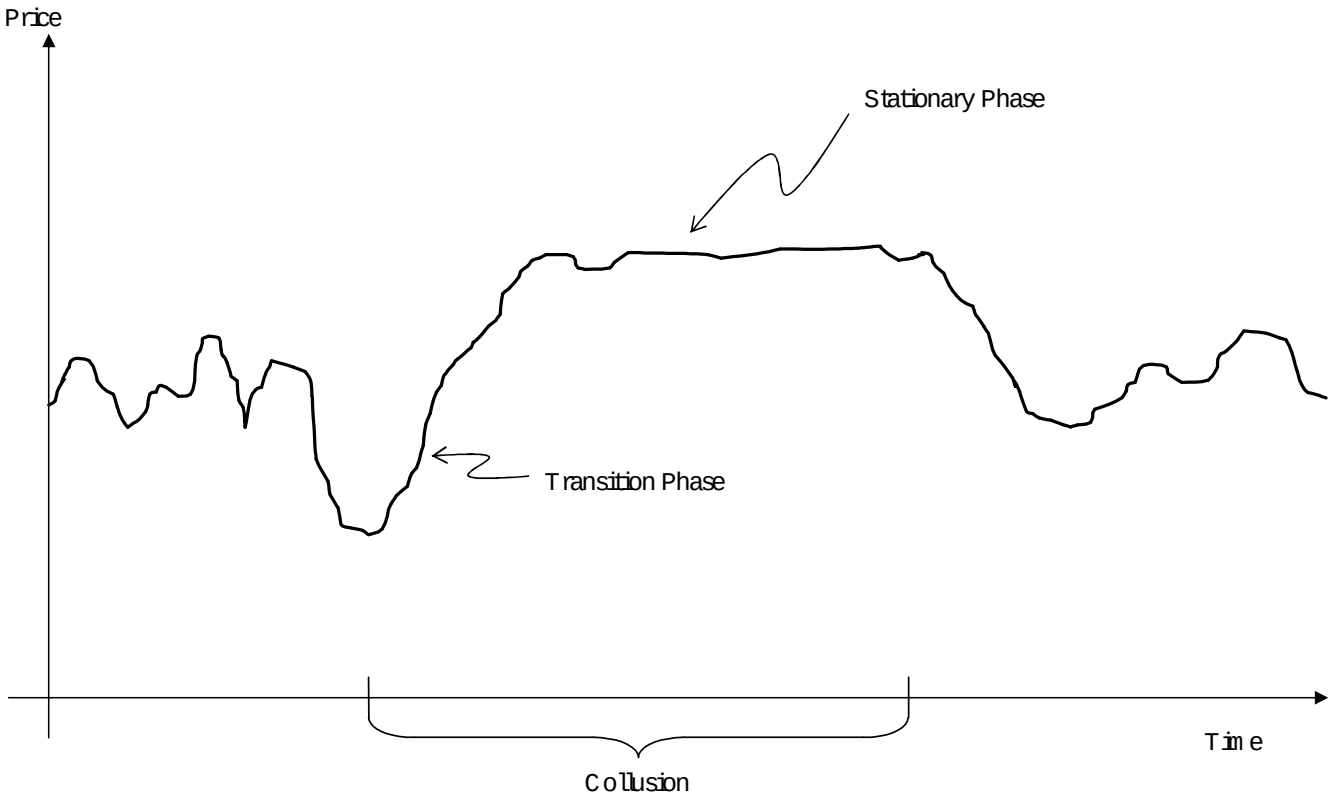
Introduction

Overview of talk

1. What do cartel price paths look like?
2. Review theory that generates realistic-looking cartel price paths.
3. Present preliminary results when cartel formation is endogenous.

Properties of Cartel Price Paths

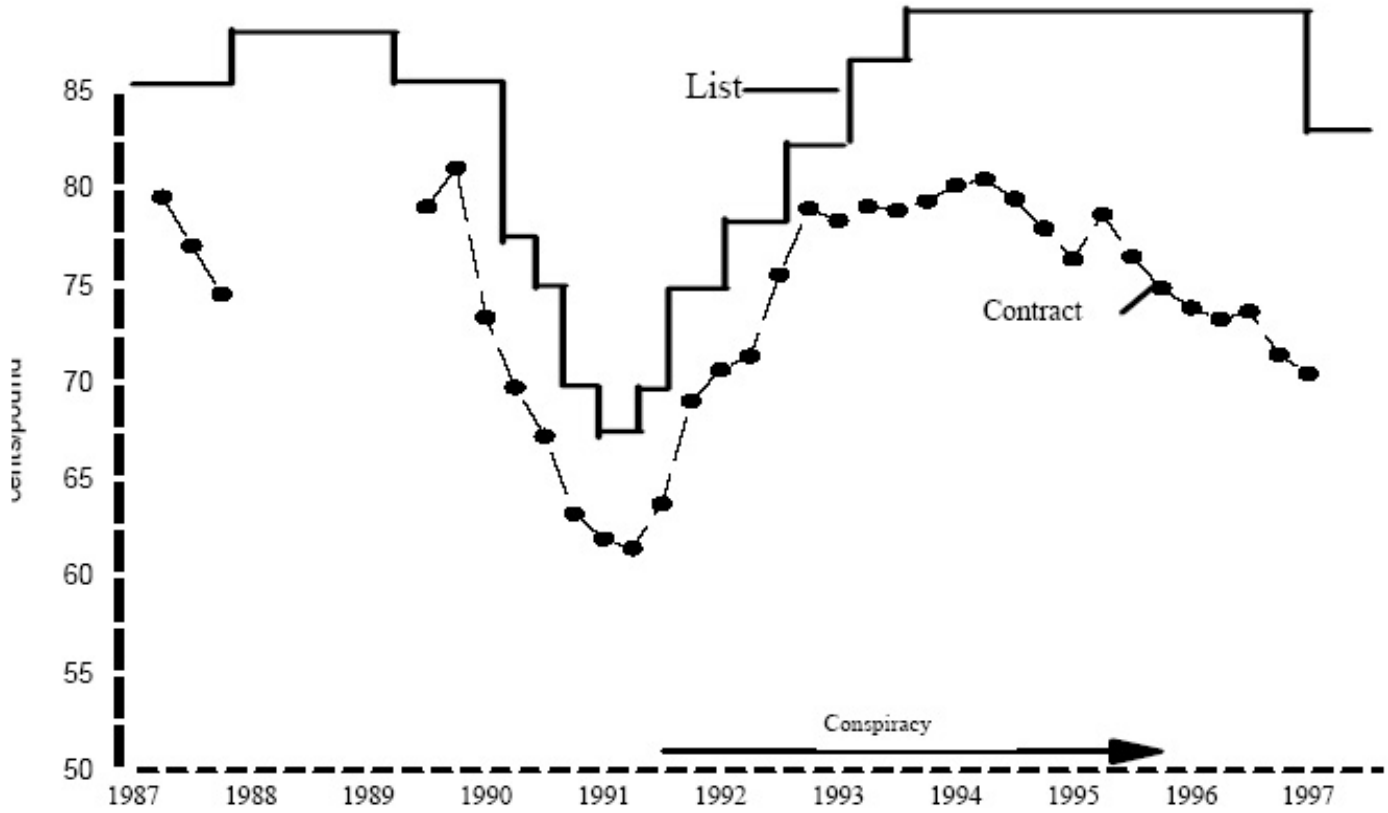
Canonical Cartel Price Path



Properties of Cartel Price Paths

Citric Acid Cartel

- Transition phase in which price gradually rises.
- Cartel formation is preceded by price decline.

