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OPTIMAL CARTEL PRICING IN THE PRESENCE OF AN ANTITRUST AUTHORITY*

BY JOSEPH E. HARRINGTON, JR.[1]

Johns Hopkins University, Baltimore, U.S.A.

The dynamic behavior of a price-fixing cartel is explored when it is concerned about creating suspicions that a cartel has formed. Consistent with preceding static theories, the cartel's steady-state price is decreasing in the damage multiple and the probability of detection. However, contrary to those theories, it is independent of the level of fixed fines. It is also shown that the cartel prices higher when a more competitive benchmark price is used in calculating damages.

1. INTRODUCTION

Since the beginning of FY 1997, the Antitrust Division has prosecuted international cartels affecting over $10 billion in U.S. commerce . . . [These cartels] have been bigger, in terms of the volume of affected commerce and the amount of harm caused to American businesses and consumers, than any conspiracies previously encountered by the Antitrust Division. [*Annual Report, Antitrust Division, United States Department of Justice*, 1999: pp. 5–6]

International cartels are estimated to represent a drain of hundreds of millions of euros on the European economy . . . . Since 1998, the number of cartel cases investigated by the Commission has increased dramatically. [*European Community Competition Policy, XXXth Report on Competition Policy*, 2000: pp. 24–25]

As these quotes from American and European antitrust authorities suggest, price-fixing remains a perennial problem, which makes it all the more important that we understand when cartels form and, when they do form, how they behave. Though there is a voluminous theoretical literature on collusive pricing, an important dimension to price-fixing cartels has received little attention. In light of the illegality of price-fixing, *a critical goal faced by a cartel is to avoid the appearance that there is a cartel*. Firms want to raise prices but not suspicions that they are

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coordinating their behavior. If high prices or rapidly increasing prices or, more generally, anomalous price movements may make customers and the antitrust authorities suspicious that a cartel is operating, one would expect this to have implications for how the cartel prices.

This article is the initial step in a research project whose objective is to explore cartel pricing in the presence of detection considerations. Some of the questions to be addressed include: What are the intertemporal properties of the collusive price path? How does the decision to form a cartel and the properties of the collusive price path respond to various instruments of antitrust policy? What types of industry traits make detection more difficult and what are the implications of those traits for cartel pricing?

Towards beginning to address these questions, this study makes two contributions. It is the first study to characterize the transitional dynamics associated with a cartel moving price from its noncollusive level to its steady-state level. At work are two dynamics: Higher prices increase penalties in the event of detection and bigger price changes make detection more likely. In spite of the potential complexity of these dynamics, the cartel price path is shown to be monotonically increasing under fairly general assumptions. The cartel gradually raises price as it balances off increasing profit with increasing the probability of detection. Having established the global stability of the cartel's steady-state price, the second contribution is to explore how antitrust policy impacts that price. Whereas some results confirm existing intuition about the influence of antitrust policy, others yield a new intuition. Comparative statics on the steady-state price reveal that it is decreasing in the damage multiple and the probability of detection, both of which are consistent with previous results. However, the steady-state price is independent of the level of fixed fines. Furthermore, if fines are the only penalty, price is the same as in the absence of antitrust laws. The equivalence between fines and damages found in previous work is then shown to break down in the context of a dynamic model. This article also raises a question that has not been previously considered. In determining damages, an overcharge is calculated that is the cartel's price less a competitive benchmark known as the but for price. I explore how the cartel's steady-state price responds to the but for price and find, quite surprisingly, that lowering the but for price induces the cartel to price higher. Thus, a more stringent benchmark induces *greater* welfare distortions.

Previous work has explored optimal cartel pricing under the constraint of possible detection though using static formulations or highly restricted dynamic models. Block et al. (1981) considered a static oligopoly model in which the probability of detection depends on the price–cost margin and the penalty is a multiple of above-normal profits. They show that the optimal cartel price is below the monopoly price and that the cartel price is decreasing in the penalty multiple and the probability of detection. This model is modified in Spiller (1986), Salant (1987), and Baker (1988) to allow buyers to know a cartel is active though they are uncertain about whether prosecution will be successful. This creates a strategic incentive to manipulate their purchases in anticipation of possibly collecting multiple damages in the future. If the probability of successful prosecution is sufficiently insensitive to price, there is a neutrality result: The buyers consume the simple monopoly quantity so that antitrust laws have no welfare impact. Though surprising, Besanko and Spulber (1990) identify several institutional features that cause the neutrality result to disappear.

