Fluidity of Social Norms in a Hierarchical System*

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Abstract

This paper explores how rigid norms - that involve agent behavior not responding to the environment - can persist in response to permanent changes in the aggregate environment. Agents deploying a flexible rule are shown to be crucial in the system transiting between rigid norms in response to such changes. A rigid norm is found to be more prevalent when aggregate changes are less frequent, larger, and more abrupt.

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1 Introduction

Why do some governments fail to respond to large economic and social changes and ultimately lose power? Why are some elected officials unresponsive to their constituents? Why do some corporations remain rigid in their practices as market conditions change? Such ossification may reflect the embodiment of a norm of being resistant to change, of acting in the same manner regardless of the state of the environment. But what are the processes that would generate such a norm of behavior? Are certain features of the external environment conducive to people being unresponsive to a change in their circumstances? What about characteristics of the internal structure of the social system?

To gain some preliminary insight into these issues, a series of simple models have been developed that, while not yet accurate representations of governments, political systems, or corporations, do embody some essential features of those systems. One such feature is their hierarchical structure. Agents who hold important high-level positions are the product of an implicit selection process that chose them from the masses at the lower levels. A second feature is that, like any type of society, agents with status are imitated by young agents in their striving to succeed within the system. In the context of a hierarchy, status is closely related to rank. There is then social learning between new agents and high-ranking old agents. With these two dynamics - selection and social learning - a number of interesting insights have been derived.

In Harrington (1998), the social selection process is examined to understand what types of individuals are selected for advancement within a hierarchy. At each level, agents compete with one another for advancement with relative performance determining who advances. After observing the realization of a stochastic environment, each agent selects an action. Some agents are endowed with a flexible rule - they choose the action that is best for that environment - while other have a rigid rule - they choose the same action irrespective of the environment. Performance depends on the appropriateness of one's action for the current environment and how proficient one is with that action. Proficiency is presumed to come from past experience as a result of such processes as learning-by-doing.[1] It is shown that if a hierarchical social system has sufficiently many levels then its highest levels are dominated by agents whose behavior is relatively unresponsive to the environment.[2]

The preceding analysis yielded some insight about the tendencies of those at high levels to be rigid. It cannot, however, help us understand how a mode of behavior can

[1] Another interpretation is that agents choose messages (for example, stances on political issues) and consistency in one's messages (for example, with respect to a political ideology) lends advantage through enhanced credibility.

[2] An equilibrium approach to these issues is examined in Harrington (1999b). Strategic behavior is shown to be a counteracting force as it results in agents being more flexible as they approach the top level.

become ubiquitous throughout a society or, in other words, become a social norm. To do that requires taking account of how rules get passed from agent to agent. This social learning feature is modelled in Harrington (1999a) along with social selection. In that set-up, new agents enter the system at the bottom rung. At that point, they make an initial adoption decision - whether to deploy a rigid rule or a flexible rule in trying to scale the hierarchy. This decision is determined by two factors: an innate tendency to be rigid or flexible and imitation of those who have been successful. Specifically, a new agent randomly selects a mentor from the highest level and learns that mentor's history of actions and environments though, quite importantly, does not observe his behavioral rule. He then attempts to draw inferences about his mentor's underlying behavioral rule and uses those inferences in deciding on a rule to adopt. After adopting behavioral rules, this population of new agents is subject to the social selection process. This is then a feedback system whereby new agents arrive, imitate old successful agents, and out of that cohort of new agents comes a new set of role models who are then imitated by the next generation of new agents and the process continues. Agents at the top not only affect the rigidity of the system by themselves being rigid or flexible but may have a pervasive and lasting influence through what the next generation infers about "what it takes to move up." It is shown that a norm of being unresponsive to one's environment is more prevalent and robust than one of being adaptive. The reason is that, among those agents who get to the top, it is easier for new agents to infer their mode of behavior if they used a rigid rule than if they used a flexible rule. In the terminology of Boyd and Richerson (1985), a rigid rule is of "higher fidelity" than a flexible rule and this makes it more imitable. The source of the lower fidelity of a flexible rule is that selection tends to promote those flexible agents whose history is consistent with them using a rigid rule while it does not tend to promote those rigid agents whose history is consistent with them using a flexible rule. It is then easier for new agents to identify those role models who deployed a rigid rule and this results in a higher fraction of them adopting a rigid rule. In brief, a rigid rule is more imitable than a flexible rule.

In the preceding model, an individual's environment was allowed to stochastically change. However, the aggregate environment was kept fixed in the sense that the frequency of environments for the population of agents did not change. Or, more to the point, what was the most common environment was fixed over time. As a result, the rigid rule that performed best on average did not change. This property would seem quite important for the stability of a rigid norm in that a rigid norm is always the rigid rule that performs best, on average. If the aggregate environment changes (and with it what is the best performing rigid rule) then an existing rigid norm may become quite dysfunctional and this could allow a successful invasion by a flexible rule. It is then not clear that we would observe a rigid norm if there are aggregate or large-scale changes. The objective of this paper is to explore that issue.

While aggregate fluctuations do indeed make it more difficult for rigid norms to occur, a rigid norm is found to be capable of surviving such shocks. Under certain conditions, rigid norms are quite resilient. Though, when a rigid norm is in place, few agents may be deploying a flexible rule, those agents are shown to have a very important role in the persistence of rigid norms. Specifically, agents deploying a flexible rule are instrumental in transiting between rigid norms in response to changes in the aggregate environment. The analysis also shows that a rigid norm is more likely to occur when large-scale changes are less frequent, bigger in size, and more abrupt.

While, to our knowledge, this line of research is unique in asking how a norm of being resistant to change can develop in a social system, research on cultural transmission is similar in its interest in how behavioral norms develop. This work includes Cavalli-Sforza and Feldman (1981), Boyd and Richerson (1985), Harrison and Carroll (1991), Epstein and Axtell (1996), and Bisin and Verdier (1997). Relatedly, there has been considerable work in economics on social learning. Our modelling approach differs in both the space of traits over which learning occurs and in the transmission process. Previous work on social learning is concerned with learning about actions. This may involve agents learning from the actions and associated payoffs of others — as in Ellison and Fudenberg (1993,1995), Rhode and Stegeman (1993), and Bala and Goyal (1994) — or may have agents learning only from the population distribution of actions — as in Banerjee (1992,1993), Bikhchandani et al (1992), Bergstrom and Stark (1993), Kirman (1993), Vega-Redondo (1993), Banerjee and Fudenberg (1995), and Smith and Sorensen (1996). In contrast, our focus is learning about rules, learning about how to respond to one's environment. This is a substantive difference in that rules are not directly observable, at best one observes realizations of a rule as it interacts with the environment. In terms of modelling the transmission process, previous research has modelled *horizontal* transmission in that agents are learning from their peers. Our transmission process is *vertical* in that it is predicated upon a hierarchical society in which agents learn from those above them. Vega-Redondo (1998) also encompasses vertical transmission in an evolutionary-style model. We feel this is a critical and largely unexplored feature of such social systems as organizations and, more generally, ones in which status is important. In terms of the process by which traits are transferred between agents, it is standard in previous work to assume that imitation is perfect. A central assumption in our model is that an agent's behavioral rule is private information so that imitation is problematic. Novel within the social learning literature, the imitability of an agent's trait is endogenized.

2 A Dynamical Model of a Social System

Our formulation of the process determining behavioral norms in a social setting is comprised of four elements. A description of an agent's meta-environment and how it changes. The space of behavioral rules over which the population is evolving. The selection process by which agents compete and social status is determined. And the social learning process by which traits are transmitted across generations.

2.1 Structure of Society

The most crucial element of our conception of a society is that it is hierarchical. This means that it is comprised of a set of $k(\geq 2)$ ordered levels that run from a "lowest" level, denumerated level 1, to a "highest" level, denumerated level k. At each level there is a large (countably infinite) population of agents. A key presumption is that agents strive to advance to higher levels. Whether it is a politician trying to advance from the state legislature to the House of Representatives or a regional manager striving to become a vice-president in a corporation, advancement typically requires performing relatively better than a subset of one's peers who faced comparable circumstances. We model this process by assuming that, at each level, agents are randomly matched into pairs. Each of these matchings is faced with a stochastic environment. Once the environment is revealed to the agents, they choose actions. The agent with greater relative performance is promoted to the next level while the other agent is assumed to exit the system. Though this "up-or-out" structure is extreme, casual observation suggests that most candidates who lose do not run again while those corporate employees who are "passed over" when their time has come may no longer be on the "fast track" which makes them less likely to be considered for promotion.[3] Eventually, each agent "expires" in that we constrain agents to being in the system for at most k periods where a period equals the length of time spent at a level. A generation equals the length of a single lifetime (which is k periods). The first generation of the model then encompasses periods $1, \ldots, k$, the second generation is periods $k+1, \ldots, 2k$, and so forth.

At each level, agents face two random events: the agent with whom they compete for advancement and the external environment. The latter is assumed to be of two possible types, $\{0, 1\}$, where the probability an agent faces environment 1 in generation g is denoted b^g. An agent's environment is assumed to be independently distributed across levels. Though there is individual uncertainty, there is no population uncertainty in the sense that, at every level, a proportion b^g of all matchings have environment 1. This lack of population uncertainty greatly simplifies the analysis. In responding to one's environment, agents have two feasible actions at their disposal, $\{0, 1\}$. In a manner to be described below, action 0 (1) is the best myopic response to environment 0 (1). When $b^g > (<) 1/2$, action 1 (0) is then more frequently the appropriate response to the environment and, in that sense, is the best overall action.

The aggregate environment is subject to change in the form of movements of b^g, the frequency with which environment 1 occurs. This we model by assuming that it deterministically oscillates between $b \in (0, 1)$ and $1 - b$ with the switch occurring

[3] Schlesinger (1991) documents the progressive paths taken to higher office while Rosenbaum (1984) documents the fast-track feature of corporations.

every T generations.

$$b^g = \begin{cases} b \text{ if } g \in \{1, \ldots, T, 2T+1, \ldots, 3T, 4T+1, \ldots\} \\ 1-b \text{ if } g \in \{T+1, \ldots, 2T, 3T+1, \ldots, 4T, 5T+1, \ldots\} \end{cases}$$

Note that the time between changes in the aggregate environment is at least one generation. If change occurs within generations then imitation can be quite dysfunctional as new agents are routinely imitating agents who were successful for a different aggregate environment. In that case, preliminary analysis suggests that a flexible rule prevails as a norm. Thus, we limit our attention to inter-generational changes where it is not so clear as to what norms will develop.

2.2 Space of Behavioral Rules

Associated with each agent is a behavioral rule. For simplicity, we limit our attention to behavioral rules that condition only on the current environment. It is then the set of functions that map the set of environments, $\{0, 1\}$, into the set of actions, $\{0, 1\}$, with the exception that we exclude the pathological case of always choosing an action inappropriate for the current environment. We believe this simplifies the analysis without any loss of generality. A flexible agent is defined to be one who always selects the action best suited for the environment: he chooses action 0 (1) when the environment is 0 (1). A rigid agent chooses the same action irrespective of the environment. There are two types: those who always use action 0 and those who always use action 1.