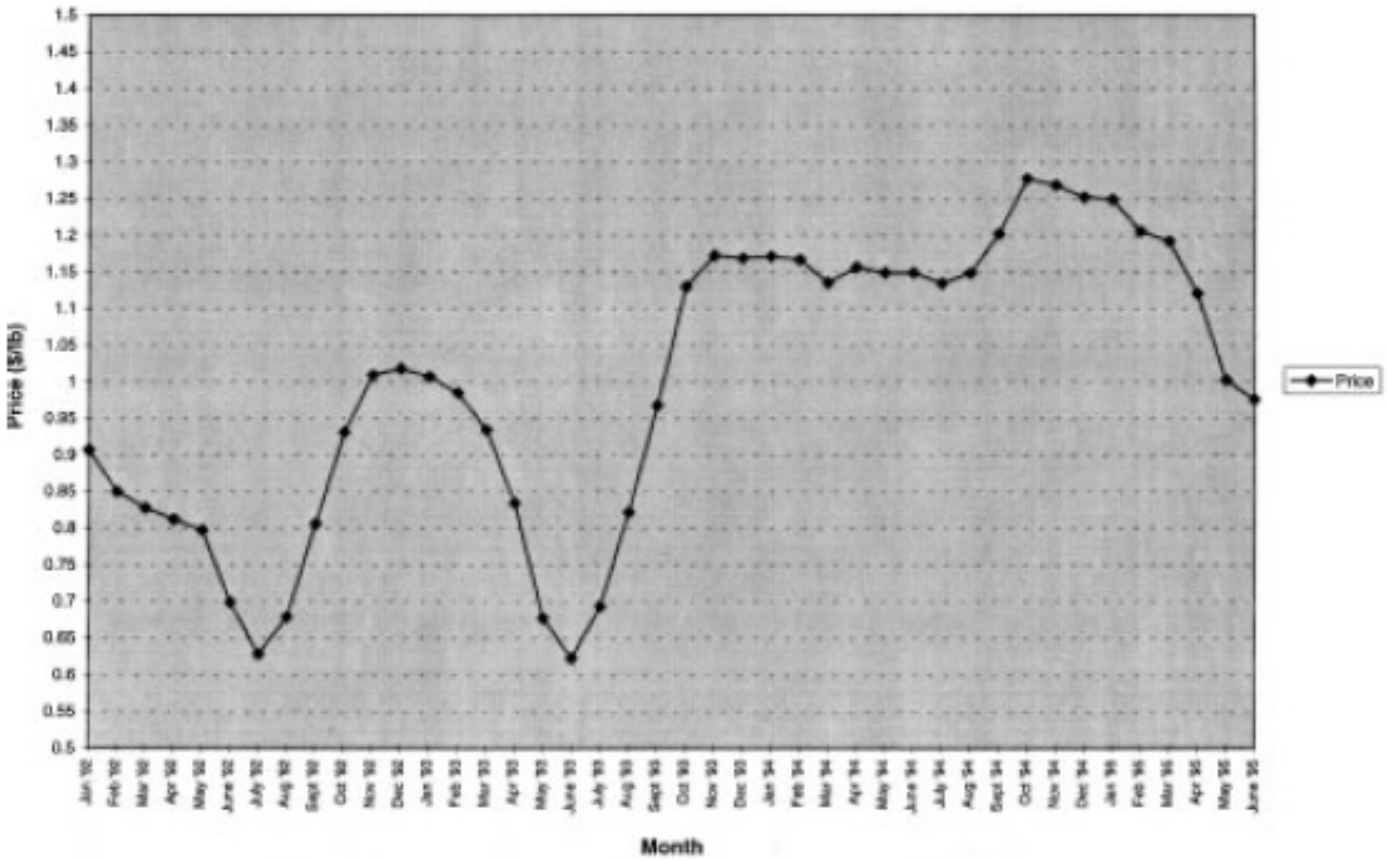


Connor (2001)

Properties of Cartel Price Paths

Lysine Cartel

- Transition phase in which price gradually rises.

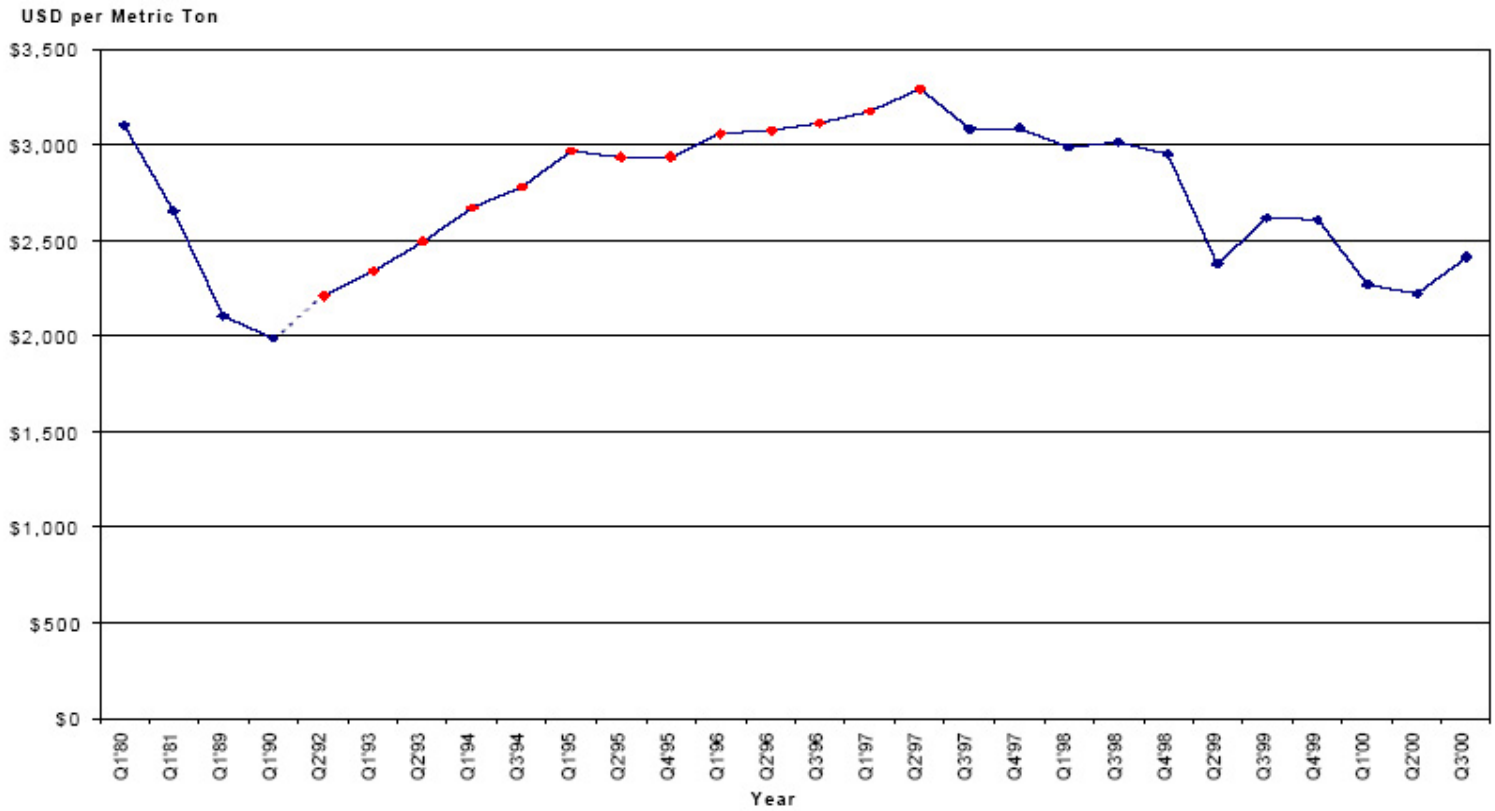


Connor (2001)

Properties of Cartel Price Paths

Graphite Electrodes Cartel

- Transition phase in which price gradually rises.
- Cartel formation is preceded by price decline.



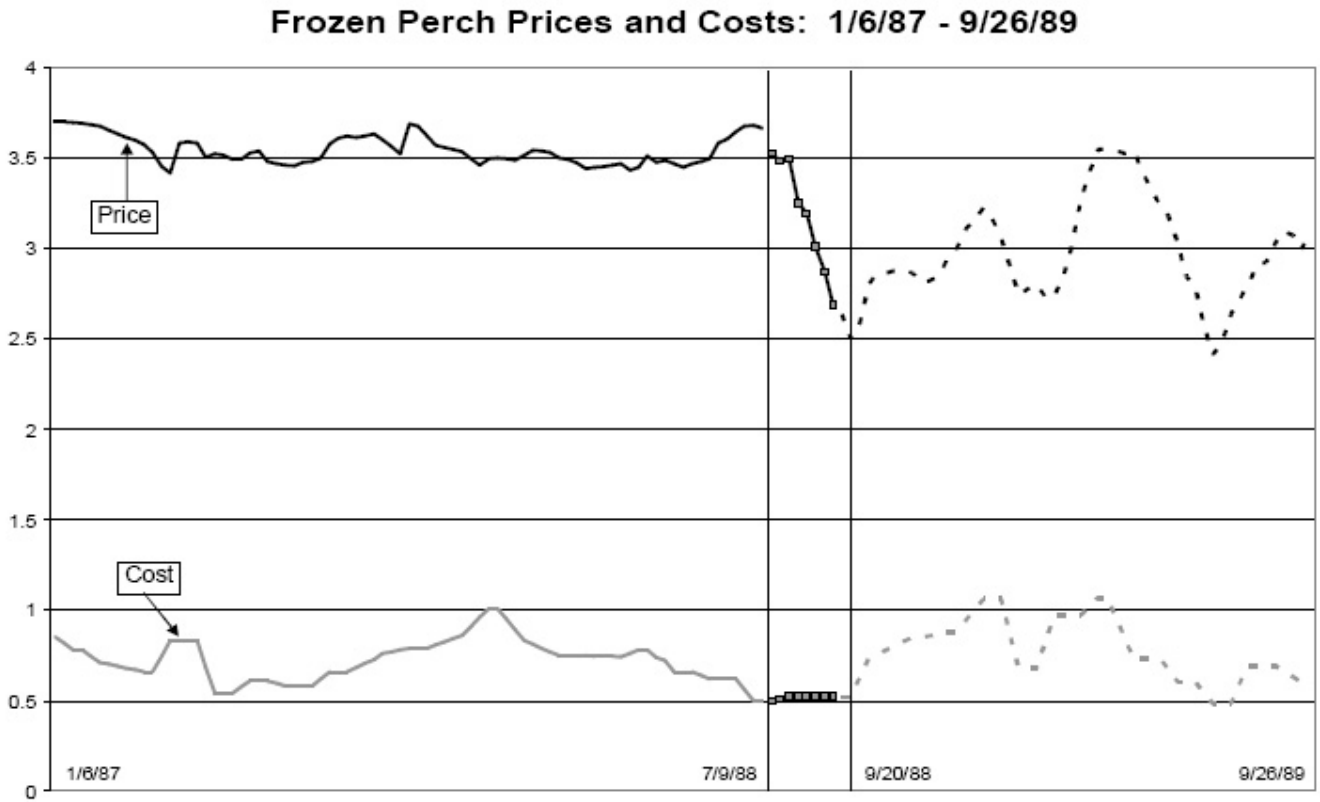
Cartel years indicated in red.

