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The Equilibrium Level of Rigidity in a Hierarchy*

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A hierarchy is considered in which those agents who perform better advance to higher levels. When agents are heterogeneous and endowed with simple behavioral rules, Harrington (1998a) showed that agents at high levels tend to be rigid, in the sense that their behavior is unresponsive to their environment, relative to agents at low levels. In the current paper, agents are homogeneous but sophisticated as their behavior is required to be consistent with a subgame perfect equilibrium. Agents at high levels are found instead to be flexible relative to agents at low levels. *Journal of Economic Literature* Classification Numbers: D00, D23, D72. © 1999 Academic Press

1. INTRODUCTION

How should we expect behavior to vary within a hierarchical system such as a corporation, an army, or a political system? More specifically, if we imagine heterogeneity in terms of behavioral plasticity—the degree to which an agent tailors his behavior to his environment—should we expect agents who occupy high levels in a hierarchy to be rigid—tending to pursue the same course of action even when the environment changes—or flexible? This question was posed in Harrington (1998a) where it was shown that, if the hierarchy has sufficiently many levels, agents at high levels tend to be rigid. In that model, agents were endowed with simple behavioral rules in that their behavior was restricted to be independent of

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the level at which they reside. There were two generic rules or agent types in the population—those who always use the same action (or approach) and those who choose the best (myopic) action for the current environment. Given these behavioral rules, any variation in behavior across levels is due solely to how the mix of agents varies across levels. It is assumed that those agents with better relative performance advance to higher levels in the hierarchy. At sufficiently high levels in the hierarchy, this advancement process was shown to select rigid agents for promotion. It is then predicted that those occupying the high levels of a hierarchy will be rigid relative to agents at low levels.[1]

The question of how behavior varies within a hierarchy is re-examined here by considering a model which is a polar extreme. First, agents are not endowed with behavioral rules but rather have preferences and are strategic in the sense that their behavioral rules are required to form a subgame perfect equilibrium. Second, agents are not heterogeneous in that they have identical preferences and, furthermore, we focus on symmetric subgame perfect equilibria so that agents deploy the same behavioral rule. Given the lack of heterogeneity in the agent population, selection will not be an operative force. Any variation in behavior across levels will be for strategic reasons as agents modify their behavior depending on where they are in the hierarchy.

We find that selection and strategic behavior generate qualitatively different predictions as to how behavior varies within a hierarchy. When there are sufficiently many levels, Harrington (1998a) showed that high levels are characterized by rigid behavior so that agents at the top are more rigid than those at the bottom. In contrast, an equilibrium theory predicts that high levels may be characterized by flexible behavior so that the agents at the top are less rigid than those at the bottom.

2. A MODEL OF A HIERARCHY

Consider a hierarchy with $k \geq 3$ levels. At each level, there is a countably infinite population of agents. These agents are randomly matched into pairs to compete for advancement up the hierarchy. After two agents are matched, a stochastic environment is realized which is observed by the two agents. For simplicity, there are just two possible environments, $\{0, 1\}$, and it is assumed that environment 1 is more common in that it occurs with

[1] Related questions have been examined in Rosen (1986) and Vega-Redondo (1998). Also, a variant of this model is used in Harrington (1998b) to show that pure office-seeking politicians can look like ideologues if one views the electoral system as a hierarchy of offices.

probability $b \in (\frac{1}{2}, 1)$. In response to this environment, agents simultaneously choose actions from the set $\{0, 1\}$. In a manner specified below, one of these agents will advance to the next level while the other agent exits the system. Defining a period as the length of time spent at a level, an agent lives for at most k periods (which only occurs when the agent traverses all k levels in the hierarchy). Environments are assumed to be iid across pairings and across levels. Hence, at each level, a proportion b of all pairings face environment 1 and, regardless of the environments that an agent has faced at previous levels, the probability he faces environment 1 at the current level is b. There is then individual uncertainty in this model but no aggregate uncertainty.

A strategy is a sequence of mappings, one for each level, which maps from the set of feasible personal histories over the preceding levels into the space of functions which map from the set of feasible (current) environments into the set of feasible (current) actions. Let a_i^h and e_i^h denote the action chosen by agent i and the environment faced by agent i at level h, respectively. Given the assumed anonymity of one's partners (and that a partner's history is private information), a history for an agent at level h is an element of $\{0, 1\}^{3(h-1)}$ in that it includes the past environments he faced, his past actions, and the past actions of the agents with which he was matched. Given the space of possible actions and environments is $\{0, 1\}$, a strategy maps this history into the space of mapping from $\{0, 1\}$ into $\{0, 1\}$ which can be represented by $\{0, 1\}^2$. We then have that a strategy is a sequence of $k-1$ functions $\{\phi^h\}_{h=1}^{k-1}$ where $\phi^h : \{0, 1\}^{3(h-1)} \to \{0, 1\}^2$.