As agents advance up through the system, they accumulate a personal history. Though there are many history-dependent types at any given level in the system, it is sufficient for our purposes to partition the population into the following five types: rigid agents endowed with action i (denoted Ri), $i \in \{0, 1\}$; flexible agents who have always chosen action i (denoted Fi), $i \in \{0, 1\}$; and flexible agents who have chosen both action 0 and action 1 (denoted FN).

2.3 Selection

The process by which people advance in this social system and thereby gain social status is specified as follows. If the two matched agents choose distinct actions then the agent whose action matches the environment survives and advances to the next level. If both agents select the action which matches the environment, the agent who has chosen that particular action more frequently in the past advances. If they have chosen that action equally frequently then an agent is randomly selected to survive. Who advances if both agents choose the less appropriate action need not be specified as the set of equations which describe the population dynamics is independent of it.

There are several notable features of this selection process. First, competition for advancement is local in that an agent competes with only one other agent (as opposed to competing with the population at large). Second, advancement depends only on an agent's current performance where, implicitly, an agent's current performance is determined by the current environment, his current action, and his proficiency with the action used. Third, proficiency comes from experience, from having done the same thing over and over. Note that survival depends lexicographically on one's current action and one's experience with that action so that the incremental effect from choosing a better action exceeds the incremental effect from more experience.[4]

2.4 Social Learning

The state of the social system is represented by k populations, one at each of the system's k levels. At the end of each period, those who were at the highest level are assumed to exit the system (there is "mandatory retirement" after k periods in the system). For the population at level $h \in \{1, ..., k-1\}$, half of them advance to level $h+1$ according to the selection process described above while the other half exit the system. A fresh cohort of new agents is assumed to enter level 1.

The behavioral rule that a new agent adopts is determined by a confluence of two forces: imitation of those who were successful and natural predisposition. There are two aspects to imitation: identifying who is worth emulating and inferring their code of conduct. The former is relatively easy as it only requires identifying people with high rank. The latter is problematic. An agent does not wear her behavioral rule on his chest and one cannot look into the heart of another. What an agent is presumed capable of doing is observing the past behavior of an agent and the context in which she had to act. The following imitation process is specified. Each new agent randomly chooses one agent from the current level k population (those with the highest social status) to act as his role model or mentor. We do not have them observe the entire population because limited information seems more natural. An incoming agent observes his mentor's history in terms of the actions she selected and the environments she faced. If a new agent's mentor always chose the same action and, furthermore, chose this action when it was inappropriate for the environment, this is unequivocal evidence that the mentor is rigid in that action. In that case, the new agent adopts that rigid rule. If a new agent's mentor chose both actions

[4] Note that there is presumed to be no cost to being flexible. There could be an *ex ante* cost to being flexible (for example, becoming familiar with both actions) and/or an *ex post* cost incurred whenever one switches actions. One way in which to model an *ex ante* cost is as follows. If a rigid agent and flexible agent both choose the same action and have the same proficiency then the rigid agent is promoted with probability $q \in \left(\frac{1}{2}, 1\right)$. In that the current model assumes $q = \frac{1}{2}$, this specification would seem to enhance the survivability of agents who deploy a rigid rule. In a sense, the model has an implicit *ex post* cost to being flexible in that an agent is less proficient with some actions.

during her lifetime then this is unequivocal evidence that she is flexible. A new agent then adopts the flexible rule. The problematic case is when a new agent's mentor always chose the same action but always faced the environment for which that action was appropriate. Such behavior is consistent with both flexible and rigid rules. We assume that new agents have natural predispositions to being flexible or rigid and this predisposition breaks the indeterminacy. A proportion $w \in [0,1]$ of new agents are predisposed to being rigid which means that they adopt a rigid rule in that case. A proportion $1 - w$ are predisposed to be flexible which means that they adopt a flexible rule in that case. A new agent then adopts the rule that he is predisposed to unless the behavior of one's mentor is evidence to the contrary. One could imagine w being determined by what types of norms are prevailing outside of this particular social system. Once a new agent adopts a rule, it persists with him throughout his time in the system.

In this section, we have provided a narrative description of the dynamical system while the Appendix provides a formal definition. Due to its complexity, numerical analysis is used. The model in Harrington (1999a) is a special case of this model when $T = +\infty$ so that the aggregate environment does not change.

3 Norms in a Fixed Aggregate Environment

Prior to examining the general model, results for the special case of a fixed aggregate environment are reviewed. Specifically, it is assumed that $T = +\infty$ and $b^g = b > \frac{1}{2}$ for all g. After reviewing what happens within the hierarchy during a single generation (Harrington, 1998), social learning is introduced and the development of norms across generations is examined (Harrington, 1999a).

3.1 Selection of Rigid Agents

The purpose of this section is to briefly review some properties of the selection process that operates within the k-level system and which was examined in Harrington (1998). To begin, the proportion of agents who are maximally proficient in action 0 (which either means that they are rigid in action 0 or they are flexible and have always faced environment 0) steadily and rather rapidly goes to zero after level 2. Since environment 0 occurs relatively infrequently, such agents are ill-suited for this meta-environment. All of the meaningful dynamics then take place with respect to $F1$s (flexible agents who are maximally proficient in action 1 because they have always faced environment 1), FNs (flexible agents who are not maximally proficient in either action because they have faced both environments), and $R1$s (agents who are rigid in action 1). At low levels, with agents having only faced a few environments, there will be a reasonable number of flexible agents who have only faced environment 1.

Next note that when an $F1$ and an $R1$ meet to compete for advancement, an $F1$ advances with probability $\frac{b}{2} + (1 - b) > \frac{1}{2}$ because, when the environment is 1, he is equally proficient in action 1 to an $R1$ and, when the environment is 0, he adapts and chooses action 0. This differential advantage provides a force by which the proportion of flexible agents can rise. However, as flexible agents rise through the hierarchy and face more environments, an increasing proportion of them will have faced environment 0 and thus be less proficient in the better action than $R1$s. This lack of proficiency puts them at a disadvantage compared to those surviving agents who are rigid in action 1. Though $R1$s may initially be driven down, their differential proficiency in action 1 becomes increasing scarce among flexible agents so that rigid agents eventually come to dominate.[5]

Using a phase diagram analysis, several possible paths for a population are derived. First, the proportion of flexible agents steadily rises as a cohort moves from the bottom to the top of the hierarchy. This requires that k be sufficiently small. Second, the proportion of flexible agents rises as one goes from low to moderate levels but falls as one goes from moderate to high levels. This can occur when k is sufficiently large. Third, the proportion of rigid agents (who use the action most appropriate for what is the most common environment) steadily rises as one moves up the hierarchy. This is not so much dependent on k but rather on the initial population mix. While there are other possible paths, simulations show that these are, by far, the most common.[6]

Interpreting this model in the political context, let the environment be the policy preferences of voters, agents be politicians, and the action chosen be a politician's position on the issues. The hierarchy reflects the property that political aspirants might start by running for low level office and if they succeed by winning office then they become eligible to run for the next higher level office. A rigid agent can be thought

[5]It is shown in Harrington (1998): If $r_1^{1,g} > 0$ and $b^g > \frac{1}{2}$ then $\lim_{k \to \infty} r_1^{k,g} = 1$. The definitions of these variables are in the Appendix.

[6]As noted by Scott Page, this dynamic has the peculiar property that promotion may be *negatively* correlated with cumulative past performance; that is, agents whose cumulative performance is lower may be promoted at a higher rate. The argument is as follows. Promotion is based on current performance. Current performance is enhanced through greater proficiency with action 1 (what is most frequently the best action). Proficiency with action 1 is enhanced through having chosen it more often in the past. This implies that current performance is maximized by having always chosen action 1 in the past. For the same history of environments (and where both environments are realized), cumulative past performance for an agent who always chose action 1 is lower than that of an agent whose action always matched the environment because the incremental value from choosing the best action is assumed to be infinitely larger than being more proficient with an action. We then find that agents with a history of having always chosen action 1 will (eventually) be promoted at a higher rate than agents with a history of having matched the environment even though the latter has higher past performance. This property is partially due to assuming that promotion is based only on current performance but also partly an artifact of the simplifying assumption that choosing a better action is infinitely better than being more proficient with an action. If we made this effect finite then eventually the past performance of rigid agents would overtake agents who deploy a flexible rule as rigid agents come to reap the benefits of investing in proficiency.

of as an ideologue in that his positions are unresponsive to voters' preferences while a flexible agent is an office-seeker as his positions are tailored to match those policies supported by voters (which also serves to maximixe the probability of winning office). Given this interpretation, Harrington (1998) shows that ideologues can survive and dominate high-level offices. A closely related model in Harrington (2000) is specifically tailored to the political setting and it shows how pure office-seeking politicians can end up looking like ideologues.

3.2 Stability of Rigid Norms

Let us now consider the development of stable norms when both selection and social learning are operative but when the aggregate environment is fixed. This is the special case of our model when $T = +\infty$ and was examined in Harrington (1999a). For our discussion, suppose $b^g = b > .5$ for all g so that environment 1 is always the most frequent environment and thereby action 1 is always the best single action.

A behavioral rule is said to be a *locally stable norm* if there exists a local attractor for which a large fraction of agents use that rule. The key finding is that a rigid rule is more prevalent than a flexible rule as a locally stable norm. When enough new agents are predisposed to a particular mode of behavior, it is generally the case that that mode of behavior can dominate. In other words, for a wide range of values for (b, k), it is a local attractor for a high fraction of agents to use a rigid (flexible) rule when enough incoming agents are predisposed to be rigid (flexible); that is, w is close to 1 (0). The more problematic and interesting case is when new agents are not predisposed to a rule. How likely is it that such a rule can dominate? Here we found a notable asymmetry: a rigid rule fares much better than a flexible rule when faced with an incoming population which is biased against it. It is common for a rigid rule to be a locally stable norm regardless of the value of w. Even when $w = 0$, so that all incoming agents are predisposed to be flexible, there is a local attractor with a high fraction of agents using a rigid rule for a wide range of values for (b, k). In contrast, a flexible rule is generally not a locally stable norm when most incoming agents are predisposed to be rigid (that is, w is sufficiently close to one).

Selection favors those agents who have always chosen action 1 for then they are maximally proficient in the action that is most frequently the best response to the environment. For agents who use a flexible rule, this implies that selection favors those who have always faced environment 1. Hence, many of those agents who use a flexible rule and rise to the top are indistinguishable from agents who use a rigid rule. This inability to differentiate themselves results in those new agents who have an innate bias to be rigid adopting a rigid rule and, in essence, "hard-wiring" their mentor's history of having always chosen action 1. When there are enough new agents predisposed to be rigid, a flexible norm is unlikely to develop. In contrast, since rigid agents (in action 1) are proficient in action 1 regardless of the history of their environments, selection does not favor those rigid agents who have exclusively faced environment 1. This allows them to differentiate themselves from agents who use a flexible rule and is why a rigid rule is a locally stable norm even when all new agents are predisposed to be flexible.

For a behavioral rule to thrive, it need not be sufficient that those agents deploying that rule rise to the top but what may also be required is that they have differentiated themselves from those who use other rules. A rigid rule does this more effectively than a flexible rule. In the terminology of Boyd and Richerson (1985), a rigid rule has "higher fidelity" than a flexible rule and this makes it more likely to become ubiquitous.