Levenstein and Suslow (2001)

Properties of Cartel Price Paths

Frozen Perch Cartel

- Stationary phase in which price variance is low.



Abrantes-Metz, Froeb, Geweke, and Taylor (2005)

Properties of Cartel Price Paths

Summary

1. Transition phase in which price gradually rises.
2. Stationary phase in which price variance is low.
3. Cartel formation is preceded by price decline.

Cartel Pricing Dynamics

Model

- Harrington and Chen (IJIO, 2006)
- Infinite horizon oligopoly game with perfect monitoring.
- Linear demand function:

$$D(P) = a - bP.$$

- Common and stochastic linear cost function,

$$C^t(q) = c^t q,$$

where c^t is a random walk over $[\underline{c}, \overline{c}]$,

$$c^t = c^{t-1} + \varepsilon^t,$$

$\varepsilon^t \sim N\left(\mu_\varepsilon, \sigma_\varepsilon^2\right)$ and *iid*.

- Non-collusive profit

$$\widehat{\pi}\left(c^t\right) \equiv \left(\widehat{P}\left(c^t\right) - c^t\right)\left(a - b\widehat{P}\left(c^t\right)\right)$$

where $\widehat{P}\left(c^t\right)$ is the non-collusive price.

- Non-collusive value

$$W\left(c^t\right) = \widehat{\pi}\left(c^t\right) + \delta \int\int W\left(v\left(c^t + \varepsilon\right)\right) f\left(\varepsilon; \mu_\varepsilon, \sigma_\varepsilon^2\right) d\varepsilon$$

where

$$v\left(c^t + \varepsilon\right) = \max\left\{\underline{c}, \min\left\{c^{t-1} + \varepsilon, \overline{c}\right\}\right\}$$

Cartel Pricing Dynamics

Detection of Collusion

- If the cartel is currently active then
  - Firms agree to a common price and realize profit.
  - With some probability, the cartel is detected.
    - Each firm pays a penalty and receives non-collusive profit thereafter.
  - If the cartel is not detected then collusion continues to the next period.
- Endogenizing the probability of detection
  - Buyers are pure empiricists and become suspicious when the price series is "unlikely."
  - Prior information of buyers
    - Price is a random walk:

$$P^t = P^{t-1} + \eta^t.$$

    - $\eta^t \sim N(?, ?)$ is normally distributed.
    - Buyers do not know the moments of the distribution on η^t.

Cartel Pricing Dynamics

Detection of Collusion

- Moments of buyers' beliefs in period t
  - Finite memory of k periods:

$$\left\{\eta^{t-k},\ldots,\eta^{t-1}\right\}$$

    where $\eta^{\tau}\equiv P^{\tau}-P^{\tau-1}$.
  - Use the sampling moments so buyers' distribution on η^t is

$$N\left(m_1^{t-1},m_2^{t-1}-\left(m_1^{t-1}\right)^2\right)$$

    where

$$m_i^{t-1}\equiv\left(\frac{1}{k}\right)\sum_{\tau=t-k}^{t-1}\left(\eta^{\tau}\right)^i.$$

- Approximate the equation of motion on buyer's moments:

$$m_i^t=\lambda_i m_i^{t-1}+\left(1-\lambda_i\right)\left(\eta^t\right)^i.$$

- Buyers assess the "reasonableness" of recent price changes.
  - Buyers "test" a sequence of the $z\,(<k)$ most recent price changes.
  - Likelihood of these z price changes is a "moving" likelihood:

$$l^t \equiv \Pi_{\tau=t+1-z}^{t} f\left(\eta^\tau; m_1^{\tau-1}, m_2^{\tau-1} - \left(m_1^{\tau-1}\right)^2\right).$$

  - ml^t is the maximum likelihood

$$ml^t \equiv \Pi_{\tau=t+1-z}^{t} \max_{y^\tau} f\left(y^\tau; m_1^{\tau-1}, m_2^{\tau-1} - \left(m_1^{\tau-1}\right)^2\right).$$

- Detection depends on relative likelihood:

$$L^t \equiv \frac{l^t}{ml^t} = \frac{\Pi_{\tau=t+1-z}^{t} f\left(\eta^\tau; m_1^{\tau-1}, m_2^{\tau-1} - \left(m_1^{\tau-1}\right)^2\right)}{\Pi_{\tau=t+1-z}^{t} \max_{y^\tau} f\left(y^\tau; m_1^{\tau-1}, m_2^{\tau-1} - \left(m_1^{\tau-1}\right)^2\right)}$$

- Approximate the equation of motion on the relative likelihood with:

$$\begin{aligned} L^t &= \left(L^{t-1}\right)^\xi \left[\frac{f\left(\eta^t; m_1^{t-1}, m_2^{t-1} - \left(m_1^{t-1}\right)^2\right)}{\max_y f\left(y; m_1^{t-1}, m_2^{t-1} - \left(m_1^{t-1}\right)^2\right)}\right] \\ &= \left(L^{t-1}\right)^\xi \varphi\left(\eta^t, m_1^{t-1}, m_2^{t-1}\right) \end{aligned}$$

- Probability of detection, $\phi\left(L^t\right)$, is decreasing in L^t:

$$\phi\left(L^t\right) \equiv \alpha_0 + \alpha_1\left(1 - L^t\right)^{\alpha_2}$$

.

- Evolution of penalties
  - X^t is penalty to be paid if caught in period t.

$$X^t = \beta X^{t-1} + \gamma x\left(P^t, c^t\right) \text{ where } \gamma \geq 0 \text{ and } \beta \in (0,1).$$

  - $x\left(P^t, c^t\right)$ is the damages incurred in period t where

$$x\left(P^t, c^t\right) = \left(P^t - \widehat{P}\left(c^t\right)\right)\left(a - bP^t\right).$$

- Cartel's problem
  - Fix the variance of buyers' beliefs at the non-collusive price variance.
  - Equations of motion:

$$P^t = P^{t-1} + \eta^t$$

$$c^{t+1} = v\left(c^t + \varepsilon^{t+1}\right)$$

$$X^t = \beta X^{t-1} + \gamma x\left(P^{t-1} + \eta^t, c^t\right)$$

$$m_1^t = \lambda m_1^{t-1} + (1-\lambda)\eta^t$$

$$L^t = \left(L^{t-1}\right)^{\xi} \varphi\left(\eta^t, m_1^{t-1}\right)$$

$$
\begin{aligned}
& V\left(P^{t-1}, X^{t-1}, c^t, m_1^{t-1}, L^{t-1}\right) \\
= & \max_{\eta^t} \pi\left(P^{t-1}+\eta^t, c^t\right)+\delta\phi\left(\left(L^{t-1}\right)^{\xi}\varphi\left(\eta^t, m_1^{t-1}\right)\right)\times \\
& \left[\int W\left(v\left(c^t+\varepsilon\right)\right) f\left(\varepsilon;\mu_\varepsilon,\sigma_\varepsilon^2\right) d\varepsilon-\beta X^{t-1}-\gamma x\left(P^{t-1}+\eta^t, c^t\right)\right] \\
& +\delta\left[1-\phi\left(\left(L^{t-1}\right)^{\xi}\varphi\left(\eta^t, m_1^{t-1}\right)\right)\right]\times \\
& \int V\left(P^{t-1}+\eta^t, \beta X^{t-1}+\gamma x\left(P^{t-1}+\eta^t, c^t\right),\right. \\
& \left. v\left(c^t+\varepsilon\right), \lambda m_1^{t-1}+(1-\lambda)\eta^t, \left(L^{t-1}\right)^{\xi}\varphi\left(\eta^t, m_1^{t-1}\right)\right) f\left(\varepsilon;\mu_\varepsilon,\sigma_\varepsilon^2\right) d\varepsilon
\end{aligned}
$$