In specifying the selection (or advancement or promotion) rule, let

$$\Omega_i^h(a) \equiv \left|\left\{h' \in \{1, \ldots, h-1\} \middle| a_i^{h'} = a\right\}\right|$$

denote the number of times that agent i has chosen action a prior to level h. Suppose agents i and j meet at level h and e^h is their environment. If $a_i^h = e^h \neq a_j^h$, then agent i advances to level $h+1$ with probability one. If $a_i^h = a = a_j^h$ and $\Omega_i^h(a) > \Omega_j^h(a)$, then agent i advances to level $h+1$ with probability one, $a \in \{0, 1\}$. If $a_i^h = a = a_j^h$ and $\Omega_i^h(a) = \Omega_j^h(a)$, then agent i advances to level $h+1$ with probability $\frac{1}{2}$, $a \in \{0, 1\}$.[2] Agents who do not advance exit the system.

The motivation for this selection rule is that an agent's performance is presumed to depend on the appropriateness of his action given the current

[2]With the selection model in Harrington (1998a), the main result only required: (i) if $a_i^h = e^h \neq a_j^h$, then agent i advances with probability one; (ii) if $a_i^h = e^h = a_j^h$ and $\Omega_i^h(e^h) > \Omega_j^h(e^h)$, then agent i advances with probability one; and (iii) if $a_i^h = e^h = a_j^h$ and $\Omega_i^h(e^h) = \Omega_j^h(e^h)$, then agent i advances with probability $\frac{1}{2}$. It was unnecessary to specify who advances when both agents chose the inappropriate action for the current environment.

environment (where it is assumed that action e^h is best when the environment is e^h) and his proficiency with that action. Proficiency is a strictly monotonic function of the number of times with which an action has been used. One rationale is learning-by-doing; the more an agent uses an action, the better he gets at using it. Alternatively, if these actions are messages, then the credibility of an agent's message may be greater if the history of his messages is more consistent, and such credibility may aid in convincing other people to do as one desires (thereby enhancing one's performance). Performance is assumed to be primarily determined by one's current action and secondarily by one's proficiency with the action deployed. Advancement occurs for the agent with higher performance.

Agents are assumed to have identical preferences and to care only about the final level that they reach with higher levels producing (weakly) higher satisfaction. An agent's utility function is denoted $V(\cdot): \{1, 2, \ldots, k\}^2 \to R$ with $V(h; \theta)$ being the payoff when an agent's maximal level is h and θ is a parameter (which we will periodically suppress). The following structure is placed on it:

$$V(k; \theta) \geq V(k-1; \theta) \geq \cdots \geq V(\theta; \theta) > V(\theta - 1; \theta) = \cdots = V(1; \theta) = 0.$$

Thus, agents only value progressing to at least level θ. They are indifferent between levels 1 and $\theta - 1$ and any level in between.[3] This parameterization is intended to capture the property that agents attach significantly more value to high levels than to low levels with this effect being more pronounced, the higher is the value of θ. Its role in stating our main result will become apparent momentarily.

All of the above structure is common knowledge to the agents.

3. EQUILIBRIUM BEHAVIOR WITHIN A HIERARCHY

We show that there exists a symmetric subgame perfect equilibrium such that agents are rigid at low levels in the hierarchy—choosing action 1 irrespective of the environment—and flexible at high levels—choosing the action which is best for the environment. This occurs when there are sufficiently many levels within the system (k is sufficiently high) and high levels are sufficiently more valued than low levels (θ is sufficiently high).[4]

[3]The critical assumption is that $V(\theta - 1; \theta) = \cdots = V(1; \theta)$. That it equals zero is a normalization.

[4]While this result is derived for when agents are indifferent as to progressing to levels below level θ, the proof suggests that a continuity argument would imply the same result holding when a small but positive value is attached to progressing from level $h - 1$ to h for $h \in \{1, \ldots, \theta - 1\}$.

The essential step of the proof is showing that it is optimal to choose action 1 when the environment is 0 given other agents will also be choosing action 1. Choosing action 1 in such a situation means doing exactly what everyone else is doing so that the probability of advancing to the next level is $\frac{1}{2}$ and, given the symmetry of agents' future behavior, the probability of advancing j levels is $(\frac{1}{2})^j$ (assuming this agent acts optimally thereafter). The virtue of instead choosing action 0 when the environment is 0 is that it results in advancement to the next level with probability one since the agent with which one is matched will be choosing action 1. The downside to having chosen action 0 is that, from this point onward, such an agent will be less proficient in action 1 than all other agents with which he will be matched at higher levels. Thus, whenever he faces environment 1, he will lose for sure. Analogously, he will be more proficient at action 0 than all other agents so, at higher levels, he will advance for sure when the environment is 0. It follows that, when the current environment is 0 and all other agents are expected to choose action 1, the selection of action 0 (with optimal behavior thereafter) by an agent will generate a probability of advancing another j levels equal to $(1-b)^{j-1}$. Next note that