4 Norms in a Changing Aggregate Environment

In this section, T is finite so that what is the most common environment changes across generations. The main objective is to understand how such large-scale changes affect the stability of rigid norms. In conducting numerical analysis, values must be specified for the system's seven variables: the four parameters defining the meta-environment, (b, k, w, T), and the three initial conditions, (r_1, r_0, b^1), where, in the initial generation, r_i is the proportion of agents using a rigid rule in action i at the system's first level and b^1 is the frequency with which environment 1 occurs.[7] Without loss of generality, $b^1 = b$. Simulations were run for $(b, w, T) \in \{.05, .10, ..., .95\} \times \{0, .1, ..., 1\} \times \{1, 2, \ldots, 20\}$ and: i) $k \in \{5, 10, \ldots, 25\}$ and $(r_1, r_0) \in \{(.25, .25), (.33, .33)\}$; and ii) $k \in \{5, 10\}$ and

$$(r_1, r_0) \in \{(.05, .05), (.25, .25), (.33, .33), (.45, .45), (.05, .45), (.45, .05)\}.$$

In that qualitative results were independent of the initial population, results are reported only for $(r_1, r_0) = (.25, .25)$. For reasons of visual presentation, results are reported for $T \in \{2, 3, \ldots, 20\}$.[8] When $T = 1$, so that what is the most common environment changes once every generation, a flexible rule prevails because social learning is dysfunctional; new agents are always imitating agents who were successful for when a different aggregate environment prevailed. Finally, note that the cases

[7] Actually, there are $k-1$ initial populations for we start with an empty hierarchy. An exogenous population enters level 1 in period 1 with population mix (r_1, r_0). In period 2, this cohort moves up to level 2 and a new population enters level 1. However, if $k \geq 3$ then there is no level k population for them to imitate so we assume that this new population is also exogenous with mix (r_1, r_0) (though all results are robust to the mix being not too different from (r_1, r_0)). Come period k, the population that entered level 1 in period 1 will have reached level k so that the new population entering level 1 will have a level k population to imitate. From period k onward, the entering level 1 population will be endogenous as it is determined by the level k population and w through (11)-(12) (see the Appendix).

[8] Inclusion of $T = 1$ tends to obscure viewing portions of the three-dimensional surface.

of, say, $b = .6$ and $b = .4$ both involve a cycle in which the frequency with which environment 1 occurs alternates between .6 and .4. What differs between them is the starting value which is .6 when $b = .6$ and .4 when $b = .4$.

In that the aggregate environment (specifically, the frequency of environment 1) is subject to a cycle of length $2T$, a local attractor is defined to be a cycle of length $2T$. After specifying values for (b, k, w, T, r_1, r_0), the system was run until convergence was achieved where the criterion is that the state variables between the start of one cycle and the start of the next cycle are sufficiently close.[9] The system almost always converged to a cycle of length $2T$ though there were some incidents of a longer cycle.

To gauge the societal presence of a particular rule in a social system, the proportion of agents using that rule is calculated at each level and averaged across all levels. Let $\rho(Q, h, g)$ denote the proportion of agents using rule $Q \in \{R1, R0, Flexible\}$ at level h in generation g. The presence of rule Q in the social system in generation g is measured by $\phi(Q, g) \equiv \left(\frac{1}{k}\right) \sum_{h=1}^{k} \rho(Q, h, g)$.[10] Assume the system has converged by generation g'. The presence of a rule at a local attractor is then measured by $\Phi(Q) \equiv \left(\frac{1}{2T}\right) \sum_{g=g'}^{g'+2T-1} \phi(Q, g)$ which is the proportion of agents using rule Q averaged over all levels for a given generation and then averaged over the $2T$ elements of a cycle. Also reported is the range of values over a steady-state cycle:

$$Range(Q) \equiv [\min\{\phi(Q, g'), \ldots, \phi(Q, g' + 2T - 1)\}, \max\{\phi(Q, g'), \ldots, \phi(Q, g' + 2T - 1)\}]$$

4.1 Properties of Attractors

In Figure 1-3, the height of a surface is $\Phi(Flexible)$. Its dependence on T and b is depicted. Figure 1 is for when the system has five levels while Figures 2 and 3 consider systems with 10 and 15 levels, respectively. First note that these figures appear symmetric around .5 which indicates that where one starts a cycle is not important for the long-run properties of the system. The following property is gleaned from these figures and the results for other values of k.

Property 1: When enough new agents are predisposed to be flexible (w is sufficiently close to zero), a flexible rule is a locally stable norm and a rigid rule is not. When enough new agents are predisposed to be rigid (w is sufficiently close to one), a rigid rule is a locally stable norm when an individual's environment

[9] The system is said to have converged by generation g if $\left|r_1^{1,g} - r_1^{1,g+2T}\right| < .000001$ and $\left|r_0^{1,g} - r_0^{1,g+2T}\right| < .000001$ (where the definitions for these variables are in the Appendix).

[10] Of course, $\phi(Q, g)$ is not the proportion of agents in the system using rule Q as there are more agents at lower levels than higher levels. This would argue to having a measure which gives more weight to lower levels. On the other hand, agents at higher levels have more power so each high-level agent is equivalent to several low-level agents. Given these two countervailing forces, we chose to give all levels equal weight. We do not believe qualitative results are sensitive to the measure used.

is relatively stable (b is near 0 or 1) and a flexible rule is a locally stable norm when an individual's environment is relatively volatile (b is near $\frac{1}{2}$).

When there are no aggregate fluctuations (that is, what is the most common environment does not change over time), Harrington (1999a) found that a rigid rule could easily be the prevailing norm even when all incoming agents were predisposed to be flexible. By Property 1, this does not occur when what is the most common environment changes. In that case, the identity of the best rigid rule changes across generations. Hence, a rigid rule that became a social norm when, say, environment 1 was most common would be highly inadequate if environment 0 became most common. If sufficiently many new agents are biased to be flexible, a rigid rule would not survive such shocks and a flexible rule would be the locally stable norm. A necessary condition for a rigid norm to develop is that there are sufficiently many new agents whose inclination is to be rigid. Given that does hold, Figures 1-3 show that it is quite common for most agents to be deploying a rigid rule, on average. Thus, a rigid norm can survive aggregate fluctuations.

What are the dynamics that allow a rule of being locked into a particular action to persist even when what is the appropriate action to be locked into changes over time? One possibility is the following. Suppose $b^g = b > \frac{1}{2}$ and further suppose that a rule of always choosing action 1 (R1) is thriving. With environment 0 being relatively infrequent, those agents who use a rule of always choosing action 0 (R0) do quite poorly in progressing up the hierarchy. Still, there will be a latent population of such agents in that some agents at the top will either have used or look like they have used an R0 rule. Some agents who use an R0 rule will rise to the top due to either always having always faced environment 0 or having always been matched with another agent using rule R0. Some agents who use a flexible rule will rise to the top having always chosen action 0 due to having always faced environment 0. With agents who have always used action 0 as role models, some new agents will adopt an R0 rule so that there are always such agents in the population. When b^g switches to $1 - b$, so that the best action is action 0, one possibility is that the latent population of agents who use a R0 rule thrives and ultimately dominates the set of agents at the top. In imitating them, the next generation would widely use an R0 rule. However, all this does not happen.

While there is indeed a latent population of agents who use an R0 rule, the latent population of agents who use a flexible rule is much much larger because, when environment 1 is more common, a flexible rule vastly outperforms a rule of always choosing action 0. When b^g switches to $1 - b$, agents who use a flexible rule end up with a much greater share of the top level than agents who use an R0 rule because their presence at the time of the change is proportionately so much bigger. This can be seen in Figure 4a which plots a cycle for $(b, k, w, T) = (.65, 10, .7, 15)$. In the transitional generation (denoted generation 16), the fraction that uses an R1 rule begins to decline. Even though the R1 rule is quite ineffective, there are initially so many agents deploying such a rule that it has a substantive presence for two generations after the transition. During generations 16-18, the fraction using an R1 rule shrinks and the fraction using a flexible rule grows (as they are doing very well against agents who always play action 1). However, the proportion of agents using a flexible rule peaks in generation 18 with about half of the population and steadily declines thereafter until the next large-scale change. These agents who are using a flexible rule are being replaced with agents who use an R0 rule. What is happening is that many of those agents who deploy a flexible rule and get to the top will have done so having always chosen action 0 because they have always faced environment 0. (Recall from Section 3.2 our argument that selection tends to promote those flexible agents who have always faced the most common environment because they are then maximally proficient in what is most frequently the best action.) A new agent who is biased to be rigid and gets one of those agents as his role model will adopt a rule of always choosing action 0. When w is close enough to one, this results in a sizable fraction of new agents deploying an R0 rule even though almost all of their role models use a flexible rule. It is this beachhead that allows rigid behavior to ultimately take over again.

This story is confirmed by examining the relationship between the average (Figures 1-3) and the range (Figures 5-7). When the average proportion of agents using a flexible rule is low, the range is often high. This high range reflects the large presence of flexible agents during the transitional generations and the small presence outside of that transition. This is strikingly clear when $k = 15$ and $w = .2$ (Figures 3 and 7). The average is lowest when b is between .15 and .3 (and .7 and .85) and this is also when the range is highest (thus generating these "cadillac tailfins"). This story also explains why a flexible rule prevails when a large fraction of incoming agents are predisposed to be flexible. Given a mentor who always chose action 0 (because he always faced environment 0), a large fraction of new agents will adopt a flexible rule rather than an R0 rule so that flexibility begets flexibility.

Animation: A QuickTime movie of Figures 2, 3, 5, and 6 - as w is raised from zero to one - is available at:

$$http://www.econ.jhu.edu/People/Harrington/Fluidity.htm$$

4.2 Comparative Statics

Property 2: Less frequent aggregate fluctuations and more severe aggregate fluctuations conduce a rigid rule to be a locally stable norm.

Figures 1-3 reveal that the prevalence of a rigid rule is increasing in both the size of the change, which is larger when $\left|b - \frac{1}{2}\right|$ is larger, and the infrequency of the change, which is increasing in T. The latter is most apparent for values of b that lie at the cusp of a flexible rule being dominant and a rigid rule being dominant. For example, examine the contours around $(b,k,w) \in \{(.4,5,1)\,(.35,10,6)\,,(.35,15,.6)\}$ that hold b fixed. The reason is quite intuitive. Examination of Figure 4a reveals that the proportion of agents using an R1 rule is steadily growing with the number of generations for which environment 1 is most common. The more time until a transition, the more that agents deploying a rigid rule can grow.

A bit more interesting is the result that more severe aggregate fluctuations are conducive to a rigid norm prevailing. By having $\left|b-\frac{1}{2}\right|$ be larger, the rigid rule not in favor is driven down more relative to agents who use a flexible rule. This, however, is not relevant to survival (or, more to the point, revival) of a rigid rule. The persistent prevalence of rigid behavior is not predicated upon the latent population of, say, agents who use an R0 rule thriving when the most common environment switches from 1 to 0 but rather from agents who use a flexible rule thriving when the transition occurs and many of those agents getting to the top looking like they use an R0 rule. A more extreme change then helps the cause of rigidity because it reduces the fidelity of a flexible rule; that is, it reduces the ability of agents who use a flexible rule and rise to the top to differentiate themselves from agents who use a rigid rule. The closer b is to 1 (0), the more likely it is that flexible agents at the top will have only faced environment 1 (0) and chosen action 1 (0) and thereby are observationally equivalent to an agent deploying a rigid rule. This means a higher fraction of new agents (who are predisposed to be rigid) adopts a rigid rule.