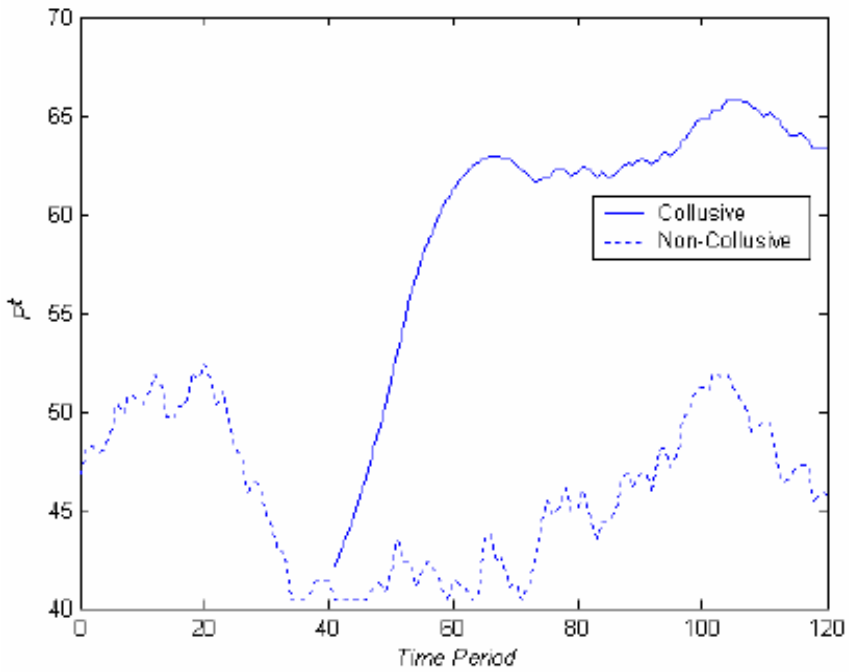
Cartel Pricing Dynamics

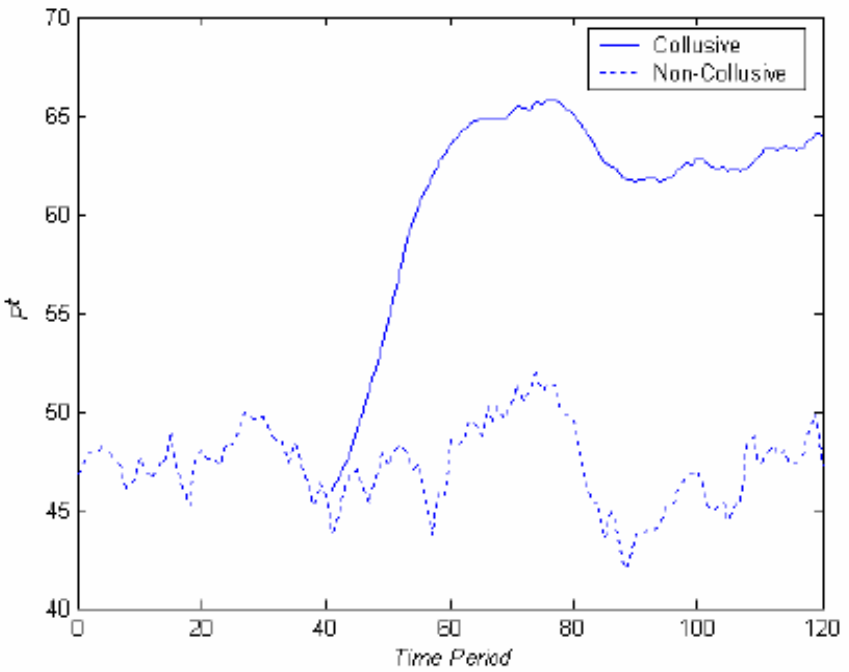
Simulating cartel price paths

- Numerically solve for the value function.
- Use value function to produce the policy (price) function.
- Generate price path.
  - Randomly choose initial values for the state variables.
  - Run model for 40 periods when firms are competing.
  - "Turn on" collusion in period 41.

Cartel Pricing Dynamics

Sample collusive price paths





Cartel Pricing Dynamics

Properties of Collusive Price Path

1. Transitory phase - price rises largely independent of cost.
2. Stationary phase - price is responsive to cost.
3. If cost variance is higher, transition path is shorter.

- Numerical analysis
  - Each 200-period run has random initial conditions and cost realizations.
  - Data is from the stationary phase (periods 101-200).
  - Results are the average of 10 runs.

Average Variance of Price Changes

Cost Variance	Non-collusive, σ^2_{nc}	Collusive, σ^2_c	σ^2_{nc}/σ^2_c
$\sigma^2_\varepsilon = 1$	0.485	.029	16.72
$\sigma^2_\varepsilon = 2$	0.967	.078	12.40
$\sigma^2_\varepsilon = 3$	1.576	.144	10.94
$\sigma^2_\varepsilon = 4$	1.980	.255	7.76

- Property: During the stationary phase, collusive price variance is much lower than the non-collusive price variance.

Cartel Formation

Introduction

- Objectives of research
  - Develop collusive markers
    - Previous research identified patterns during collusion.
    - What patterns arise prior to collusion?
  - Produce a "test bed" to identify effective screening methods.
- Key modelling step:
  - Allow intensity of competition to be stochastic.

- Non-collusive price

$$\widehat{P}\left(c^t, w^t\right) = \left(\frac{a}{b}\right)\left(1 - w^t\right) + w^t c^t,$$

  - $w^t \in [.5, 1]$
  - $w^t = .5$ is joint profit maximum, $w^t = 1$ is perfect competition.
  - w^t follows a random walk over $[\underline{w}, \overline{w}]$:

$$w^t = w^{t-1} + \zeta^t,$$

where $\zeta^t \sim N\left(\mu_\zeta, \sigma_\zeta^2\right)$.

Cartel Formation

Non-Collusion

- Non-collusive profit

$$\widehat{\pi}\left(c^t, w^t\right) \equiv \left(\widehat{P}\left(c^t, w^t\right) - c^t\right)\left(a - b\widehat{P}\left(c^t, w^t\right)\right)$$

- Non-collusive value

$$\begin{aligned} W\left(c^t, w^t\right) &= \widehat{\pi}\left(c^t, w^t\right) + \\ &\quad \delta \int\int W\left(v\left(c^t + \varepsilon\right), \omega\left(w^t + \zeta\right)\right) \times \\ &\quad f\left(\varepsilon; \mu_\varepsilon, \sigma_\varepsilon^2\right) f\left(\zeta; \mu_\zeta, \sigma_\zeta^2\right) d\varepsilon d\zeta \end{aligned}$$

where

$$\omega\left(w^{t-1} + \zeta^t\right) \equiv \max\left\{\underline{w}, \min\left\{w^{t-1} + \zeta^t, \overline{w}\right\}\right\}.$$

Cartel Formation

Damages

- Damages in period t:

$$x\left(P^t, c^t; \widehat{w}\right) = \left(P^t - \widehat{P}\left(c^t, \widehat{w}\right)\right)\left(a - bP^t\right).$$

  where $\widehat{w}$ is the conduct variable in the period of cartel formation.
- Variance of buyers' beliefs is set at the pre-cartel price variance, s_p^2.

$$
\begin{aligned}
& V\left(P^{t-1}, X^{t-1}, c^t, m_1^{t-1}, L^{t-1}; s_p^2, \widehat{w}\right) \\
= \quad & \max_{\eta^t} \pi\left(P^{t-1}+\eta^t, c^t\right) + \delta\phi\left(\left(L^{t-1}\right)^{\xi}\varphi\left(\eta^t, m_1^{t-1}; s_p^2\right)\right) \times \\
& \left[\int W\left(v\left(c^t+\varepsilon\right); \widehat{w}\right) f\left(\varepsilon; \mu_\varepsilon, \sigma_\varepsilon^2\right) d\varepsilon\right. \\
& \left. -\beta X^{t-1} - \gamma x\left(P^{t-1}+\eta^t, c^t; \widehat{w}\right) - F\right] \\
& +\delta\left[1-\phi\left(\left(L^{t-1}\right)^{\xi}\varphi\left(\eta^t, m_1^{t-1}; s_p^2\right)\right)\right] \times \\
& \int V\left(P^{t-1}+\eta^t, \beta X^{t-1}+\gamma x\left(P^{t-1}+\eta^t, c^t; \widehat{w}\right), v\left(c^t+\varepsilon\right),\right. \\
& \left.\lambda m_1^{t-1}+(1-\lambda)\eta^t, \left(L^{t-1}\right)^{\xi}\varphi\left(\eta^t, m_1^{t-1}; s_p^2\right); s_p^2, \widehat{w}\right) f\left(\varepsilon; \mu_\varepsilon, \sigma_\varepsilon^2\right) d\varepsilon.
\end{aligned}
$$

- A cartel forms in period t when:

$$V\left(P^{t-1}, X^{t-1}, c^t, m_1^{t-1}, L^{t-1}; s_p^2\left(t\right), w^t\right) > W\left(c^t, w^t\right).$$