$$\left(\frac{1}{2}\right)^j \gtreqless (1-b)^{j-1} \text{ as } j \lesseqgtr \frac{-\ln(1-b)}{\ln(1/2)-\ln(1-b)} (>1)$$

so that choosing action 1 results in a lower probability in advancing over the next j levels when j is low but a higher probability when j is high. Thus, if agents attach sufficiently more value to advancing to high levels, it is preferable to choose action 1 in such a situation. This is the basic strategy of the proof. This argument shows how it can be optimal for an agent to be rigid when everyone else is rigid. It is trivially optimal for an agent to be flexible when everyone else is flexible in that choosing an action which is inappropriate for the current environment will result in failure to advance for certain given all other agents are choosing the appropriate action.

In stating the main result, define the following:

$$\lambda(b) \equiv \frac{\ln b - 2\ln(1-b)}{\ln(1/2)-\ln(1-b)}$$

and let $\lceil x \rceil$ denote the minimal integer greater than or equal to x and $\lfloor x \rfloor$ denote the maximal integer less than or equal to x, $x \in R$. It is straightforward to show that $\lambda(b) > 1$.

THEOREM 3.1. *If $k \geq \lceil \lambda(b) \rceil$ and $\theta \in \{\lceil \lambda(b) \rceil, \ldots, k\}$ then $\exists \overline{h} \in \{1, \ldots, k-2\}$ such that it is a symmetric subgame perfect equilibrium outcome for $a_i^h = 1$ for $h \in \{1, \ldots, \overline{h}\}$ and $a_i^h = e_i^h$ for $h \in \{\overline{h}+1, \ldots, k-1\}$.*

Proof. Consider the following symmetric strategy profile:

$$\phi_i^h = \begin{cases} 1 \text{ if } \Omega_i^h(1) = h-1 \text{ and } h \in \{1, \ldots, \overline{h}\} \\ 0 \text{ if } \Omega_i^h(1) < h-1 \text{ and } h \in \{1, \ldots, \overline{h}\} \\ e_i^h \text{ if } \Omega_i^h(1) \geq \overline{h} \text{ and } h \in \{\overline{h}+1, \ldots, k-1\} \\ 0 \text{ if } \Omega_i^h(1) < \overline{h} \text{ and } h \in \{\overline{h}+1, \ldots, k-1\} \end{cases} \tag{1}$$

Notice that an agent's action depends only on what he has done in the past and what level he is at. It does not depend on what his partners have done. This seems reasonable in that, given a large population and a finite number of levels, there is probability zero that an agent will meet another agent again or meet anyone that has met anyone he has met and so forth.

(1) Let the current level be denoted h' and suppose that $h' \in \{1, \ldots, \overline{h}\}$.

(1a) Suppose $\Omega_i^{h'}(1) = h' - 1$ (so that the history is along the equilibrium path).

(1ai) Suppose $e_i^{h'} = 1$. Since his partner is expected to choose action 1, agent i's payoff from choosing action 0 is $V(h')$ since he is sure to lose. His strategy calls for him to choose action 1 which yields an expected payoff of at least $\frac{1}{2}V(h') + \frac{1}{2}V(h'+1)$ since, in equilibrium, he and his partner have identical histories and choose identical actions.

(1aii) Suppose $e_i^{h'} = 0$. His strategy calls for him to choose action 1. Assuming that he acts according to his strategy at all future levels, his expected payoff from action 1 is

$$\sum_{j=1}^{k-h'} \left(\frac{1}{2}\right)^j V(h'+j-1) + \left(\frac{1}{2}\right)^{k-h'} V(k). \tag{2}$$

For $h \in \{h', \ldots, \overline{h}\}$, agents will have chosen identical actions at past levels and will choose identical actions at the current level (specifically, action 1) so that the probability of advancing from level h to $h+1$ is $\frac{1}{2}$. For $h > \overline{h}$, agents will have chosen identical actions over levels $1, \ldots, \overline{h}$ but there will be heterogeneity in their actions after level $\overline{h}$ since agents will have chosen the action that corresponds to the environment and different agents may have faced different environments. For example, a proportion b of agents at level $\overline{h}+2$ will have chosen action 1 $\overline{h}+1$ times while a proportion 1-b will have chosen action 1 $\overline{h}$ times as the former faced environment 1 at level $\overline{h}+1$ (and chose action 1) while the latter faced environment 0 at level $\overline{h}+1$ (and chose action 0). However, note that all agents start with the same history of actions at level $\overline{h}+1$ and that the probability distribution over an agent's environment is the same for all agents. This means that the probability distribution over histories of actions is the same. Hence, the conditional probability that an agent who has survived to level h will survive until level $h+1$ is the same for all agents. Since half of all agents advance, this probability is $\frac{1}{2}$. The probability that an agent survives until level h, given he has reached level h', is then $(\frac{1}{2})^{h-h'}$. The probability that an agent reaches level h and goes no further is then $(\frac{1}{2})^{h-h'+1}$ which is the probability of winning at levels $h, \ldots, h'-1$ and losing at h'. This argument gives us the expression in (2).