Taking a different approach to this problem, Besanko and Spulber (1989, 1990) use a game of incomplete information to model firms and those who are engaging in detection. Firms' common cost is private information and either the antitrust authority (1989) or the buyers (1990) do not observe cartel formation. Instead, they draw inferences from observed price and decide whether or not to pursue a case, which may be costly. Consistent with Block et al. (1981), they find that the cartel's equilibrium price is decreasing in antitrust penalties. Further work using this approach includes LaCasse (1995), Polo (1997), Souam (2001), and Schinkel and Tuinstra (2002).

All of the above-mentioned work uses a static model. There are three papers that consider price-fixing and detection in a dynamic setting. Cyrenne (1999) modifies Green and Porter (1984) by assuming that a price war, and the ensuing raising of price after the war, results in detection for sure and with it a fixed fine. Spagnolo (2000) and Motta and Polo (2003) explore the effects of leniency programs on the incentives to collude when the probability of detection and penalties are both fixed. Though considering collusive behavior in a dynamic setting with antitrust laws, these papers exclude the sources of dynamics that are the foci of the current analysis; specifically, they do not allow detection and penalties to be sensitive to firms' current and past pricing behavior. As a result, these papers have little to say about the transitional pricing dynamics associated with a newly formed cartel. That is a unique contribution of this study.

One of the primary objectives of this literature is understanding the impact of various antitrust policy instruments on cartel behavior. With the exception of the neutrality result, the general findings are that the cartel price is decreasing in the probability of detection, (fixed) fines, and the damage multiple (where the damage penalty equals this multiple times the damages caused by collusion). In many contexts, one can then use either damages or fines to restrain cartel pricing and, for some models, there is an equivalence between these two instruments. In a rich dynamic model, this article also shows that the steady-state price is decreasing in the damage multiple and the probability of detection. However, contrary to previous work, if fines are the only penalty, the cartel's steady-state price is the same as in the absence of antitrust laws. Thus, I find a long-run neutrality result with respect to fixed penalties.

The problem of a cartel setting price to raise profit while trying to avoid detection and penalties has many similarities with other criminal activities and, in particular, tax evasion. Within this broader literature, the current article is the only one to model infinitely lived agents where both detection and penalties depends on the history of criminal activity. Davis (1988) considers an infinitely lived criminal but where the probability of detection depends only on the current level of criminal activity and the penalty is fixed. As a result, there are no meaningful dynamics—the optimal solution is a constant rate of criminal activity. Leung (1995) allows dynamics through recidivism—a caught criminal can commit future crimes—but the likelihood of being caught and the penalty depend only on activity in the current period though the penalty is influenced by past convictions. Macho-Stadler et al. (1999) endogenize the probability of detection in that it depends on the aggregate amount of tax evasion, though an individual takes that probability as fixed and independent of his own behavior. In an overlapping generations model of tax evasion, Olivella (1996) has agents who live for two periods and allows the probability of an audit and the associated penalty to depend on behavior in both periods. With infinitely lived agents, Engel and Hines (1999) consider tax evasion where the penalty depends on evasion in the current and previous period, though detection depends only on the current evasion rate. They do generate some meaningful dynamics as the evasion rate gradually declines to some steady-state value.

2. MODEL

The representative firm's profit when all firms charge a price of $P \in \Omega$ is denoted $\pi(P)$ where Ω is the set of feasible prices. If market demand is $D(\cdot)$ and a firm's cost function is $C(\cdot)$ then the profit function is $\pi(P) = P(D(P)/n) - C(D(P)/n)$, given $n \geq 2$ firms. In the absence of the formation of a cartel, a symmetric equilibrium is assumed to exist that entails a price of $\hat{P}$ and firm profit of $\hat{\pi} \geq 0$.

If firms form a cartel, they meet to determine price. Assume these meetings, and any associated documentation, provides the "smoking gun" if an investigation is pursued.[2] The cartel is detected with some probability and incurs penalties in that event. Detection can be viewed as the end of the horizon with a terminal payoff of $[\hat{\pi}/(1-\delta)] - X^t - F$ where X^t is a firm's damages in the event the cartel is detected, F is any (fixed) fines, and $\delta \in (0, 1)$ is the discount factor.[3] In this model, damages refers to any penalty that is sensitive to the prices charged and fines refer to penalties that are fixed with respect to the endogenous variables. If not detected, collusion continues on to the next period. There is an infinite number of periods. Penalties are assumed to be sufficiently bounded from above for all histories so that the expected present value of a firm's income stream is always positive and thus bankruptcy is avoided.