4.3 Effects of Gradualism

Thus far, the size and frequency of aggregate changes on social norms have been explored. In this sub-section, the model is enriched to introduce a third characteristic of large-scale change - abruptness. Consider the following intertemporal path for the frequency of environment 1 over a single cycle:

$$b^g = \begin{cases} b \text{ if } g \in \{1,\ldots,T-z\} \\ b-(2b-1)\left(\frac{g-T+z}{1+z}\right) \text{ if } g \in \{T-z+1,\ldots,T\} \\ 1-b \text{ if } g \in \{T+1,\ldots,2T-z\} \\ 1-b+(2b-1)\left(\frac{g-2T+z}{1+z}\right) \text{ if } g \in \{2T-z,\ldots,2T\} \end{cases}$$

This cycle repeats itself every $2T$ generations. The parameter space is now (b,k,w,T,z) where $z \in \{0,1,\ldots,T-1\}$ is the number of generations in the transition of b^g from b to $1-b$ (and similarly from $1-b$ to b). The preceding model is the special case of $z=0$. For example, let $b=.8$ and $T=10$. If $z=1$ then a cycle has b^g equalling .8 for nine generations, .5 for one generation, .2 for nine generations, and .5 for one generation. If $z=2$ then b^g equals .8 for eight generations, .6 for one generation, .4 for one generation, .2 for eight generations, .4 for one generation, and .6 for one generation. Raising z then eases the transition.

We examined the dependence of $\Phi\left(Flexible\right)$ and $Range\left(Flexible\right)$ on $z \in \{0, 1, \ldots, T-1\}$ for $(b, k, w, T) \in \{.05, .10, ..., .95\} \times \{5, 10, 15\} \times \{0, .1, ..., 1\} \times \{10\}$ and $(r_1, r_0) = \{(.05, .05), (.25, .25), (.45, .45), (.05, .45), (.45, .05)\}$. We also considered $T \in \{5, 15\}$ and $(r_1, r_0) = (.25, .25)$. The case of $(k, T) = (10, 10)$ is shown in Figure 8 and is representative of the results. Flexible norms are more likely to develop when the transition is more gradual and less abrupt.

Property 3: A more abrupt transition conduces a rigid rule to be a locally stable norm.

This finding is consistent with preceding analysis which established that more extreme changes in environmental frequency are conducive to a rigid norm. Figure 9 shows what a cycle looks like when $(b, k, w, T) = (.7, 10, .5, 10)$ for $z \in \{0, 3, 6, 9\}$. When $z = 0$ so that the transition is maximally abrupt, the range of proportion of agents deploying a flexible rule is $[.04, .65]$. When $z = 9$ so that the transition is very smooth, the range is instead $[.67, .94]$ so that the minimum presence of agents using a flexible rule exceeds the maximum presence when $z = 0$.

4.4 Efficiency of Norms

While our model does not specify the performance of the social system (only the performance of individual agents as it pertains to their promotion within the system), some insight can be had into it if one supposes that an agent's contribution to the performance of the social system depends on the appropriateness of his action and his proficiency with that action. A flexible norm ensures that agents always respond appropriately to the environment. The cost, however, is that flexible rules involve lower proficiency. A rigid norm achieves proficiency but then behavior may often be a poor response to the environment. Indeed, this cost could be quite substantial if the rigid norm involves an action that is infrequently a good match to the environment. In light of this trade-off, the best solution might be to have an evolving rigid rule where agents use a rule that is rigid in the action which is most frequently the best response to the environment. It must be evolving because what is the most common environment - and thereby what is most frequently the best response to the environment - changes over time.

Our previous analysis showed that a rigid norm could persist over time and the reason is that it does evolve with the aggregate environment. However, the efficiency of a rigid norm will depend very much on how quickly it evolves. One that evolves slowly may entail extended periods of time in which agents are acting inappropriately for the environment at a high rate. To gain some insight into this issue, define $e^{h,g}$ as the proportion of agents at level h in generation g whose action matches the environment:

$$e^{h,g} = br_1^{h,g} + (1-b)\, r_0^{h,g} + \left(1 - r_1^{h,g} - r_0^{h,g}\right)$$

Next define $\theta(g)$ as that proportion averaged across all k levels:

$$\theta(g) = \left(\frac{1}{k}\right) \sum_{h=1}^{k} e^{h,g}$$

Finally, we define Θ to be this system-wide average after it is averaged across $2T$ generations (so that it encompasses an entire cycle):

$$\Theta = \left(\frac{1}{2T}\right) \sum_{g=g'}^{g'+2T-1} \theta(g)$$

where g' is some generation for which the system has converged. Θ is said to be the *efficiency* of the system. However, recall that we are only measuring the frequency with which the appropriate action was chosen and are not taking account of the proficiency of agents.

Figures 10 reports Θ for various values of (b, w, T) where $(k, z) = (10, 0)$. Note that Figure 10 corresponds to Figure 2; for the same parameter values, the latter measures the presence of agents using a flexible rule while the former measures the efficiency of the system. The first property to note is that efficiency can be high even if the proportion of agents using a flexible rule is not. Consider $w = .8$ and b around .25 or .75 (the results are the same). The proportion of agents using a flexible rule is less than .5 while efficiency is around .75. This indicates that the rigid rule is evolving rather rapidly.

Next note that the efficiency of norms is non-monotonic in the stability of the environment, as measured by $\left|b - \frac{1}{2}\right|$. Generally, efficiency is highest when the environment is very volatile (b is close to $\frac{1}{2}$) or very stable (b is close to 0 or 1). Since

$$\frac{\partial \Theta}{\partial b} = \left(\frac{1}{2Tk}\right) \sum_{g=g'}^{g'+2T-1} \sum_{h=1}^{k} \left(r_1^{h,g} - r_0^{h,g}\right),$$

if the population mix was fixed then, depending on the sign of $\frac{\partial \Theta}{\partial b}$, efficiency would either monotonically increase or decrease as, say, b moves from $\frac{1}{2}$ to 1. Of course, the mix is not fixed and depends on the value of b. The non-monotonicity follows from the property that there is a large presence of agents using flexible rules when the environment is very volatile and a large presence of agents using the appropriate rigid rule when the environment is very stable. Efficiency is lowest when the environment is moderately volatile because it allows for many agents to deploy a rigid rule but this results in the inappropriate action often being chosen, typically with a frequency of around $\min\{b, 1-b\}$. The exception to this result is when $w = 1$ and T is low (so that the aggregate environment changes rapidly) in which case efficiency is monotonically decreasing as the environment becomes more stable. This is due to the rigid rule not evolving fast enough.

Property 4: Generally, the efficiency of the system is highest when the environment is very volatile (b is close to $\frac{1}{2}$) or very stable (b is close to 0 or 1). Efficiency is lowest when the environment is moderately volatile (b is close to .25 or .75).

Figure 11 tracks efficiency over a cycle for various values of b and w with $(k, T, z) = (10, 10, 0)$. There are two distinct patterns. The first pattern occurs when the environment is sufficiently volatile and sufficiently many new agents are predisposed to be flexible; for example, $(b, w) = (.6, .5)$. The efficiency of the system is quite high due to the high proportion of agents using a flexible rule. When the aggregate environment switches in generation 11, efficiency spikes up. Presumably, this is because there was a small proportion of agents using a rigid rule in the best action for generations 1-10. When was the best action switched, these agents perform poorly and are replaced with agents using a flexible rule. Note that efficiency gradually declines after generation 11 as a rigid rule in what is the new best action has a small but growing presence. The second pattern occurs when the environment is relatively stable and sufficiently many new agents are predisposed to be rigid; for example, $(b, w) = (.7, .5)$. Efficiency is noticeably lower due to a rigid rule prevailing. Contrary to the first pattern, the efficiency of the system falls when what is the most common environment (and action) changes. When $(b, w) = (.7, .5)$, efficiency drops from around .7 in generation 10 to about .45 in generation 11. Initially there are few flexible agents to replace these poorly equipped rigid agents so the system is dominated by agents who most frequently choose the wrong action for the environment. As we previously showed, the presence of flexible rules grows after such a switch and this is reflected in efficiency rising after generation 11. However, agents deploying a flexible rule are gradually replaced with new agents who use a rule rigid in what is now the best action. At that point, efficiency falls and approaches the frequency with which that action is the best action. To more closely observe how the presence of a flexible rule corresponds with the efficiency of the system, see Figure 4.

5 Rambling Remarks

One of the more interesting insights is the role of those agents deploying a flexible rule in transiting the system from one rigid norm to another in response to a large-scale change. If one presumes, as we have modelled, that there are benefits from becoming good at one thing then a rigid norm may be desirable. The trick is that, in response to large-scale changes, the properties of that rigid norm must adapt. There are two likely consequences if they do not. First, a rigid norm may become unstable and be replaced by a flexible norm. That is hardly disastrous and indeed may be best for certain contexts. However, for some situations, that may be less than best. Second, the social system persists with a rigid norm but one in which the action does not adapt. In that situation, the social system is periodically quite inefficient as agents are locked into the action that is inappropriate most of the time. However, the best of both worlds - specialization and adaptation to large-scale changes - can be achieved through the proper use of flexible agents. When a big change occurs, those few agents deploying a flexible rule in a population dominated by agents who are rigid (in what is now the inappropriate action) will quickly rise to the top. Some of them will have histories observationally equivalent to an agent deploying a rigid rule (in what is now the appropriate action). Those new agents who are predisposed to be rigid and have such agents as role models will "hardwire" that behavior in the form of a rigid rule and thus a new rigid norm develops.

Thinking about this issue more generally in the context of a corporation, the point is to have flexibility in promotion procedures whereby someone who deviates from the norm but performs well in some objective sense is allowed to move up. In our model, advancement depended only on one's action, environment, and proficiency. However, in a corporation or a government, promotion is also likely to depend on the characteristics of one's superior and, in a system dominated by rigid behavior, one's superior is likely to deploy a rigid rule. It is not clear that an agent who deviates from the commonly accepted action will fare well compared to other agents who stick with the traditional action. An organizational design issue then is to put into place a structure that allows mavericks to succeed but balances this with the virtue of imitation - learning from the past experiences of others.

The force just described is not present in our model but is one we are currently attempting to encompass. The idea is to make performance depend not only on objective measures - how appropriate is one's action for the external environment and how proficient is one with that action - but also subjective measures - how well does an action conform with one's superior's notion as to what is a proper action. The twist is that the traits of one's superior are the product of selection and social learning and thus the criteria for advancement deployed by superiors are evolving within the system. The performance criterion for advancement must then be allowed to evolve with the system. Rather than think of this criterion as something fixed and external to the model like profit, it is instead the evolutionary product of the past.

This idea - that the criterion which determines success in a social system may itself be endogenous - has broad parallels with work on sexual selection in the evolutionary biology literature; see, for example, Andersson (1994) and Laland (1994). There it has been shown that a female preference for a male trait may emerge even if, putting that female preference aside, that trait is detrimental to the number of offspring produced. Thus, what types of traits enhance the rate of reproduction is not necessarily something exogenous (like strength, speed, smarts) but something that can evolve (perhaps to being red-headed, spectacled, and professorial!). While it is unlikely that this literature will provide any models appropriate for the evolution of social norms, it may offer some general insights that are applicable.