Continuing with the case of $e_i^{h'} = 0$, now consider agent i choosing action 0 instead of action 1. He will win for sure and advance to level $h'+1$. By his strategy, he will choose action 0 thereafter. Since then, for $h > h'$, $\Omega_i^h(0) = h - h'$ and, according to agent j's strategy, $\Omega_j^h(0) = 0$ for $h \leq \overline{h}$ and $\Omega_j^h(0) \leq h - \overline{h} - 1$ for $h \geq \overline{h}+1$, we have that $\Omega_i^{h'}(0) > \Omega_j^{h'}(0) \forall h > h'$. Hence, when environment 0 occurs, agent i wins for sure. When instead the environment is 1, he will lose for sure since $\Omega_i^h(1) = h' - 1 < \min\{h, \overline{h}\} \leq \Omega_j^h(1)$. Given the probability that the environment is 1 is b, this agent's resulting expected payoff is then

$$\sum_{j=1}^{k-h'-1} b(1-b)^{j-1} V(h'+j) + (1-b)^{k-h'-1} V(k). \tag{3}$$

Define

$$\Gamma(h) \equiv \sum_{j=1}^{k-h} \left(\frac{1}{2}\right)^{j} V(h+j-1) + \left(\frac{1}{2}\right)^{k-h} V(k)$$
$$- \sum_{j=1}^{k-h-1} b(1-b)^{j-1} V(h+j) - (1-b)^{k-h-1} V(k) \tag{4}$$

as the difference in the payoff from choosing action 1 and choosing action 0. The optimality of action 1 at level h' when $e_i^{h'} = 0$ requires $\Gamma(h') > 0$.

LEMMA 3.2. *If $k \geq \lceil \lambda(\underline{b}) \rceil$ and $\theta \in \{\lceil \lambda(b) \rceil, \ldots, k\}$ then $\exists \overline{h} \geq 1$ such that $\Gamma(h) > 0 \forall h \in \{1, \ldots, \overline{h}\}$.*

Proof. To begin, (4) can be rearranged so that

$$\Gamma(h) = \left(\frac{1}{2}\right) V(h) + \sum_{j=1}^{k-h-1} \left[\left(\frac{1}{2}\right)^{j+1} - b(1-b)^{j-1}\right] V(h+j)$$
$$+ \left[\left(\frac{1}{2}\right)^{k-h} - (1-b)^{k-h-1}\right] V(k). \tag{5}$$

Let us derive conditions whereby $\Gamma(h) > 0$. First note that $(\frac{1}{2})^{j+1} - b(1-b)^{j-1} > 0$ iff

$$(j+1)\ln\left(\tfrac{1}{2}\right) > \ln b + (j-1)\ln(1-b) \tag{6}$$

or

$$j > \frac{\ln b - \ln(1-b) - \ln(1/2)}{\ln(1/2) - \ln(1-b)} \tag{7}$$

where it is straightforward to show that the rhs expression in (7) equals $\lambda(b) - 1$. We then have that $(\frac{1}{2})^{j+1} - b(1-b)^{j-1} > 0$ iff $j \geq \lfloor \lambda(b) \rfloor$.

By assumption, $k \geq \lceil \lambda(b) \rceil$. Let us consider two cases: (i) $k = \lceil \lambda(b) \rceil$; and (ii) $k \geq \lceil \lambda(b) \rceil + 1$.

If $k = \lceil \lambda(b) \rceil$ then, as stated in Lemma 3.2, set $\theta = k$ so that $V(h) = 0 \forall h \in \{1, \ldots, k-1\}$. Therefore, given $V(k) > 0$, $\Gamma(h) > 0$ iff $(\frac{1}{2})^{k-h} - (1-b)^{k-h-1} > 0$. Setting $h = 1$ then $\Gamma(1) > 0$ iff $(\frac{1}{2})^{k-1} - (1-b)^{k-2} > 0$. Next note that $k = \lceil \lambda(b) \rceil$ implies $k - 1 \geq \lambda(b) - 1$ which, by the analysis surrounding (6) and (7), is equivalent to $(\frac{1}{2})^k - b(1-b)^{k-2} \geq 0$. Given that $\frac{1}{2} < b$, then the preceding inequality implies $(\frac{1}{2})^{k-1} - (1-b)^{k-2} > 0$ which is what we wanted to prove. We conclude that if $k = \lceil \lambda(b) \rceil$ and $\theta = k$, then $\Gamma(1) > 0$. This proves $\exists \overline{h} \geq 1$ such that $\Gamma(h) > 0 \forall h \in \{1, \ldots, \overline{h}\}$.