A cartel member's damages, denoted X^t for period t, are assumed to evolve in the following manner:

$$X^t = \beta X^{t-1} + \gamma x(P^t), \quad \text{where } \beta \in [0, 1), \gamma \geq 0 \tag{1}$$

where P^t is the cartel price. As time progresses, damages incurred in previous periods become increasingly difficult to document and $1 - \beta$ measures the rate

[2] Though it is assumed that an investigation leads to conviction with probability one, all results would go through if the probability of conviction is only required to be positive.

[3] One could allow for the cartel to be reestablished sometime in the future and I suspect many results would not change. Of the 1,300 firms indicted by the Department of Justice over 1962–1980, 14% were recidivists (Bosch and Eckard, 1991).

of the deterioration of the evidence.[4] $x(P^t)$ is the level of damages incurred in the current period where γ is the multiple of damages that a firm can expect to pay if found caught colluding. While U.S. antitrust law specifies treble damages, γ could be less than three because a case is settled out-of-court. Lande (1993) shows that single damages are not unusual for an out-of-court settlement.[5] Current U.S. antitrust practice is $x(P^t) = (P^t - \hat{P})(D(P^t)/n)$.[6]

Detection of a cartel can occur from many sources, some of which are related to price—such as customer complaints—and some of which are unrelated to price—such as internal whistleblowers.[7] Hay and Kelley (1974) find that detection was attributed to a complaint by a customer or a local, state, or federal agency in 13 of 49 price-fixing cases. In the recent graphite electrodes case, it was reported that the investigation began with a complaint from a steel manufacturer who is a purchaser of graphite electrodes (Levenstein and Suslow, 2001). Anomalous pricing may cause customers to become suspicious and pursue legal action or share their suspicions with the antitrust authorities.[8] Though it is not important for my model, I do imagine that buyers (in many price-fixing cases, they are industrial buyers) are the ones who are becoming suspicious about collusion. In practice, the antitrust authorities do not actively engage in detection:

> As a general rule, the [Antitrust] Division follows leads generated by disgruntled employees, unhappy customers, or witnesses from ongoing investigations. As such, it is very much a reactive agency with respect to the search for criminal antitrust violations . . . Customers, especially federal, state, and local procurement agencies, play a role in identifying suspicious pricing, bid, or shipment patterns. [McAnney, 1991, pp. 529, 530]

This is also confirmed by Hay and Kelley (1974) and my own personal communication with antitrust economists at the U.S. Department of Justice.

In modeling the detection process, there is not much relevant evidence to offer guidance and it is not well understood how people identify anomalous events. I

[4] Assuming a depreciation rate to damages is important analytically as it bounds the penalty. An alternative approach is to impose a statute of limitations so that the damage penalty is the sum of damages incurred over a bounded number of periods into the past. I conjecture the same type of insight would emerge under such an assumption. I thank Ted O'Donoghue for making this suggestion. β can also capture the fact that the real value of the damages declines over time as defendants are not required to pay foregone interest; interest is applied only after the judicial determination of an antitrust violation. Blackstone and Bowman (1987) estimate that this reduced the real value of damage penalties by around 50% in 1975 given the average length of a cartel around that time was 8.6 years.

[5] See Connor (2001) and White (2001) for some estimates of damages associated with the lysine cartel. Also see de Roos (1999) for an analysis of the lysine cartel.

[6] "[After the] court selects a 'competitive price,' [it] . . . awards the plaintiff the difference between the competitive estimate and the amount paid, times the quantity purchased, trebled." (Breit and Elzinga, 1986, p. 21.)

[7] Bryant and Eckard (1991) estimate the chances of a price-fixing cartel being indicted in a 12-month period to be around 15%.