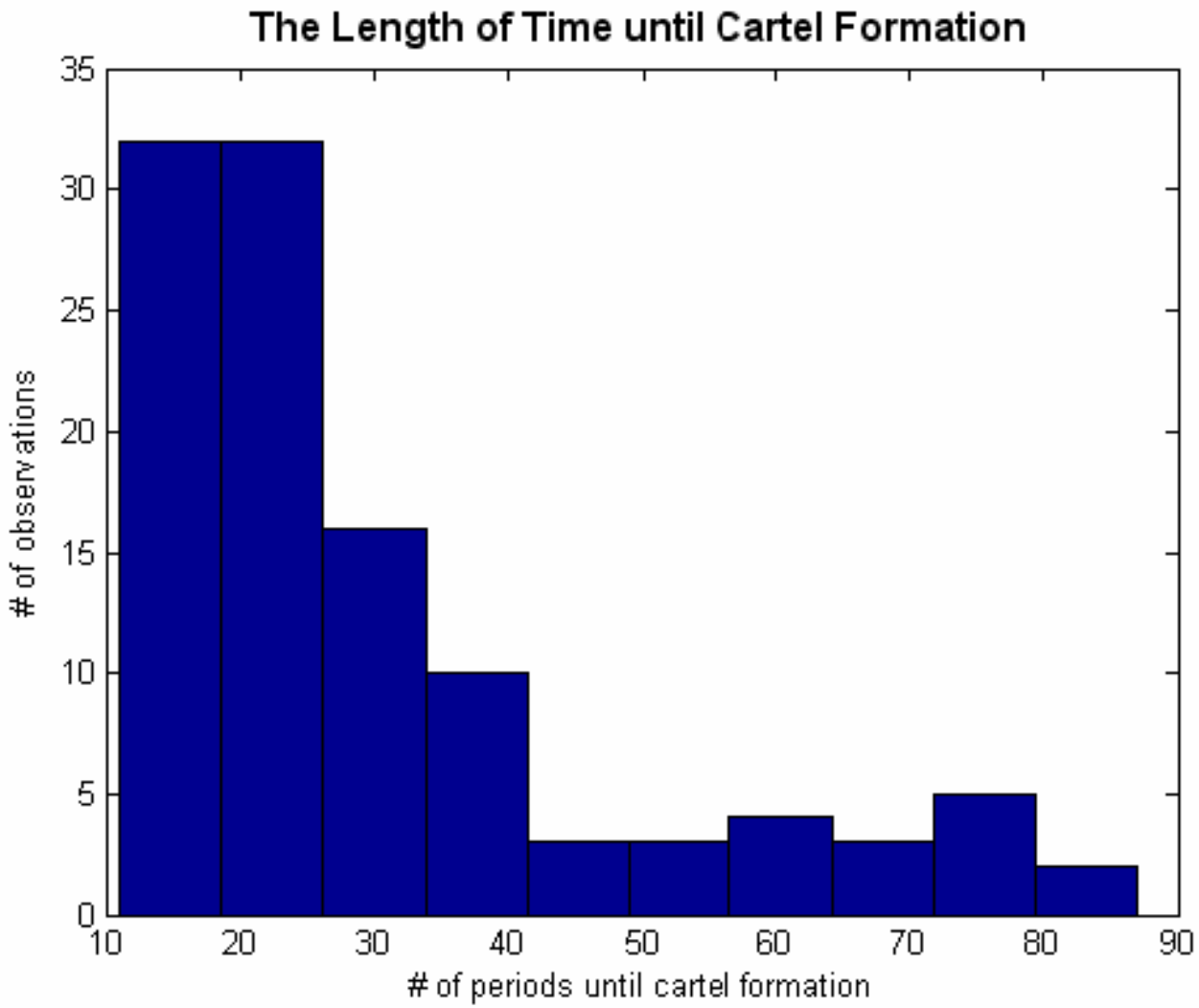
- Timing of cartel formation depends on
  - intensity of competition, w^t.
  - cost, c^t.
  - pre-cartel empirical price variance, $s_p^2\left(t\right)$.

Cartel Formation

Numerical analysis

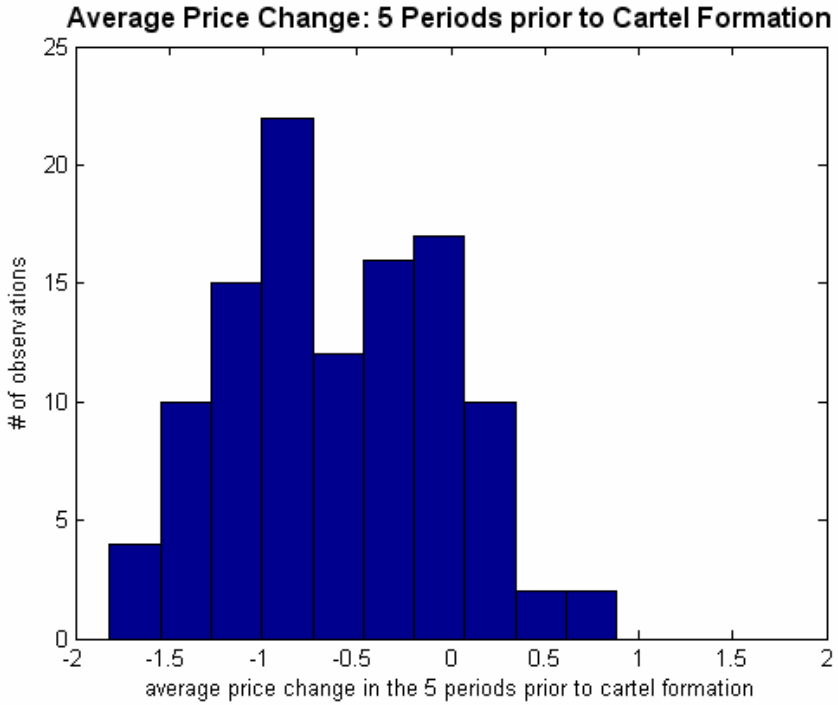
- Steps
  - Randomly choose initial values for state variables.
  - Allow a cartel to form starting in period 41.
  - Exclude runs in which a cartel formed prior to period 51.
- Parameterizations
  - 10 parameter configurations that vary in
    - size of the penalty
    - variance of cost shock
    - variance of conduct shock
    - sensitivity of detection to buyers' beliefs
  - 1000 runs for each parameter configuration
    - 110-236 runs (per parameter configuration) with cartel formation after pd. 50.

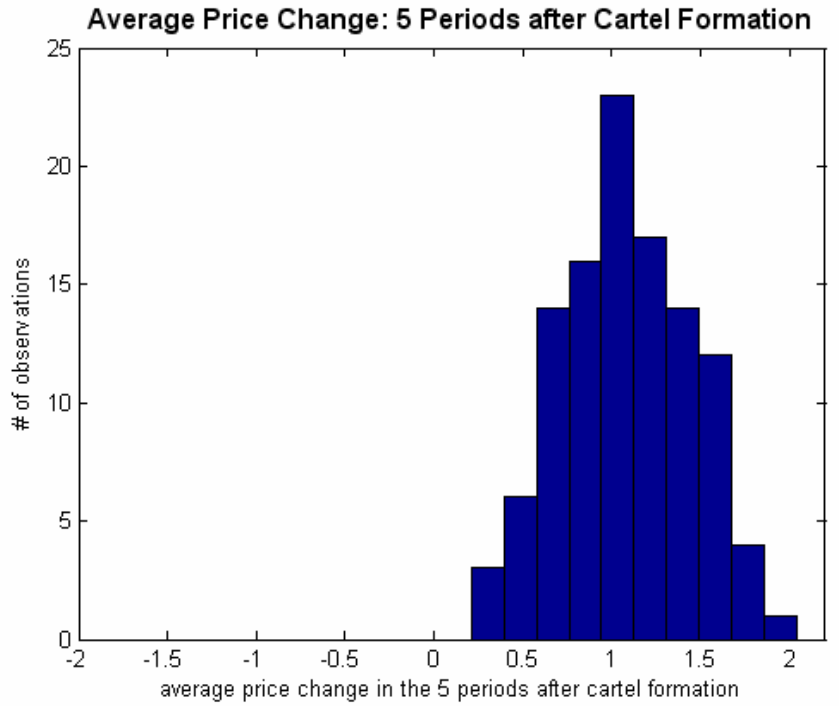
Cartel Formation



Cartel Formation

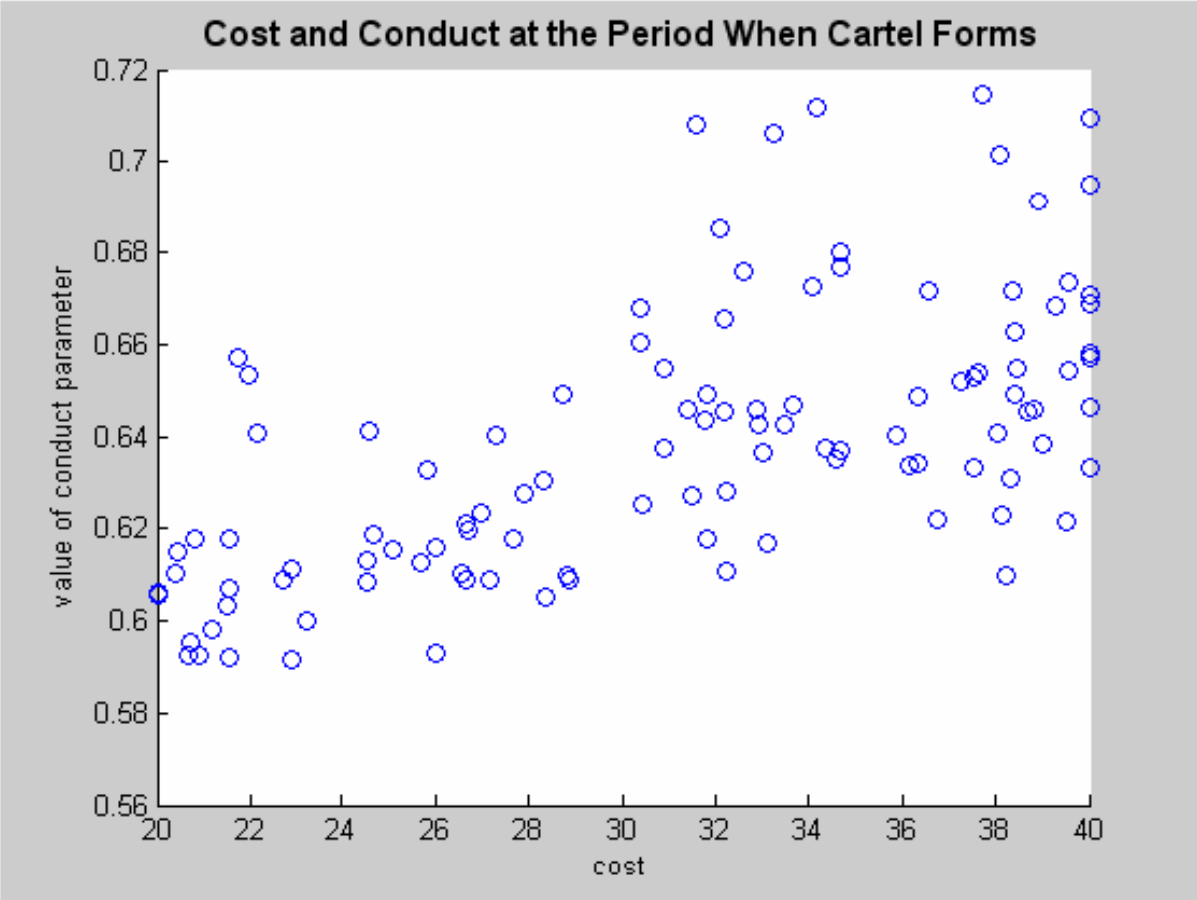
Property 1: Cartel formation is preceded by a price decrease (and followed by a price increase).