Now suppose $k \geq \lceil \lambda(b) \rceil + 1$ and further suppose h satisfies $k - h - 1 \geq \lfloor \lambda(b) \rfloor$. The first term in (5) is obviously non-negative. Turning to the second term, which is the summation, from (6) and (7) we know that $(\frac{1}{2})^{j+1} - b(1-b)^{j-1} > 0$ iff $j \geq \lfloor \lambda(b) \rfloor$. If $V(h+j) = 0$ for $j \in \{1, \ldots, \lfloor \lambda(b) \rfloor - 1\}$ and $V(h+j) \geq 0$ for $j \in \{\lfloor \lambda(b) \rfloor, \ldots, k-h-1\}$, then the second term in (5) is non-negative. Next note that if $k - h - 1 \geq \lfloor \lambda(b) \rfloor$, then (7) holds for $j = k - h - 1$ which means that $(\frac{1}{2})^{k-h} > b(1-b)^{k-h-2}$. In that the latter is equivalent to $(\frac{1}{2})^{k-h} > (b/(1-b))(1-b)^{k-h-1}$, it follows that $(\frac{1}{2})^{k-h} > (1-b)^{k-h-1}$ since $b > 1 - b$. Hence, the last term in (5) is positive since $V(k) > 0$. From these steps, we can infer that $\Gamma(h) > 0 \forall h \in \{1, \ldots, \overline{h}\}$ if $V(h+j) = 0$ for $j \in \{0, 1, \ldots, \lfloor \lambda(b) \rfloor - 1\}$ and $V(h+j) \geq 0$ for $j \in \{\lfloor \lambda(b) \rfloor, \ldots, k-h-1\}$ for all $h \in \{1, \ldots, \overline{h}\}$ or, equivalently, $V(h) = 0$ for $h \in \{1, \ldots, \overline{h} + \lfloor \lambda(b) \rfloor - 1\}$ and $V(h) \geq 0$ for $h \in \{\overline{h} + \lfloor \lambda(b) \rfloor, \ldots, k-1\}$. Therefore, if $\theta \geq \overline{h} + \lfloor \lambda(b) \rfloor$, then $\Gamma(h) > 0 \forall h \in \{1, \ldots, \overline{h}\}$. By assuming $\theta \geq 1 + \lfloor \lambda(b) \rfloor$, as is done in the statement of Lemma 3.2, it follows that $\Gamma(1) > 0$ and thus $\exists \overline{h} \geq 1$ such that $\Gamma(h) > 0 \forall h \in \{1, \ldots, \overline{h}\}$. This analysis was done under the presumption that $k - h - 1 \geq \lfloor \lambda(b) \rfloor$ for all $h \in \{1, \ldots, \overline{h}\}$ or, equivalently, $k - \overline{h} - 1 \geq \lfloor \lambda(b) \rfloor$. For this to be ensured for some value of $\overline{h} \geq 1$ requires that $k - 2 \geq \lfloor \lambda(b) \rfloor$ which is exactly our supposition that $k \geq \lceil \lambda(b) \rceil + 1$. ∎

With Lemma 3.2, we conclude that, along the equilibrium path, the prescribed action of 1 at level h', when $h' \in \{1, \ldots, \overline{h}\}$, is optimal as long as $\overline{h}$ satisfies $\Gamma(h) > 0 \forall h \in \{1, \ldots, \overline{h}\}$. Furthermore, we know that $\overline{h} \geq 1$ exists which satisfies $\Gamma > 0 \forall h \in \{1, \ldots, \overline{h}\}$. We conclude that $\exists \overline{h} \geq 1$ such that the strategy is optimal for these histories.

(1b) Suppose $\Omega_i^{h'}(1) < h' - 1$.

(1bi) Suppose $e_i^{h'} = 1$. Choosing the prescribed action of 0 yields a payoff of $V(h')$ as the agent with which he has been matched chooses action 1. If agent i instead chooses action 1, his payoff is still $V(h')$ because he will have chosen action 1 less than his partner so that he is still sure to lose.

(1bii) Suppose $e_i^{h'} = 0$. By choosing his prescribed action of 0, agent i's payoff is at least $V(h' + 1)$ while choosing action 1 yields a payoff of $V(h')$ since he will lose for sure by virtue of having chosen action 1 less frequently than the agent with which he has been matched. Thus, the prescribed behavior is optimal.

(2) Suppose the current level is h' and $h' \in \{\overline{h} + 1, \ldots, k - 1\}$.