[8] The Nasdaq case is one in which truly anomalous pricing resulted in suspicions about collusion. It was scholars rather than market participants who observed that dealers avoided odd-eighth quotes and ultimately explained it as a form of collusive behavior (Christie and Schultz, 1994). Though the market-makers did not admit guilt, they did pay an out-of-court settlement of around $1 billion.

have then decided to take a more agnostic approach by specifying a class of probability of detection functions and exploring how properties of those functions influence cartel's pricing dynamics. Letting $\phi(P^t, P^{t-1})$ denote the probability of detection in period t, it is allowed to depend on the current price and the previous period's price. One can interpret $\phi(\hat{P}, \hat{P})$ as a baseline probability of detection driven by, for example, internal whistleblowers. The inclusion of a more comprehensive price history would significantly complicate the analysis—greatly expanding the state space—without any apparent gain in insight. Though proving the existence of an optimal path only needs continuity of $\phi(P^t, P^{t-1})$, characterization of the price path will require more structure. Results will then be derived when detection is driven only by the price change, $P^t - P^{t-1}$, after which I will discuss their robustness with allowing the price level, P^t, to also matter.

This modeling of detection warrants some further discussion, since it does not explicitly model those agents who might engage in detection. The first point to make concerns tractability. Even with a single agent (i.e., the cartel), this is a complex model with two state variables, (P^{t-1}, X^{t-1}), and thereby two distinct sources of dynamics—detection and antitrust penalties. As currently formulated, the model is rich enough to provide new insight into cartel pricing, even with a simple modeling of the detection process, and a more complex model at this stage is likely to prove intractable. Tractability issues aside, there is another motivation for this approach. The objective of this article is not to develop insight and testable hypotheses about detection but rather about cartel pricing. A good model of the detection process is then defined to be one that is a plausible description of how cartel members *perceive* the detection process. It strikes me as quite reasonable that firms might simply postulate that higher prices or bigger price changes result in a greater likelihood of creating suspicions without having derived that relationship from first principles about buyers.

In period 1, firms have the choice of forming a cartel, and risking detection and penalties, or earning noncollusive profit of $\hat{\pi}$. If they choose the former, they can, at any time, choose to discontinue colluding. In that event, it is assumed they will never collude again and receive a terminal payoff of $[\hat{\pi}/(1-\delta)] - \sigma(P^{t-1}, X^{t-1})$ where the last period of collusion is period $t-1$. $\sigma(P^{t-1}, X^{t-1})$ is to be interpreted as the present value of the expected penalty when collusion is discovered after the dissolution of the cartel (e.g., incidental discovery of incriminating documents in an unrelated legal case). The cartel chooses a symmetric infinite price path so as to maximize the expected sum of discounted income. It is important to emphasize that we do not ignore the usual equilibrium conditions, which ensure that a firm will go along with the collusive price path. One can cast the preceding model as an infinite-horizon perfect monitoring (though nonrepeated) game played among the n firms. The joint profit-maximizing price path that is characterized here is then the best symmetric equilibrium price path when δ is sufficiently close to one; that is, when the equilibrium conditions do not bind. Given the complexity of the dynamics associated with detection and antitrust penalties, it makes sense to initially characterize this price path, which, as the reader will see, is a substantive task in itself. The case when incentive compatibility constraints bind is explored in Harrington (2003a).

3. EXISTENCE OF AN OPTIMAL PRICE PATH

The basic problem is one of the cartel manager choosing a price path to maximize the expected present value of the representative cartel member's income stream. To establish the existence of an optimal price path, dynamic programming is used. The state variables are yesterday's price, P^{t-1}, and accumulated damages, X^{t-1}. $V(P^{t-1}, X^{t-1})$ denotes the value function when the cartel is still functioning as of period t and is defined as the fixed point to:

(2)
$$\begin{aligned} V(P^{t-1}, X^{t-1}) = \max_{P\in\Omega} \pi(P) &+ \delta\phi(P, P^{t-1})[(\hat{\pi}/(1-\delta)) - \beta X^{t-1} - \gamma x(P) - F] \\ &+ \delta[1-\phi(P, P^{t-1})] \max\{V(P, \beta X^{t-1} + \gamma x(P)), (\hat{\pi}/(1-\delta)) \\ &\qquad - \sigma(P, \beta X^{t-1} + \gamma x(P))\} \end{aligned}$$

$(\hat{\pi}/(1-\delta)) - \beta X^{t-1} - \gamma x(P) - F$ is the terminal payoff associated with the cartel being detected. Also note that firms have the future option of dismantling the cartel and receiving a terminal payoff of $(\hat{\pi}/(1-\delta)) - \sigma(P, \beta X^{t-1} + \gamma x(P))$.

The following assumptions are sufficient for existence though additional structure will be required to characterize the price path.