6 Appendix

The initial population resides at the lowest level of the system and is comprised of three types: rigid agents who use action 1, rigid agents who use action 0, and flexible agents. Agents accumulate a personal history as they advance through society. Though there are then many types after the population has a chance to interact with the environment, it is sufficient for our purposes to partition the population into the following five types. A hierarchy is comprised of k levels while a generation equals the length of a single lifetime (which is k periods).

$r_i^{h,g} \equiv$ proportion of the level h population for generation g that are rigid agents endowed with action i (Ri), $i \in \{0, 1\}$

$f_i^{h,g} \equiv$ proportion of the level h population for generation g that are flexible agents who have always chosen action i (Fi), $i \in \{0, 1\}$

$x^{h,g} \equiv$ proportion of the level h population for generation g that are flexible agents who have chosen both action 0 and action 1 (FN).

The level h-generation g state of the system is then $(r_1^{h,g}, f_1^{h,g}, r_0^{h,g}, f_0^{h,g}, x^{h,g})$. The initial population mix is generally assumed to satisfy: $r_1^{1,1} > 0$, $r_0^{1,1} > 0$, and $1 - r_1^{1,1} - r_0^{1,1} > 0$. Abbreviating $r_1^{1,1}$ with r_1 and $r_0^{1,1}$ with r_0, the initial conditions of the system is (r_1, r_0, b^1) where b^1 is the initial frequency with which environment 1 occurs.

Since agents do not have a history at level 1, the characterization of the dynamical process is different between level 2 and higher levels. The level 2 population is determined by the following system of equations:

$$r_1^{2,g} = (r_1^{1,g})^2 + 2r_1^{1,g}r_0^{1,g}b^g + r_1^{1,g}(1 - r_1^{1,g} - r_0^{1,g})b^g \tag{1}$$

$$f_1^{2,g} = b^g[(1 - r_1^{1,g} - r_0^{1,g})r_1^{1,g} + 2(1 - r_1^{1,g} - r_0^{1,g})r_0^{1,g} + (1 - r_1^{1,g} - r_0^{1,g})^2] \tag{2}$$

$$r_0^{2,g} = 2r_1^{1,g}r_0^{1,g}(1 - b^g) + (r_0^{1,g})^2 + r_0^{1,g}(1 - r_1^{1,g} - r_0^{1,g})(1 - b^g) \tag{3}$$

$$f_0^{2,g} = (1 - b^g)[2(1 - r_1^{1,g} - r_0^{1,g})r_1^{1,g} + (1 - r_1^{1,g} - r_0^{1,g})r_0^{1,g} + (1 - r_1^{1,g} - r_0^{1,g})^2] \tag{4}$$

$$x^{2,g} = 0 \tag{5}$$

while the level $h + 1$ population, for $h \in \{2..., k - 1\}$, is determined by:

$$r_1^{h+1,g} = (r_1^{h,g})^2 + 2r_1^{h,g}r_0^{h,g}b^g + r_1^{h,g}f_1^{h,g}b^g + 2r_1^{h,g}f_0^{h,g}b^g + 2r_1^{h,g}x^{h,g}b^g \tag{6}$$

$$f_1^{h+1,g} = r_1^{h,g}f_1^{h,g}b^g + 2r_0^{h,g}f_1^{h,g}b^g + (f_1^{h,g})^2b^g + 2f_1^{h,g}f_0^{h,g}b^g + 2f_1^{h,g}x^{h,g}b^g \tag{7}$$

$$r_0^{h+1,g} = 2r_1^{h,g}r_0^{h,g}(1-b^g)+(r_0^{h,g})^2+2r_0^{h,g}f_1^{h,g}(1-b^g)+r_0^{h,g}f_0^{h,g}(1-b^g)+2r_0^{h,g}x^{h,g}(1-b^g) \tag{8}$$

$$\begin{aligned} f_0^{h+1,g} &= 2r_1^{h,g}f_0^{h,g}(1-b^g) + r_0^{h,g}f_0^{h,g}(1-b^g) + 2f_1^{h,g}f_0^{h,g}(1-b^g) \\ &\quad +(f_0^{h,g})^2(1-b^g) + 2f_1^{h,g}f_0^{h,g}(1-b^g) \end{aligned} \tag{9}$$

$$\begin{aligned} x^{h+1,g} &= 2r_1^{h,g}f_1^{h,g}(1-b^g) + (f_1^{h,g})^2(1-b^g) + 2r_0^{h,g}f_0^{h,g}b^g + (f_0^{h,g})^2b^g \\ &\quad +2r_1^{h,g}x^{h,g}(1-b^g) + 2r_0^{h,g}x^{h,g}b^g + 2f_1^{h,g}x^{h,g}(1-b^g) + 2f_0^{h,g}x^{h,g}b^g + (x^{h,g})^2 \end{aligned} \tag{10}$$

To see how these equations are derived, let us consider each term in (6). A proportion $\left(r_1^{h,g}\right)^2$ of all matchings involve $R1s$ meeting in which case an $R1$ advances for sure. This gives us the first term in (6). A proportion $2r_1^{h,g}r_0^{h,g}$ of all matchings involve an $R1$ and an $R0$ in which case an $R1$ advances only when the environment is 1 which occurs in a proportion b^g of all such matchings and thus we have the second term. A proportion $2r_1^{h,g}f_1^{h,g}$ of all matchings involve an $R1$ and an $F1$. An $R1$ advances with probability $\frac{1}{2}$ when the environment is 1 and probability zero when the environment is 0. Since $2r_1^{h,g}f_1^{h,g}[b^g\left(\frac{1}{2}\right) + (1-b^g)\,0] = r_1^{h,g}f_1^{h,g}b^g$, we have the third term. A proportion $2r_1^{h,g}f_0^{h,g}$ $(2r_1^{h,g}x^{h,g})$ of all matchings involve an $R1$ and an $F0$ (FN) and, in those matchings, $R1$ wins for sure when the environment is 1 and loses for sure when the environment is 0. This gives us the final two terms.

Let $d_i^{h,g}$ denote the proportion of the level h population for generation g that have always chosen action i and have faced at least one type $j(\neq i)$ environment. The following pair of equations determines the behavioral rules of the level one population of generation $g+1$:

$$r_1^{1,g+1} = w(r_1^{k,g} + f_1^{k,g}) + (1-w)d_1^{k,g} \tag{11}$$

$$r_0^{1,g+1} = w(r_0^{k,g} + f_0^{k,g}) + (1-w)d_0^{k,g} \tag{12}$$

The complete dynamical system then involves the embedding of (1)-(10) in (11)-(12).

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Figure 1
Average Proportion of Agents Using a Flexible Rule (k=5)

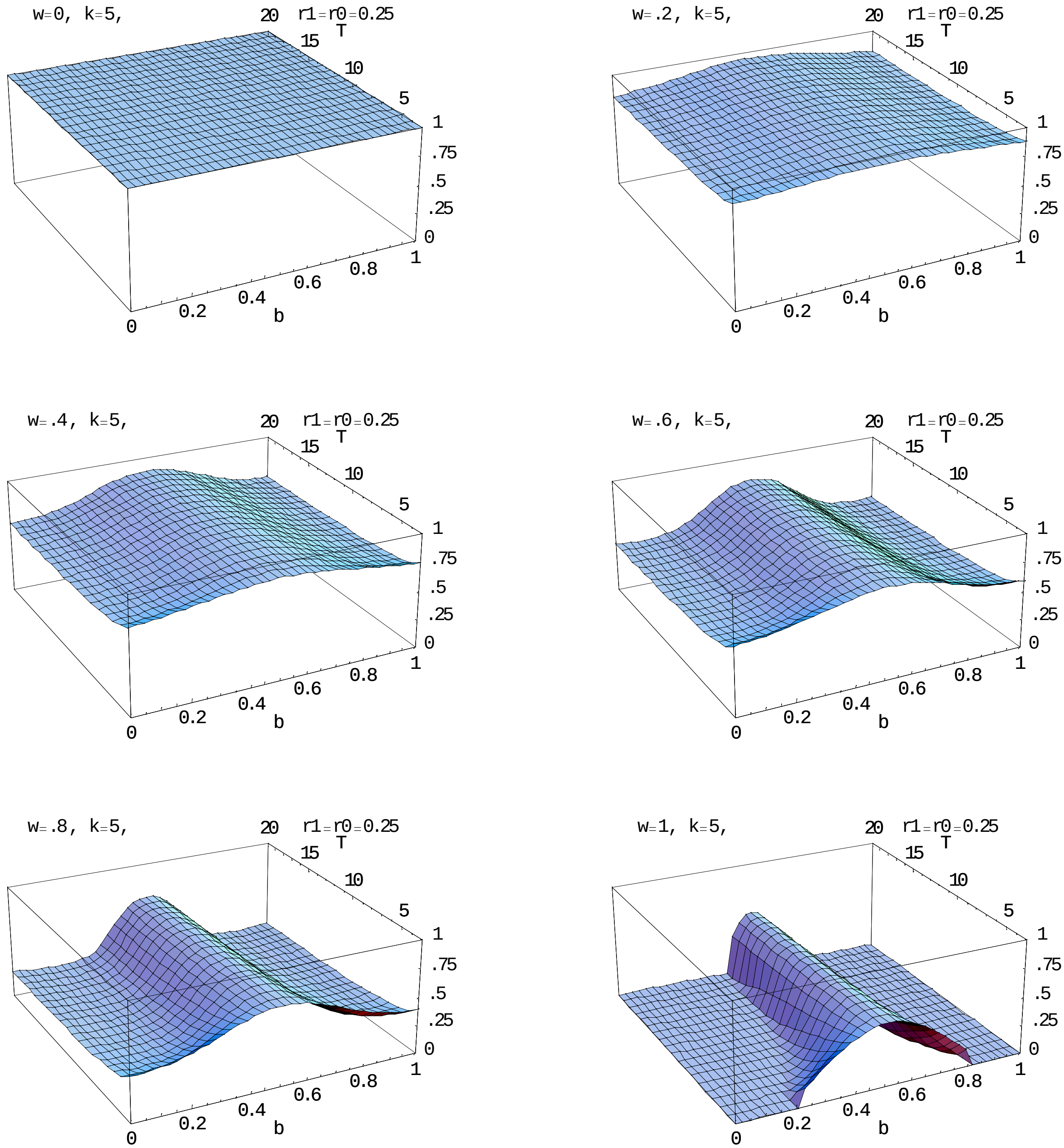

Figure 2
Average Proportion of Agents Using a Flexible Rule (k=10)