(2a) Suppose $\Omega_i^{h'}(1) \geq \overline{h}$. The prescribed action is $e_i^{h'}$. Since the other agents are being flexible, if agent i chooses an action different from $e_i^{h'}$, then he fails to advance for sure with a resulting payoff of $V(h')$. Choosing $e_i^{h'}$ gives at least as high a payoff so that it is optimal.

(2b) Suppose $\Omega_i^{h'}(1) < \overline{h}$.

(2bi) Suppose $e_i^{h'} = 1$. Action 0 yields a payoff of $V(h')$ as he loses for sure as the other agent chooses action 1. Letting agent j be the agent with which agent i is match, we know that $\Omega_j^{h'}(1) \geq \overline{h}$. Since, by supposition, $\Omega_i^{h'}(1) < \overline{h}$, then agent i fails to advance for sure if he chooses action 1 as agent j is more proficient with action 1. Thus, action 1 also yields a payoff of $V(h')$ which implies that the prescribed action of 0 is optimal.

2bii) Suppose $e_i^{h'} = 0$. Action 0 yields a payoff of at least $V(h' + 1)$ since $\Omega_i^{h'}(1) < \overline{h}$ implies $\Omega_i^{h'}(0) \geq h' - \overline{h} + 1$ which implies $\Omega_i^{h'}(0) > \Omega_j^{h'}(0)$. Since action 1 yields a payoff of $V(h')$, then action 0 is optimal. ■

Clearly, at level $k - 1$, an agent will be flexible. Given there is only one higher level, an agent wants to maximize the probability of advancing to that level which means choosing the action which is most appropriate for the environment. Therefore, in equilibrium, agents must eventually be flexible. Next note that once all other agents choose to be flexible that an agent fails to advance for sure if he is not flexible. Therefore, along a subgame perfect equilibrium outcome path, an agent must be flexible when all other agents are flexible. Where optimal behavior is not so obvious is at levels for which other agents are expected to be rigid. When is it optimal for an agent to choose action 1 irrespective of the environment when other agents are expected to act likewise? If environment 1 occurs, then choosing action 1 is clearly best as every other agent is choosing action 1 so that doing different ensures that one does not advance. The problematic case is when environment 0 occurs. In that the other agents are expected to choose action 1, an agent can advance to the next level for sure by being flexible and choosing action 0. The cost to that choice is that, at higher levels, an agent will be less proficient in action 1 than other agents he will compete with for further advancement. This lack of proficiency can be seriously detrimental because action 1 is most frequently the appropriate response to the environment. By instead choosing action 1 when the environment is 0, the probability of advancement falls from 1 to $\frac{1}{2}$ but an agent remains comparable to other agents in terms of his proficiency with the better action.

When it is sufficiently early in the hierarchy, so that there are many future levels over which an agent will compete, it is preferable to be rigid and maintain proficiency. Prudent behavior is to avoid being differentially less proficient in the better action. As the number of remaining levels shrinks, the incentive to go for the short-run strategy of being flexible mounts and eventually becomes sufficiently great that it is not an equilibrium for agents to be rigid. At such a level, equilibrium requires that all agents switch to being flexible. Underlying Theorem 3.1 is then a trade-off between investing in one's proficiency through experience—which means choosing action 1 even when the environment is 0—and improving one's chances of advancing to the next level—which means choosing action 1 when the environment is 1 and action 0 when the environment is 0. The former effect dominates when an agent is low in the hierarchy, so that many rounds of competition remain, and the latter effect dominates when he is sufficiently high in the hierarchy so that relatively few rounds of competition remain.

Theorem 3.1 shows when an equilibrium exists in which agents are initially rigid and then flexible. However, it is important to note that there is always another equilibrium which has agents being flexible at all levels. We are unaware of any basis by which to select one equilibrium over another. In that this is a constant-sum game,[5] there is no Pareto relation between any equilibria. Secondly, none of these equilibria are weakly dominated. Finally, along the equilibrium path, the prescribed behavior is uniquely optimal.[6]

[5]Note that a player's payoff depends only on his final level and a fixed fraction of agents advances to the next level. Since a fraction $(1/2)^h$ of the population will reach level $h \in \{1, \ldots, k-1\}$ and $(1/2)^k$ will reach level k, the average payoff in the population is fixed at $\sum_{h=1}^{k-1}(1/2)^h V(h) + (1/2)^{k-1} V(k)$.