- (A1) $\pi : \Omega \to \Re$ is bounded and continuously differentiable and $\exists P^m > \hat{P}$ such that $\pi'(P) \gtreqqless 0$ as $P \lesseqqgtr P^m$.
- (A2) $x : \Omega \to \Re_+$ is bounded, continuously differentiable, and nondecreasing.
- (A3) $\phi : \Omega^2 \to [0, 1]$ is continuous.
- (A4) $\sigma : \Omega \times \Re_+ \to \Re_+$ is bounded, continuous, and nondecreasing.
- (A5) Ω is a compact convex subset of $\Re_+$ and $[\hat{P}, P^m] \subseteq \Omega$.

THEOREM 1. *Assume A1–A5. An optimal price path exists.*

The proof is in Harrington (2001), which entails a modification of standard arguments in Stokey and Lucas (1989).

4. PROPERTIES OF AN OPTIMAL PRICE PATH

In order to characterize the intertemporal structure of the price path, additional structure on the probability of detection function is required. Previous static analyses of cartel pricing assume the probability of detection depends only on the price level and is increasing (e.g., Block et al., 1981). I initially explored this case and found a counterfactual result: There is a price spike in the first period of collusion with price declining thereafter (Harrington, 2001). As firms collude over time, one can show that accumulated damages on an optimal cartel price path grow, which means a higher penalty in the event of detection. Since the probability of detection is increasing in price, a natural response to a higher potential penalty is to lower price and thereby reduce the likelihood of detection. Thus, firms steadily lower price over time so as to make detection less likely. To my knowledge, there is no empirical evidence in support of such a price path. Indeed, it is quite contrary to the price paths documented in the market for citric acid (Connor, 1998), graphite electrodes (Harrington, 2003b), and bromine (Levenstein and Suslow, 2001) where, at least initially, the price path is rising over many periods. In that a decreasing price path (after an initial price spike) is the logical implication of having detection depend only on the price level, I infer that detection is not largely driven by the price level. A natural alternative is that detection is driven instead by price changes. That is the avenue I will pursue here. However, I will later discuss the robustness of these results when detection depends on both price changes and price levels.

In specifying properties for the probability of detection function, the basic story to have in mind is that the environment is perceived to be stable so that cartel members expect buyers to anticipate price being fairly stable. Thus, bigger price changes—up and even possibly down—are more likely to be perceived as anomalous and thus trigger suspicions about the presence of a cartel.[9] With this story in mind, I have sought to impose the minimal structure necessary to characterize pricing dynamics.

(A6) $\exists\hat{\phi} : \Re \rightarrow [0, 1]$ and $g : \Omega \rightarrow \Re_{++}$, where g is a strictly positive, nonincreasing, continuously differentiable function, such that

$$\phi(P^t, P^{t-1}) = \hat{\phi}((P^t - P^{t-1})g(P^{t-1})) \quad \forall(P^t, P^{t-1}) \in \Omega^2$$

(A7) If $x \geq y \geq 0$ then $\hat{\phi}(x) \geq \hat{\phi}(y)$.
(A8) $\hat{\phi}(x) \geq \hat{\phi}(0)\forall x \in \Re$ and $\hat{\phi}(0) \in [0, 1)$.
(A9) $\exists\varepsilon > 0$ such that $\hat{\phi}$ is continuously differentiable in an ε-ball around P', $\forall P' \in \Omega$, and $\hat{\phi}'(0) = 0$.

A6–A8 specify that the probability of detection depends on the change in price, is nondecreasing for price increases, and is minimized by keeping price constant. Note that if g is a constant then the probability of detection depends only on the size of price movements whereas if $g(P^{t-1}) = 1/P^{t-1}$ then it depends on the percentage change in price. A9 requires differentiability around a price change of zero and is a necessary technical condition.[10] Though we state $\hat{\phi}'(0) = 0$ as an assumption, it actually follows from A8 and assuming the derivative of $\hat{\phi}$ at a price change of zero is defined. Two additional assumptions involving the profit and damage functions are required.

[9] Here it is worth mentioning that I am attempting to model the process by which buyers come to *entertain* the hypothesis that firms are colluding. I actually imagine a multistage process is at work where, in stage 1, buyers, in response to abnormal pricing behavior, think that something is awry and possibly consider that firms may have cartelized; and, in stage 2, buyers file a complaint with the antitrust authorities who might then engage in a systematic analysis of the market to determine whether it is a case worth pursuing. My model implicitly focuses on the first stage.