Cartel Formation

Property 2: Trade-off between more intense competition and lower cost for triggering cartel formation.



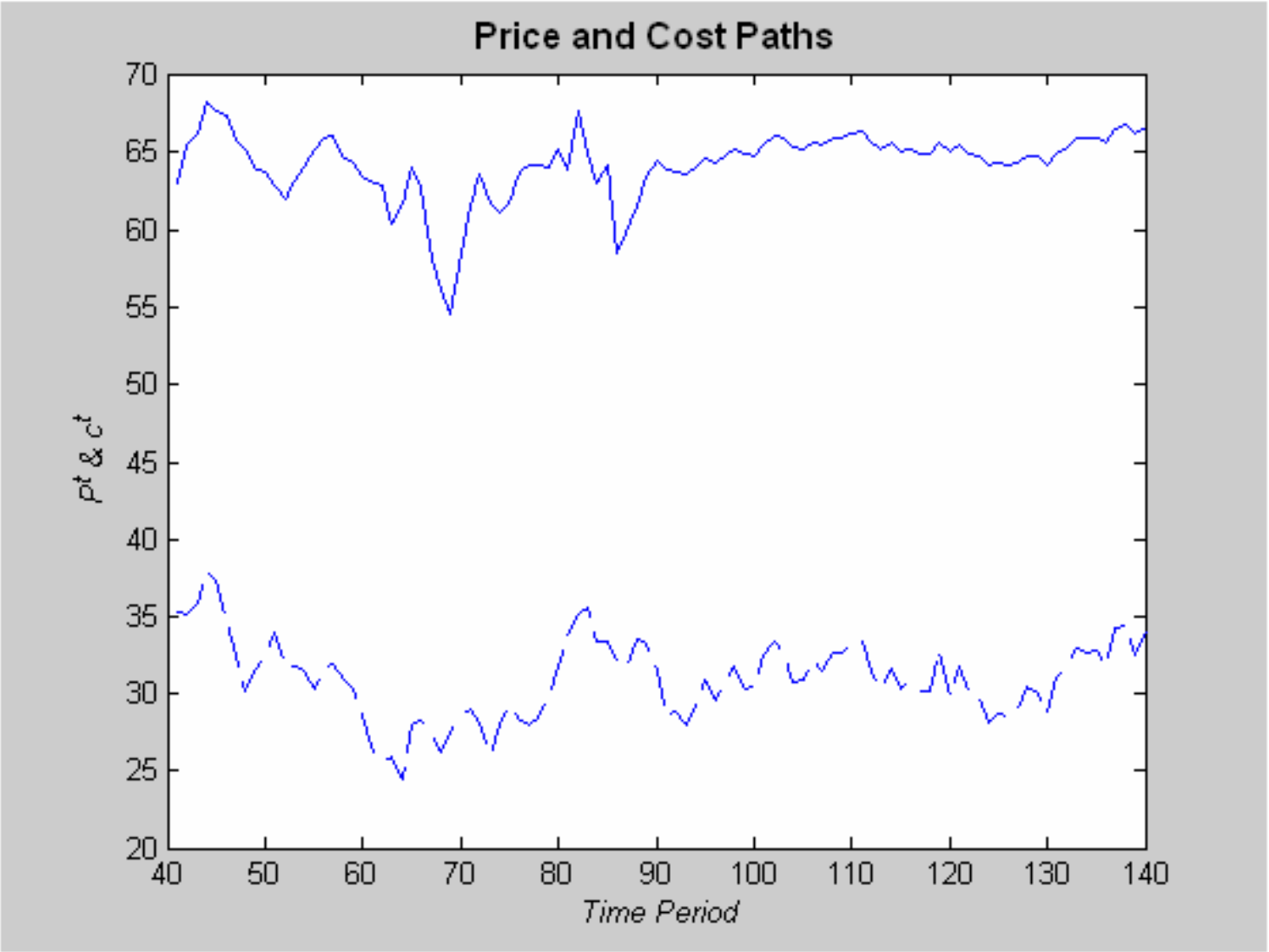
Cartel Formation

- Correlation between cost and the intensity of competition at the time of cartel formation.

	Parameter Specifications									
	PCI	PCII	PCIII	PCIV	PCV	PCVI	PCVII	PCVIII	PCIX	PCX
Correlation Coefficient	0.6288	0.7250	0.6385	0.7038	0.5701	0.7486	0.6669	0.6791	0.6925	0.7755
(p-value)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