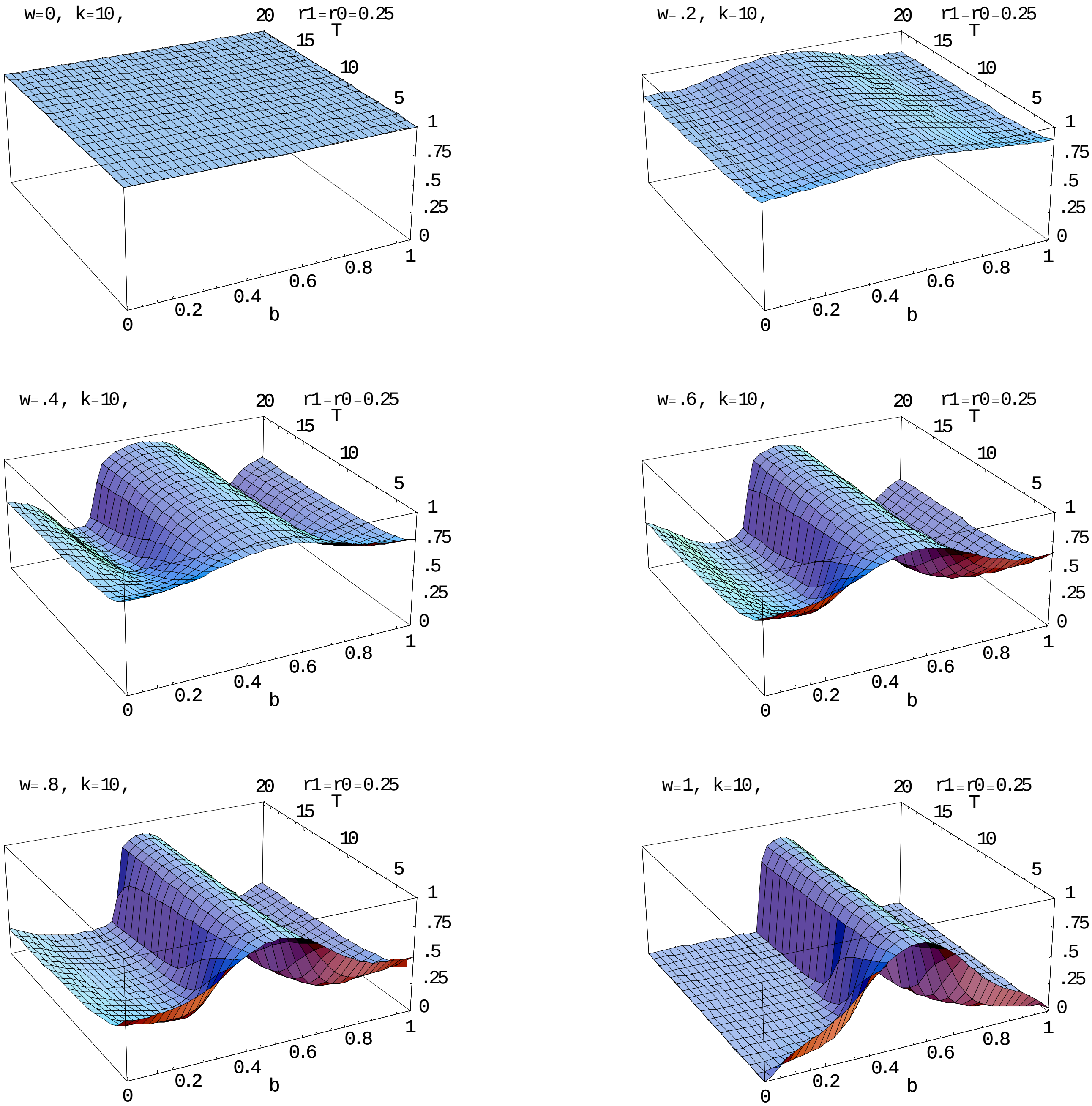

Figure 3
Average Proportion of Agents Using a Flexible Rule (k=15)

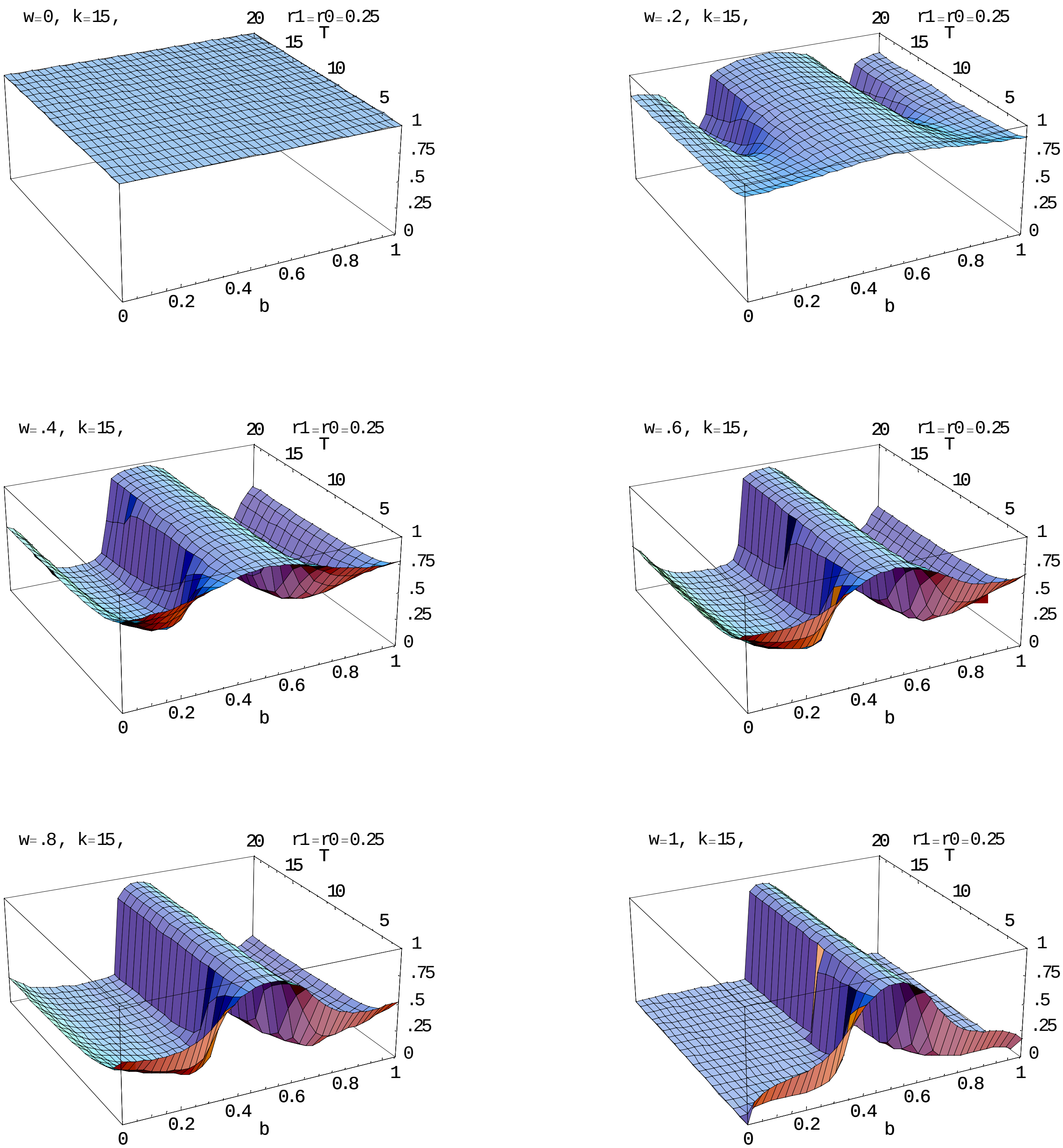

Figure 4a
Average Proportion of Agents Over a Cycle

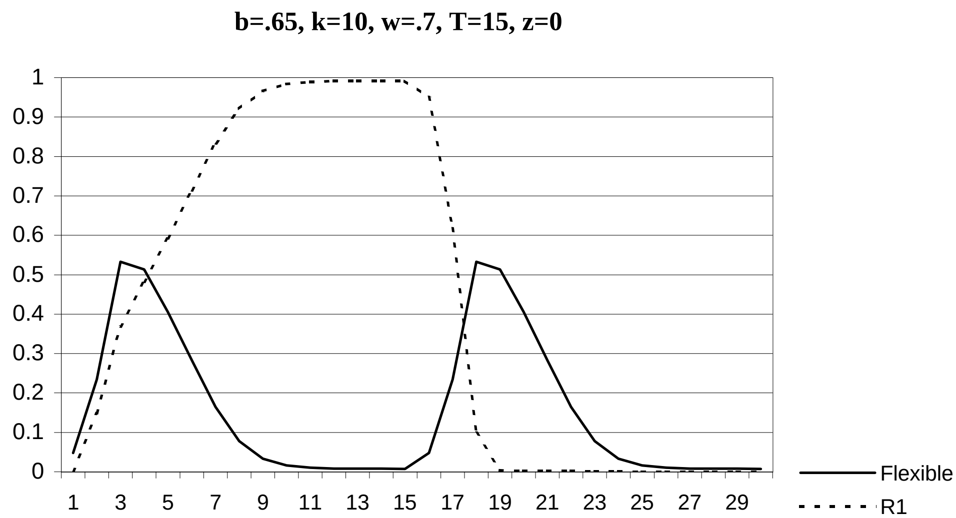

Figure 4b
Average Efficiency Over a Cycle

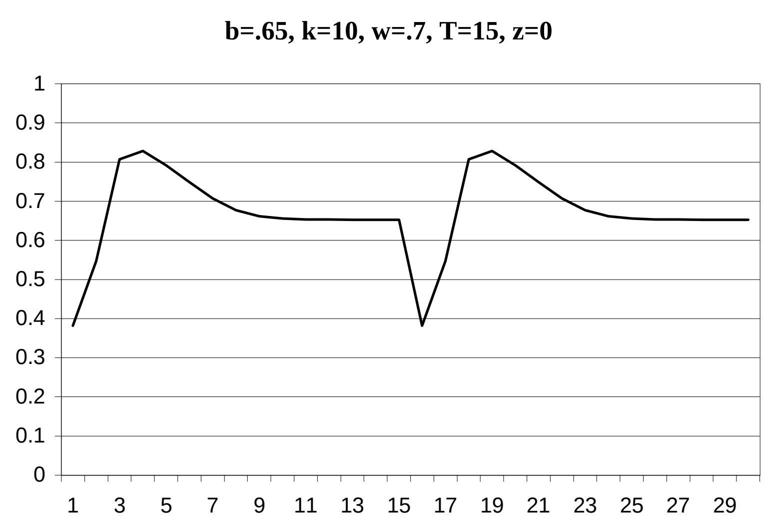

Figure 5
Range of Proportion of Agents Using a Flexible Rule (k=5)