[10] I want to acknowledge Ali Khan for the proper statement of A9. He developed an elegant example that showed that a function can be differentiable at a point but not be differentiable in an ε-ball around that point.

(A10) $\pi(P) - \delta\hat{\phi}(0)[(\frac{\gamma x(P)}{1-\beta}) + F] > \hat{\pi} \forall P \in (\hat{P}, P^m]$.
(A11) $\exists P^* \in (\hat{P}, P^m]$ such that

$$\pi'(P) - [\delta\hat{\phi}(0)/(1 - \delta\beta(1 - \hat{\phi}(0)))]\gamma x'(P) \gtreqless 0 \quad \text{as} \quad P \lesseqgtr P^*$$

In Harrington (2001), it is shown that A10 is sufficient to ensure that, at a steady-state price of P, colluding is preferable to not colluding. A11 requires quasiconcavity of an income function, which is defined to be profit less some multiple of damages. It is shown later that these assumptions are satisfied under standard conditions on demand and cost functions.

4.1. *Monotonicity of the Price Path.* Theorem 2 shows that collusion is infinitely lived, involves a nondecreasing price path, and the long-run price is P^* (as defined in A11). These properties for the price path are derived when firms choose to cartelize.[11] All proofs are in the appendix.

THEOREM 2. *Assume A1–A11 and $P^0 \in [\hat{P}, P^*)$. If it is optimal to form a cartel then it is optimal to collude in all periods and if $\{P^t\}_{t=1}^{\infty}$ is an optimal price path then (i) it is nondecreasing over time and (ii) $P^t \rightarrow P^*$ as $t \rightarrow \infty$.*

The intuition is immediate. In that larger price movements result in a higher probability of detection, the optimal price path has the cartel gradually increase price to its long-run target value of P^* with the hope of not triggering suspicions. Although the monotonicity of the price path is unsurprising, it is worth emphasizing the generality under which it is proven. Although the firm profit and damage functions are presumed to be well behaved (so as to generate the quasiconcavity in A11), a very wide class of detection functions—depending only on price changes—is allowed. Besides providing a clean characterization of pricing dynamics, Theorem 2 also serves to establish the global stability of the steady-state price, which will be intensively explored in the remainder of this section.

Though the equations characterizing the dynamic path of price are rather complex, there is a simple equation defining the long-run or steady-state cartel price, P^*, which makes it conducive for performing comparative statics. P^* is defined as the unique solution to

$$\pi'(P^*) - [\delta\hat{\phi}(0)/(1 - \delta\beta(1 - \hat{\phi}(0)))]\gamma x'(P^*) = 0 \tag{3}$$

which has a natural interpretation. In the steady state, consider the effect of a one time marginal change in price from P^*. First, there is the marginal change in current profit of $\pi'(P^*)$. Second, there is the marginal change in damages of $x'(P^*)$. Of course, higher damages are a loss only when the cartel is detected and, in the meantime, they depreciate at rate β. The expected present value of the loss

[11] Here are two sets of sufficient conditions for cartel formation to occur when $P^0 = \hat{P}$ and $X^0 = 0$. First, γ and F are sufficiently small. Second, $x(\hat{P}) = 0$ and $F = 0$. The first case is immediate and the second case is shown in Harrington (2001). The latter is robust to small changes in the assumptions.

from this marginal change in damages is $[\delta\hat{\phi}(0)/(1-\delta\beta(1-\hat{\phi}(0)))]\gamma x'(P^*)$. Third, there is the change in the expected penalty associated with previously incurred damages, $\hat{\phi}'(0)[(\gamma x(P^*)/(1-\beta))+F]$, but this equals zero since $\hat{\phi}'(0)=0$. As described in (3), the steady-state cartel price is set to equate the rise in profit from a higher price with the expected present value of the marginal rise in damages from that higher price.

If the profit function is concave ($\pi''<0$), the damage function is strictly increasing ($\gamma x'>0$), and the minimum probability of detection is positive ($\hat{\phi}(0)>0$), it follows from (3) that $P^*<P^m$ so that the cartel price is bounded below the simple monopoly price in all periods. Thus, antitrust law constrains pricing behavior even in the long run. However, note that if $\gamma=0$, so that the only penalty is fixed fines, then $P^*=P^m$. *At the steady state, fixed fines do not constrain the cartel's price*. It is true, however, that higher fines can be expected to affect the speed with which price is raised and, if fines are sufficiently high, they can deter cartel formation altogether. This is summarized as Result 1.