[6]First note that the equilibrium with flexible behavior at all levels is a strict Nash equilibrium. The proof is easy. Given all other agents are choosing the appropriate action for the current environment, the probability of advancing is zero if an agent chooses an action inappropriate for the current environment. Next note that, regardless of one's history of actions, there is a strictly positive probability of advancing to the highest level by choosing the appropriate action because there is always a strictly positive probability that the agent with which one is matched is equally or less proficient than one's self. Hence, flexibility is the unique optimal strategy when everyone else is flexible. The equilibrium with rigid behavior at low levels is not a strict Nash equilibrium in that, for some histories off of the equilibrium path, there are multiple substrategies that are optimal. In particular, when an agent deviated by choosing action 0 when the environment was 0 (during the phase in which he was supposed to be rigid in action 1), an action of either 0 or 1 at level $\overline{h}+1$ is optimal when the current environment is 1 in that both yield probability zero of advancing. This distinction between equilibria does not seem to provide a basis for discriminating between them.

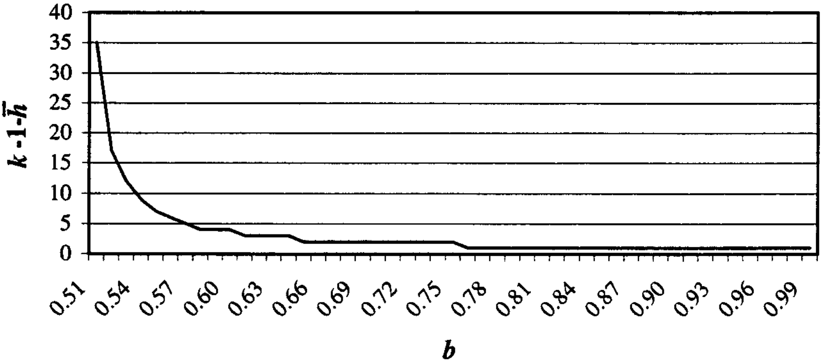

FIG. 1. Minimal number of levels of flexibility when $\theta = k$.

If agents care only about reaching the highest level, then a stronger result can be derived.

THEOREM 3.3. *Assume* $\theta = k$ *so that* $V(k) > V(k-1) = \cdots = V(1) = 0$. *It is a symmetric subgame perfect equilibrium outcome for* $a_i^h = 1$ *for* $h \in \{1, \ldots, \bar{h}\}$ *and* $a_i^h = e_i^h$ *for* $h \in \{\bar{h}+1, \ldots, k-1\}$, *if and only if* $\bar{h} \in \{1, \ldots, k - \lceil((-\ln(1-b))/(\ln(1/2) - \ln(1-b)))\rceil\}$.

Proof. The proof is analogous to the proof of Theorem 3.1. What differs is the expression for $\Gamma(h)$ which is now

$$\Gamma(h) = V(k)\left[\left(\tfrac{1}{2}\right)^{k-h} - (1-b)^{k-h-1}\right]. \tag{8}$$

It is straightforward to show that $\Gamma(h) \geq 0$ for $h \leq k - (-\ln(1-b))/(\ln(1/2) - \ln(1-b))$ and $\Gamma(h) < 0$ for $h > k - ((-\ln(1-b))/(\ln(1/2) - \ln(1-b)))$. ■

When agents care only about the highest level, equilibrium behavior can involve being rigid throughout one's time in the system with the exception of the last $\lceil((-\ln(1-b))/(\ln(1/2) - \ln(1-b)))\rceil$ levels.[7] Plotting the minimal number of levels over which agents are flexible (see Fig. 1), it is clear that, except when the environment is quite volatile (b is close to 0.5), agents can be rigid for all but the very highest levels. For example, agents are rigid except for the final two levels when environment 1 occurs 70% of the time. Rigidity can then be quite common throughout the hierarchy until the uppermost levels at which point opportunism requires that agents shed their rigidity.

[7]We are considering their behavior over levels $1, \ldots, k-1$ not $1, \ldots, k$ as agents do not act at level k but rather revel in the glory of their accomplishments.

TABLE I
Maximum Level of Rigidity when $V(h) = h^{\alpha}$

	$k = 5$	$k = 10$	$k = 15$	$k = 20$
$b = 0.60$				
$\alpha = 2$	0	0	0	0
$\alpha = 4$	0	1	1	1
$\alpha = 6$	0	2	4	4
$\alpha = 8$	0	3	6	7
$\alpha = 10$	0	4	7	10
$b = 0.70$				
$\alpha = 2$	0	0	0	0
$\alpha = 4$	1	3	4	5
$\alpha = 6$	1	5	8	10
$\alpha = 8$	2	6	9	13
$\alpha = 10$	2	6	10	14
$b = 0.80$				
$\alpha = 2$	0	2	2	2
$\alpha = 4$	2	5	8	10
$\alpha = 6$	2	6	10	14
$\alpha = 8$	2	7	11	15
$\alpha = 10$	2	7	11	16
$b = 0.90$				
$\alpha = 2$	1	4	6	7
$\alpha = 4$	2	6	11	15
$\alpha = 6$	3	7	11	16
$\alpha = 8$	3	7	12	16
$\alpha = 10$	3	7	12	17