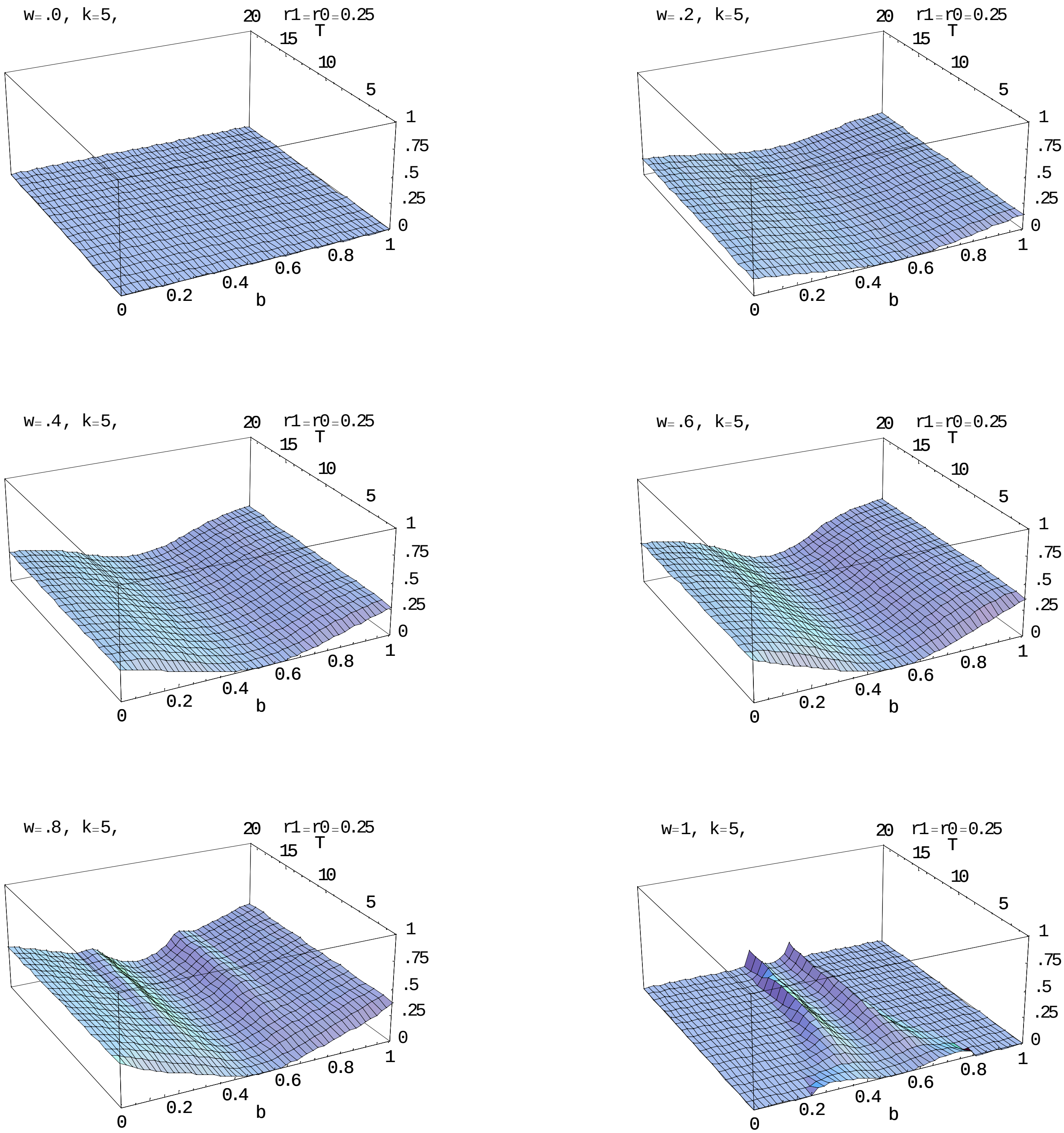

Figure 6

Range of Proportion of Agents Using a Flexible Rule (k=10)

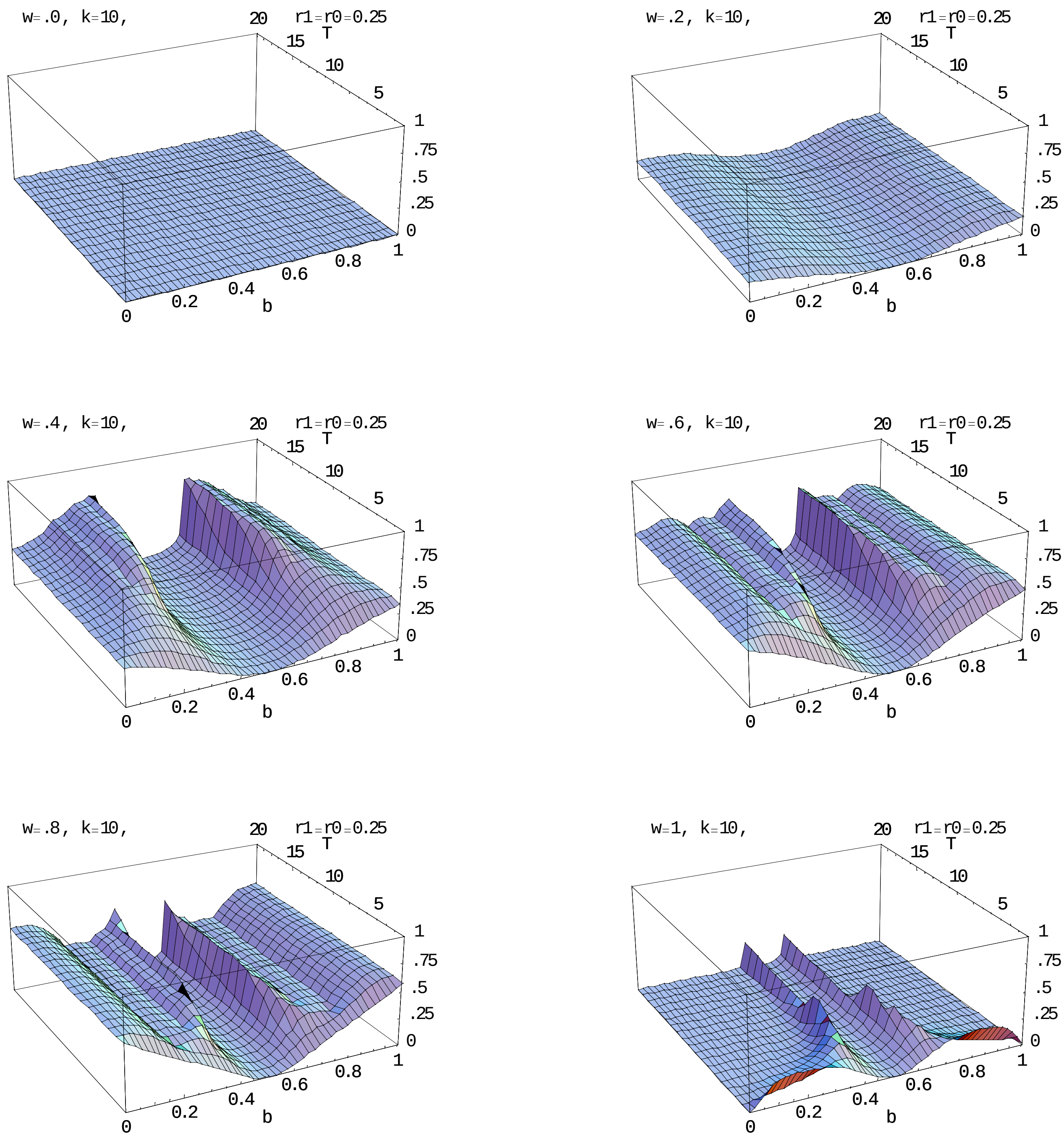

Figure 7
Range of Proportion of Agents Using a Flexible Rule (k=15)

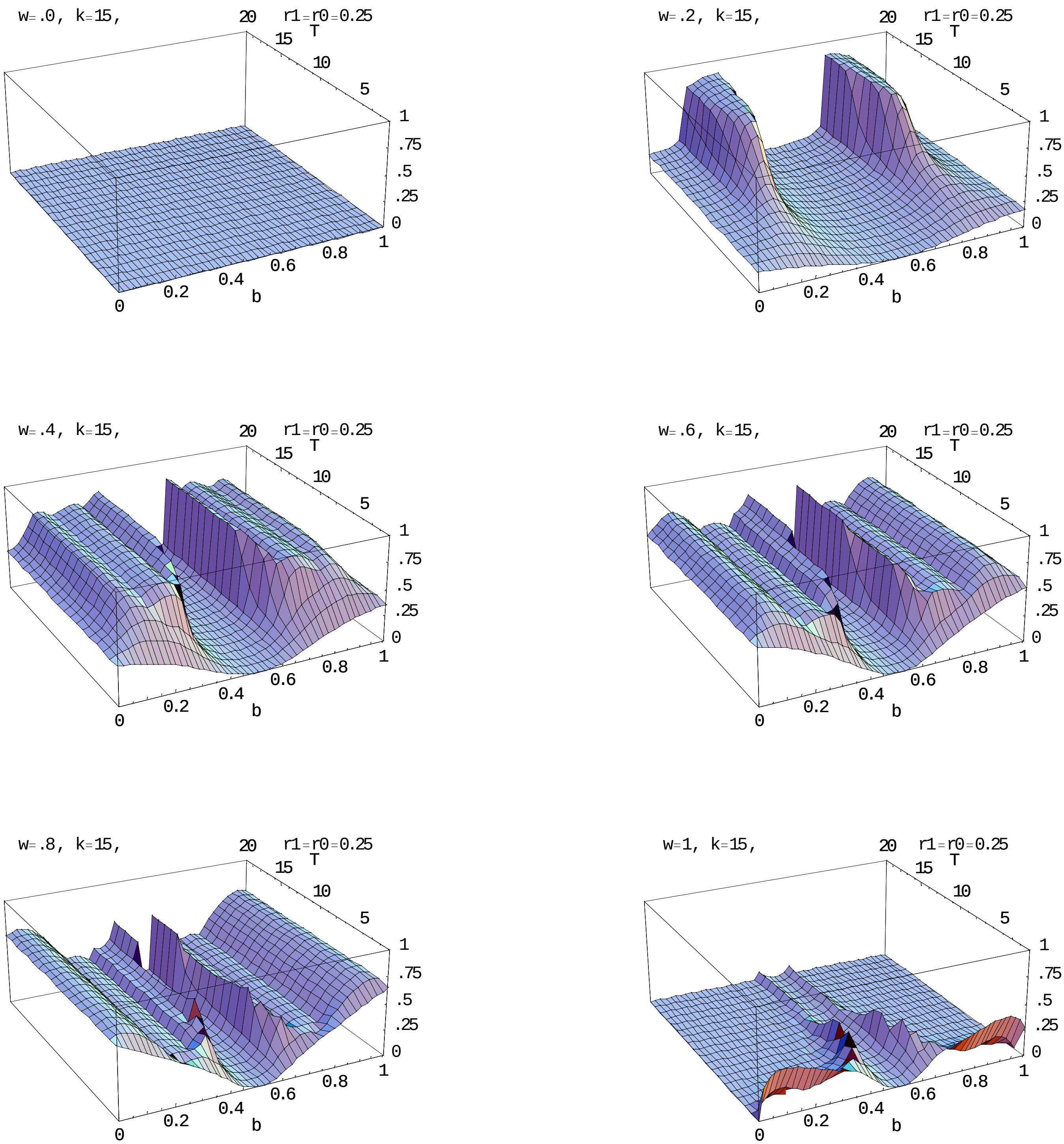

Figure 8
Average Proportion of Agents Using a Flexible Rule (T=10, k=10)