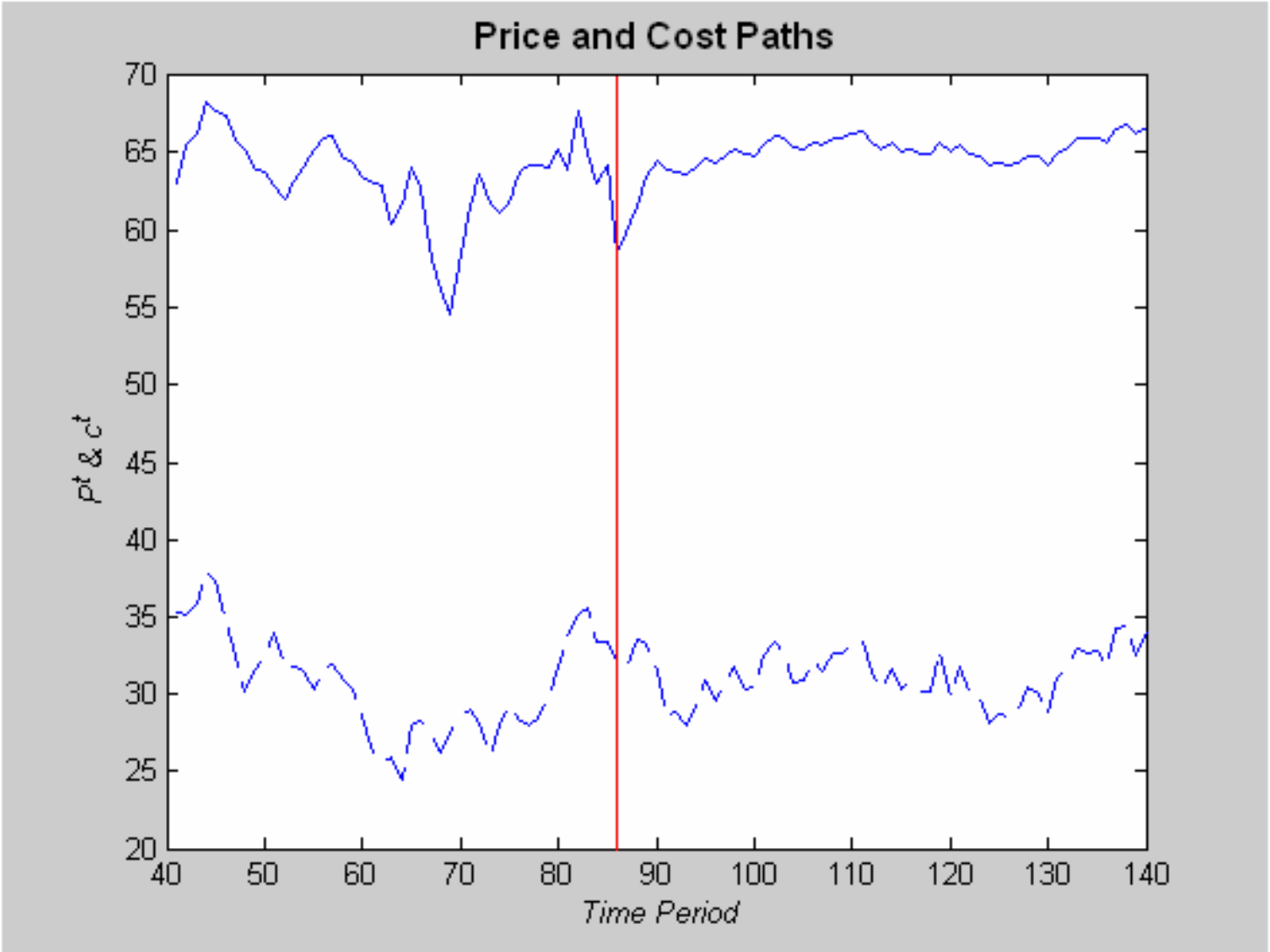
Cartel Formation

Cartel price paths



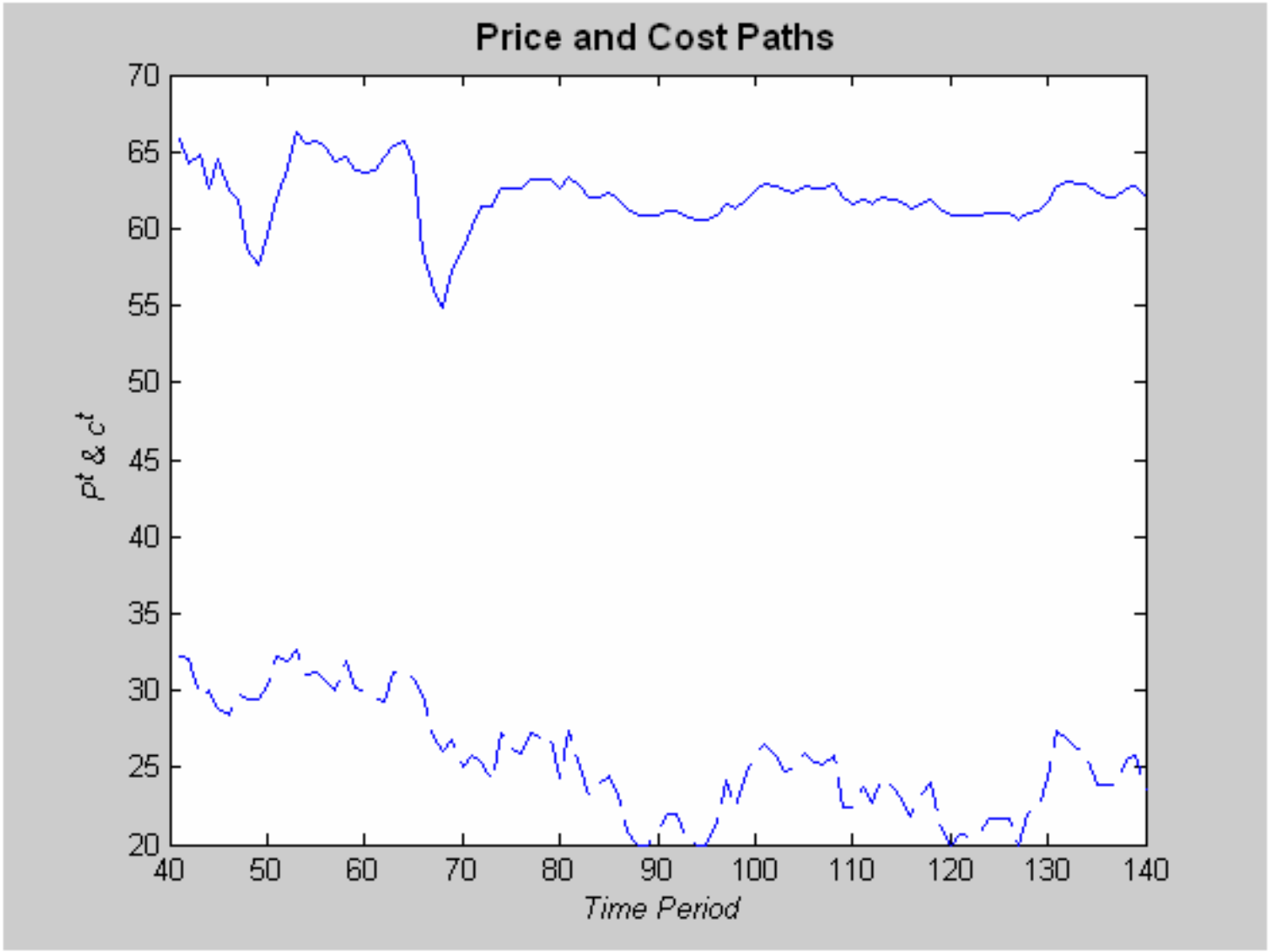
Cartel Formation

Cartel Price Paths



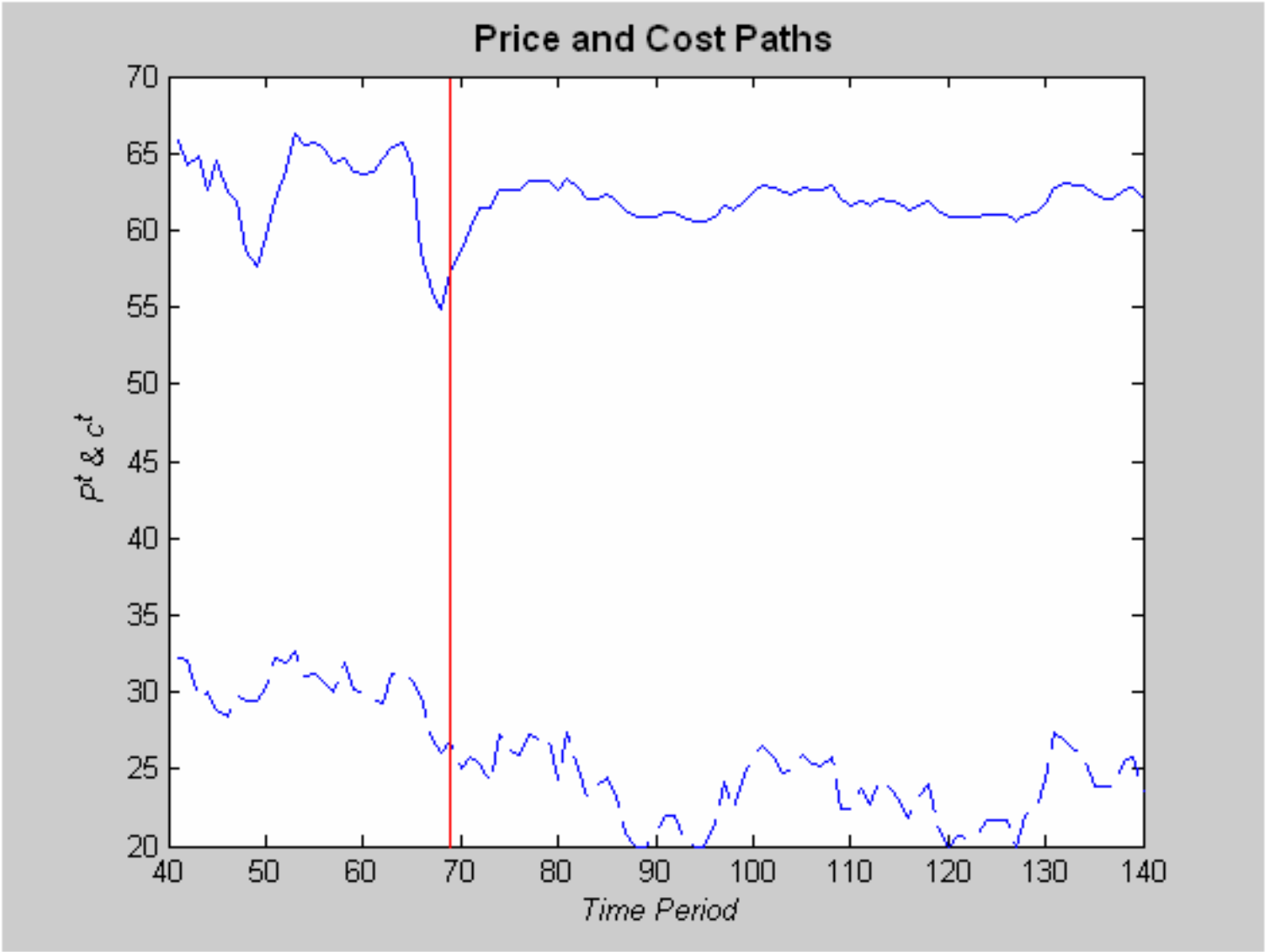
Cartel Formation

Cartel Price Paths



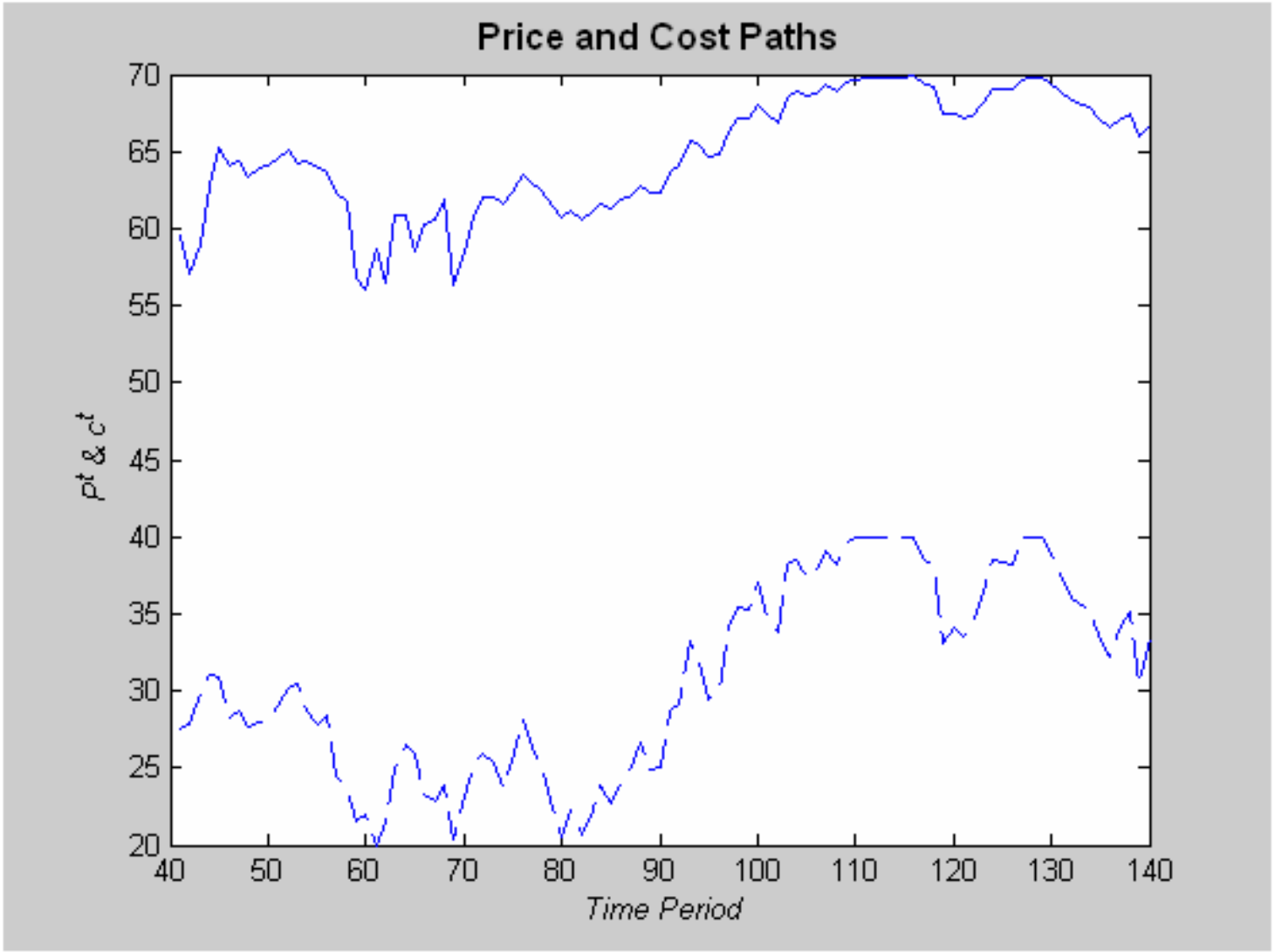
Cartel Formation

Cartel Price Paths



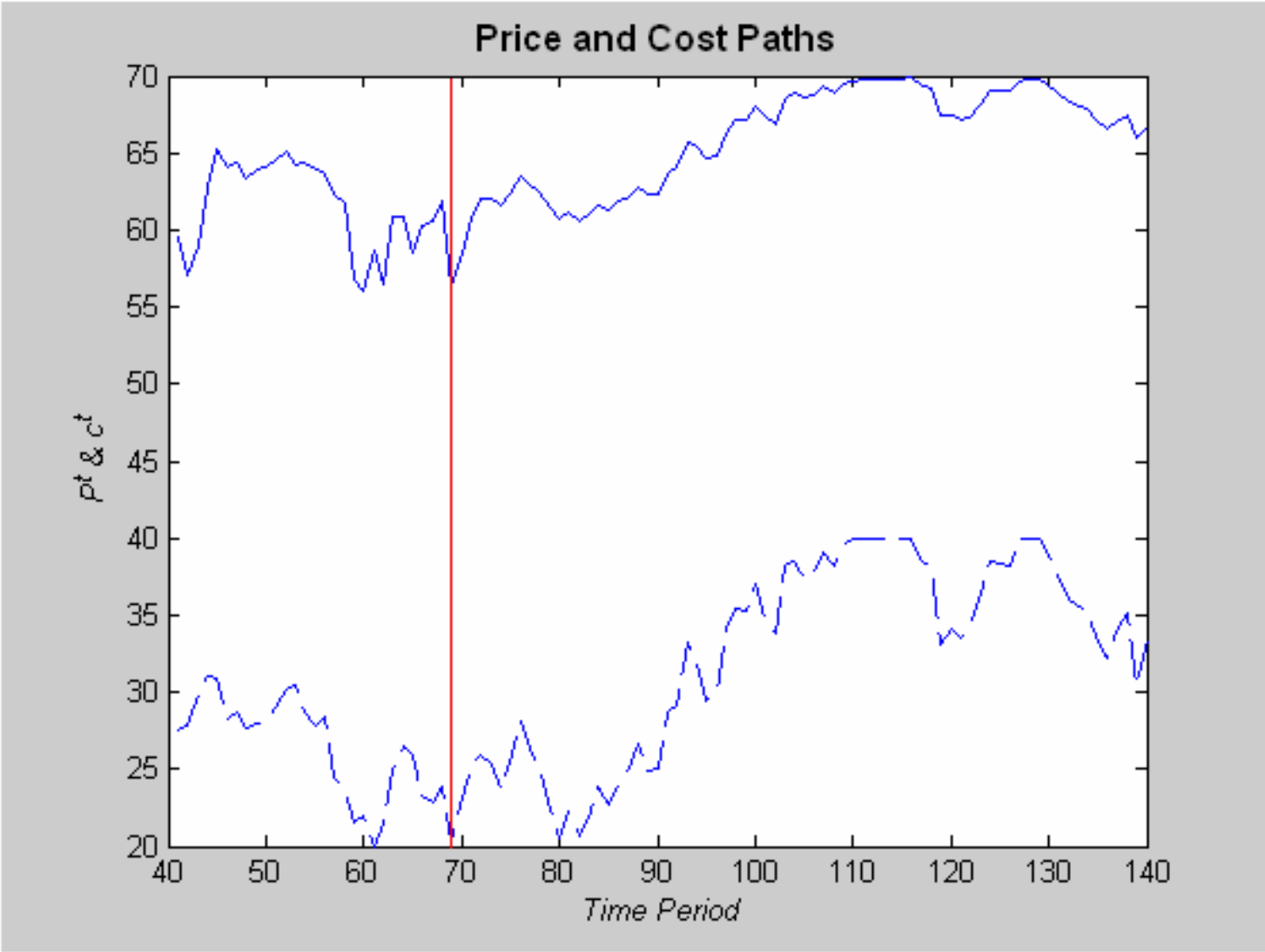
Cartel Formation

Cartel Price Paths



Cartel Formation

Cartel Price Paths



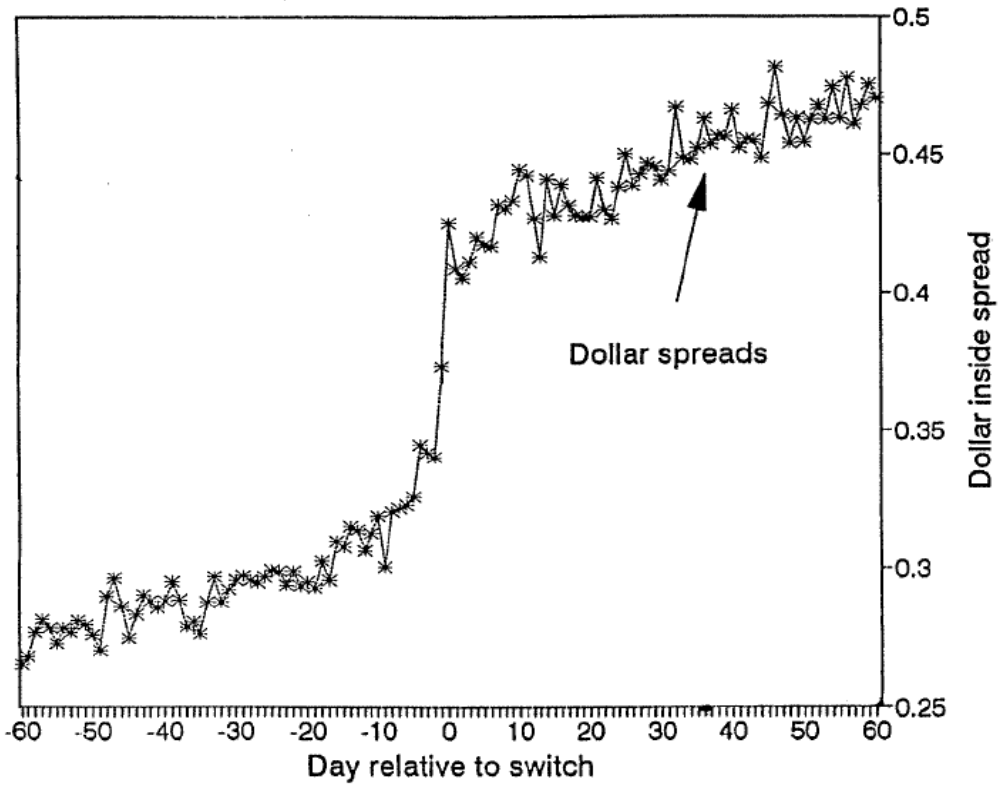
Concluding Remarks

- Next step
  - Develop data base of simulated cartel price paths.
  - Experiment with various econometric tests for identifying point of cartel formation.
    - Continuous monitoring for a structural break.
    - Runs tests (for consecutive price increases).
- Modelling collusion for the purpose of screening should
  - use the rich details we have about hard core cartels
    - European Commission decisions are informative.
    - "How Do Cartels Operate?" *Foundations and Trends in Microeconomics* (Harrington, 2006)
  - focus on *explicit* collusion.
    - Model avoidance of detection.
    - Model communication.

Concluding Remarks and Future Directions

Nasdaq: Collusion and Bid-Ask Spreads

- Could screening have identified price-fixing in Nasdaq?



Christie and Schultz (1999)