Finally, we offer some numerical results for when an agent's utility is strictly convex in his ultimate level: $V(h) = h^{\alpha}$ where $\alpha > 1$. Table I reports the maximum level at which agents' behavior is rigid for the equilibrium described in the proof of Theorem 3.1.[8] For example, when $(\alpha, b, k) = (6, 0.7, 10)$, equilibria are known to exist for which agents deploy action 1, regardless of the environment, for the first five levels and then are flexible for the remaining four levels. As intuition would suggest, the maximal level of rigidity is greater when the system is more hierarchical (k is higher) and it is more worthwhile to be proficient in action 1 (b is higher so that action 1 is more frequently the best response to the environment). In that the

[8]In other words, Table I reports the highest value of $\overline{h}$ such that $\Gamma(h) \geq 0 \forall h \in \{1, \ldots, \overline{h}\}$.

maximal level of rigidity is increasing in α, rigidity is present higher up the hierarchy when agents attach more value to progressing to higher levels. This confirms the insight associated with Theorem 3.1.

4. CONCLUDING REMARKS

In the nonequilibrium selection model of Harrington (1998a), the presence of rigid behavior was greatest at the highest levels of a hierarchy (when there are sufficiently many levels in the hierarchy). In that model, agents were endowed with behavioral rules of which there were two generic types. A rigid rule had an agent choose the same action irrespective of the environment so that a rigid agent either always chose action 1 or always chose action 0. A flexible rule had an agent always choose the best action for the current environment which meant tailoring one's action to the environment. In that an agent's behavior was not allowed to vary with the level at which an agent resides, any variation in behavior across levels is due to the evolution of the mix of agent types as a cohort is weeded out in its progression up the hierarchy. It was shown that the proportion of agents deploying a flexible rule goes to zero as the level becomes arbitrarily high. For systems with sufficiently many levels, agents at high levels are then predicted to be rigid relative to agents at low levels.

The current model considers quite different forces. First, agents are not endowed with a decision rule but rather strategically select a rule given their preferences. Second, agents are identical in that they have the same preferences and strategy sets and, furthermore, we focus on symmetric equilibria. Hence, any variation in behavior is not due to the changing mix of types—in that all agents use the same rule—but rather to the variation in each agent's behavior across different levels in the hierarchy. The previous theorems establish the existence of equilibria with very different predictions from the nonequilibrium selection model. Equilibria exist in which agents use a rule that prescribes flexible behavior at the highest levels and rigid behavior at the lowest levels. Agents at high levels are then predicted to be flexible relative to agents at low levels.

What do we conclude from these findings? An ardent believer in equilibrium might conclude that the results in Harrington (1998a) are uninteresting because they are inconsistent with equilibrium. However, our view is that the truth lies in between these two extreme specifications. While agents may be more sophisticated than always being rigid or always being flexible, they are unlikely to be as sophisticated as is presumed by assuming that their behavior is consistent with an equilibrium. Maneuvering through a hierarchy is a complex dynamic problem and it is not obvious why agents should either know or have converged to its solution. In light of that, heterogeneity across agents may not only be in terms of preferences but also in their opinions of what is the best strategy. Indeed, one interpretation of the model of Harrington (1998a) is that all agents have the same objective—whether it is to maximize expected tenure, maximize the probability of getting to the top, or yet some other goal—but differ in what they think is the best way to achieve that objective. Some may see tomorrow as the window to the future so they choose to be flexible. Others may see the need to invest in being proficient in a particular action and choose to be rigid. The point is that the information that agents are apt to have and the capabilities they are apt to be endowed with for processing that information are likely to lead different agents to different conclusions regarding how they should behave. That two methods of characterizing behavior—selection from an exogenous population and equilibrium—generate qualitatively different predictions conveys the importance of developing a model which properly takes account of both the ability and inclination of agents to strategize and the limitations on the strategizing process imposed by the complexity of the environment and the boundedness of agents' processing skills.

Of course, that agents consciously strategize and introspect their way to an equilibrium is only one view of high-level reasoning. Another perspective on such cognitive processes is that agents learn. They may not be able to derive the best rule for traversing a hierarchical social system but they can observe the past and try to infer from it what has previously proven successful in climbing a hierarchy. Though developed to pursue different questions than the one posed here, an initial exploration of that sort is conducted in Harrington (1998c) where the hierarchical selection process in Harrington (1998a) is embedded in a model of social learning. Agents are not endowed with a behavioral rule but rather have certain innate tendencies and, most importantly, are influenced by the past behavior of those who currently occupy the upper levels of the hierarchy. While that analysis does not clarify the ambiguity concerning how behavior should vary across levels in an hierarchy, it does provide some suggestions as to how learning might be introduced to address that issue.

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