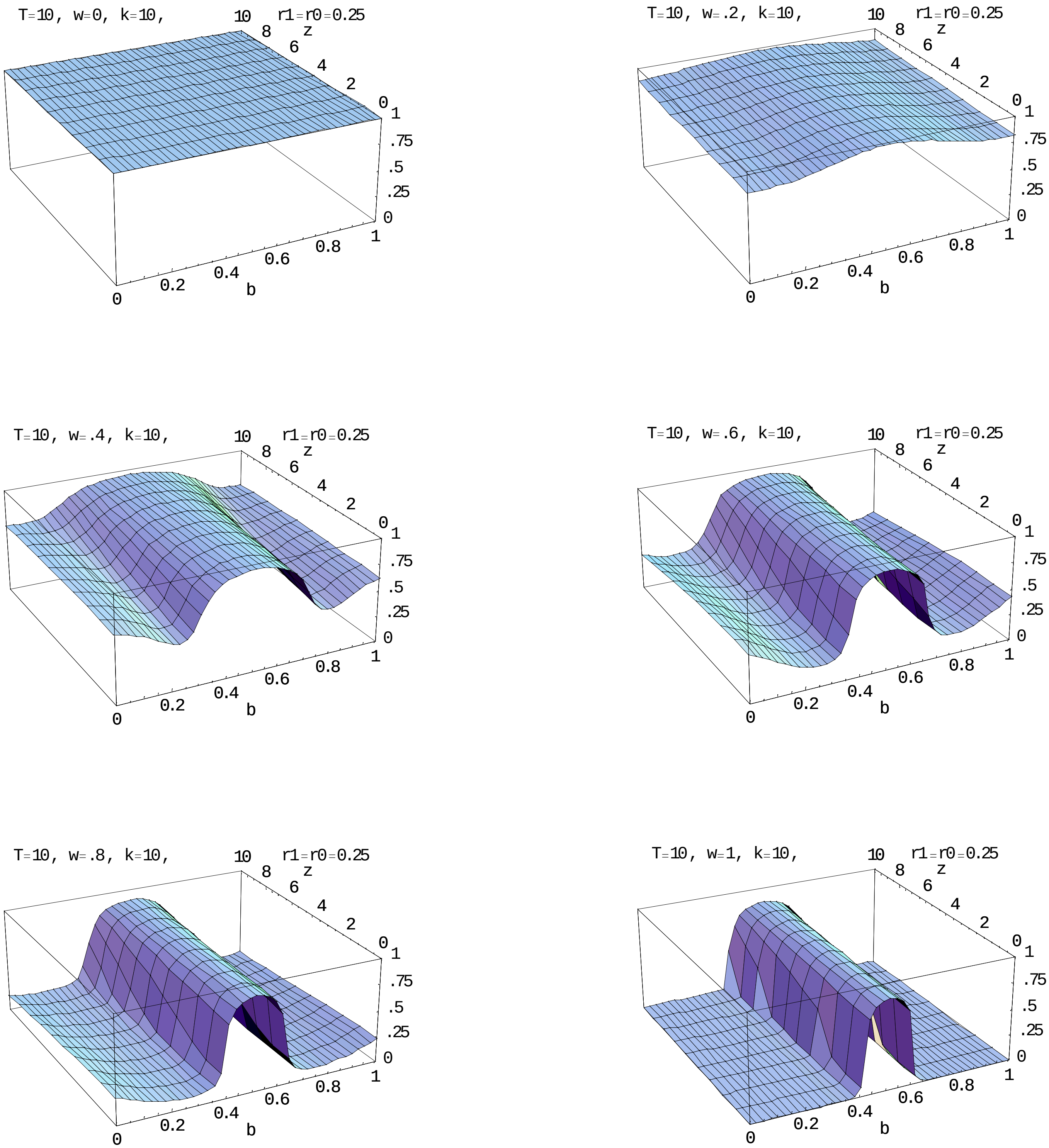

Figure 9 - Average Proportion of Agents over a Cycle

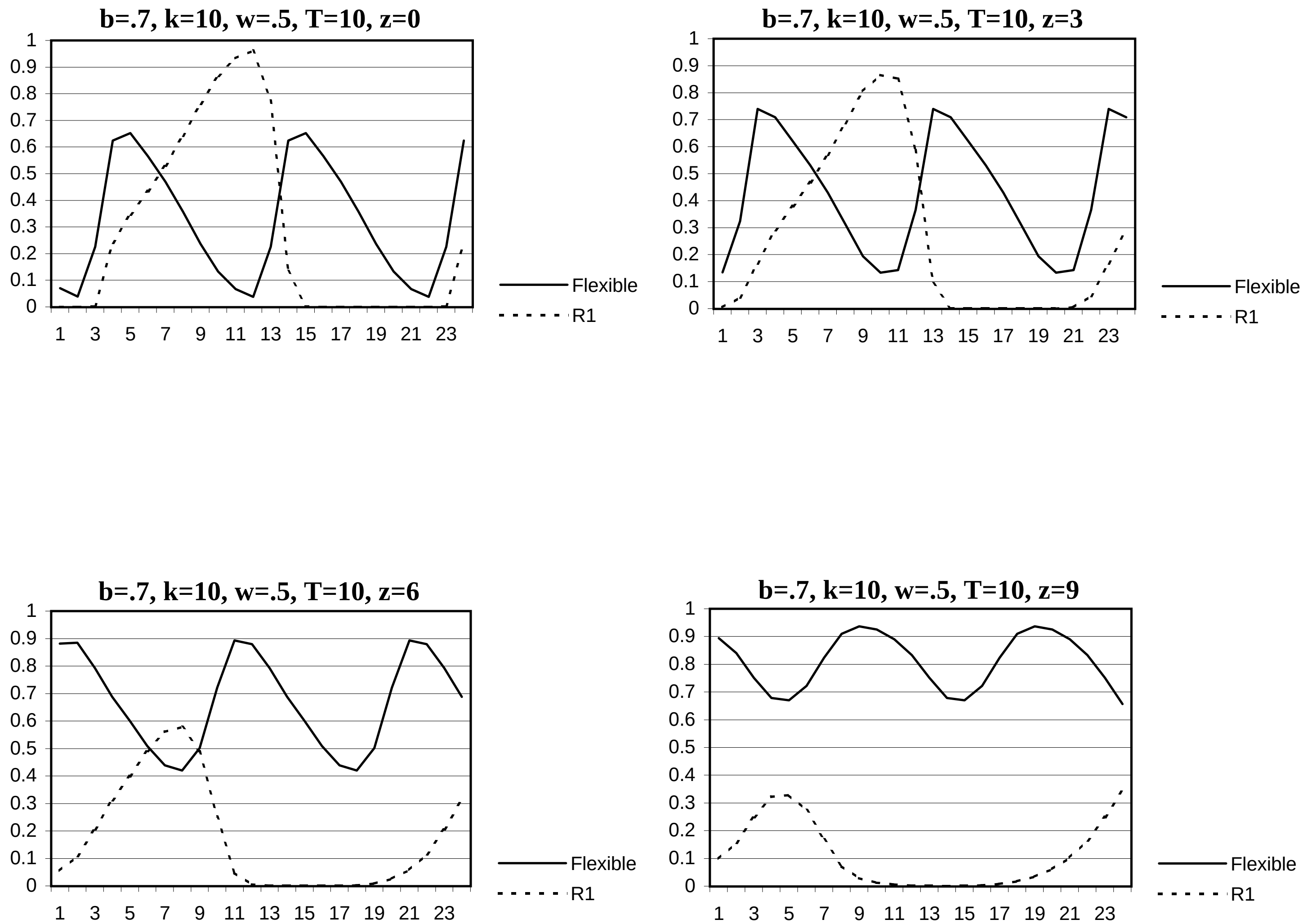

Figure 10
Average Efficiency (k=10, z=0)

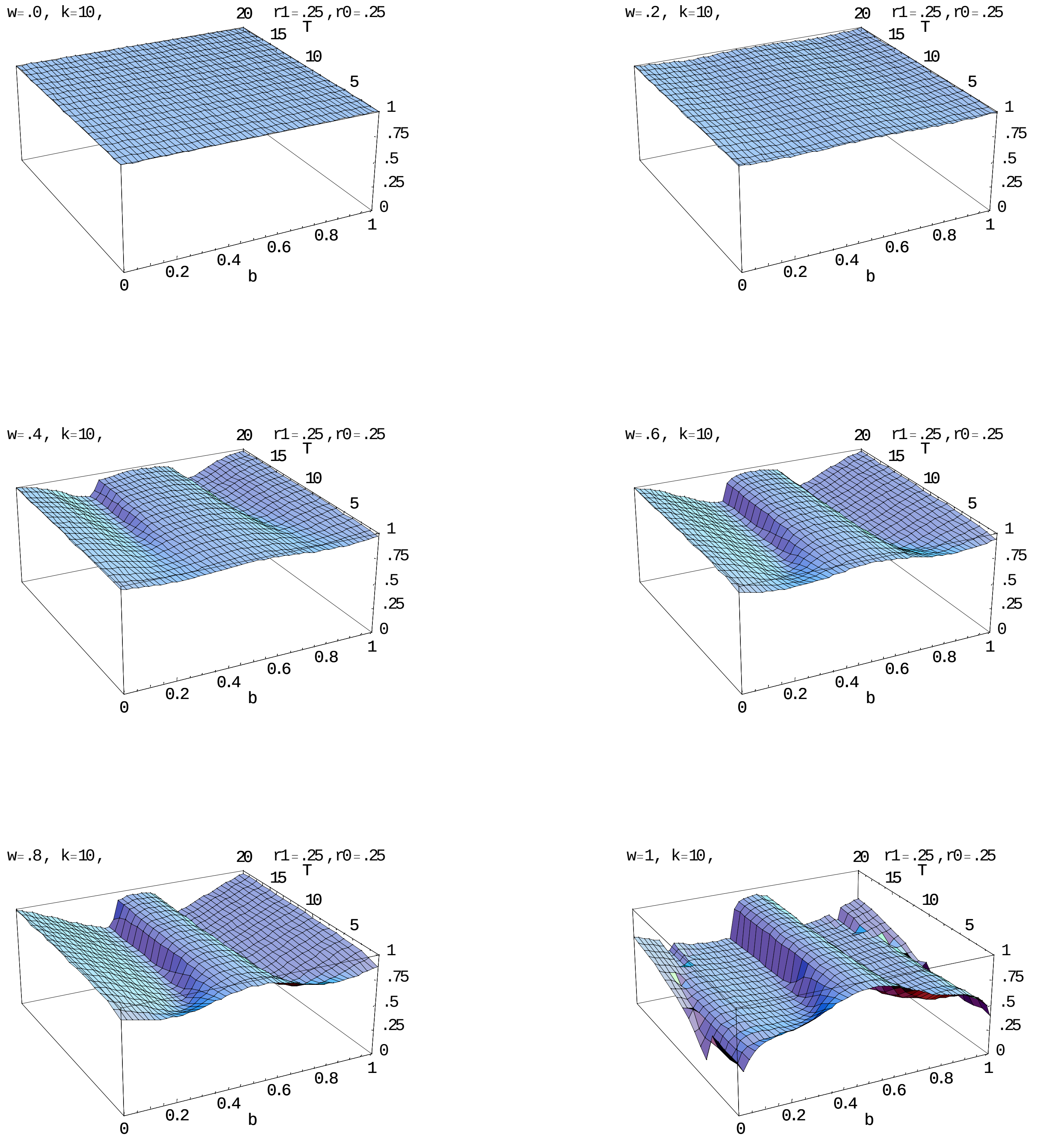

Figure 11
Average Efficiency Over a Cycle (k=10, T=10, z=0)