Result 1. The steady state cartel price is less than the simple monopoly price when penalties include damages. The steady-state cartel price equals the simple monopoly price when the only penalty is fixed fines (assuming cartel formation occurs).

This independence result with respect to fines can be explained as follows. In the long run, price settles down so that price changes converge to zero. Given that $\hat{\phi}'(0)=0$, marginal changes in price have no first-order effect on the probability of detection though they continue to have a first-order effect on the potential penalty through the damage function. Thus, factors that influence the relationship between price and the size of the penalty—the discount factor, the rate of depreciation of damages, the damage multiple, and the damage function—all influence the long-run price. As a result, if there are only fines and no damages then, as price changes go to zero, marginal changes in price have no effect on the expected penalty so that the cartel price converges to the simple monopoly price.

The independence of the steady-state cartel price with respect to fixed penalties is in stark contrast to static models of collusive pricing in the presence of antitrust laws and represents a unique implication of a dynamic approach. In those models, there is an equivalence between fines and damages in the sense that any price resulting for some damage multiple could alternatively be generated through an appropriately selected fine.[12] In contrast, when detection depends on

[12] To see this point, consider a static model in which the cartel maximizes profit less expected penalties and let $\tilde{\phi}(P)$ denote the probability of detection (note that it only depends on the price level). When the penalty is damages, the expected penalty is $\tilde{\phi}(P)\gamma x(P)$ and when the penalty is fines, the expected penalty is $\tilde{\phi}(P)F$. The optimal cartel price is defined by that price that equates marginal profit with marginal expected penalty. Next suppose a price of $\bar{P}$ is induced by a policy of damages:

$$\pi'(\bar{P})=\tilde{\phi}'(\bar{P})\gamma x(\bar{P})+\tilde{\phi}(\bar{P})\gamma x'(\bar{P})$$

We can then induce that same price with fines by setting F so that

$$\tilde{\phi}'(\bar{P})F=\tilde{\phi}'(\bar{P})\gamma x(\bar{P})+\tilde{\phi}(\bar{P})\gamma x'(\bar{P}) \Leftrightarrow F=\gamma x(\bar{P})+[\tilde{\phi}(\bar{P})/\tilde{\phi}'(\bar{P})]\gamma x'(\bar{P})$$

Thus, any price can be implemented either by fines or damages.

price changes in a dynamic model, price is bounded below the simple monopoly price when penalties include damages but converges to the simple monopoly price when damages are not deployed. Thus, if antitrust policy is intended to constrain cartel prices in the steady state, it is essential that penalties be responsive to the price charged.

It is worth noting that the steady-state price can also be independent of the damage multiple though it requires that damages are proportional to profit. If $x(P^t) = \theta\pi(P^t)$ for some $\theta > 0$ then (3) once again implies $P^* = P^m$. For example, this proportionality occurs under the standard damage formula of $x(P^t) = (P^t - \hat{P})(D(P^t)/n)$ when marginal cost is constant and the but for price is the competitive price.

4.2. *Comparative Statics.* Assume the market demand function, $D(\cdot)$, is twice differentiable and each firm has constant marginal cost of c. A firm's profit is then $\pi(P) = (P - c)(D(P)/n)$. Further assume $D''(P) \leq 0$ so that A1 holds. Next suppose that the damage function is $x(P) = (P - \hat{P})(D(P)/n)$ where $\hat{P} > c$. To ensure that A11 is satisfied, define

$$\Psi(P) \equiv \pi(P) - \kappa x(P) = (1/n)[(P-c)D(P) - \kappa(P - \hat{P})D(P)]$$

where

$$\kappa \equiv \delta\hat{\phi}(0)\gamma/(1 - \delta\beta(1 - \hat{\phi}(0)))$$

Note that if $\Psi''(P) < 0$ then P^* is defined by $\Psi'(P^*) = 0$. Taking the first two derivatives of Ψ,

$$\text{(4)} \qquad \begin{aligned} \Psi'(P) &= (1/n)\{(1-\kappa)[(P-c)D'(P) + D(P)] + \kappa(\hat{P} - c)D'(P)\} \\ \Psi''(P) &= (1/n)\{(1-\kappa)[2D'(P) + (P-c)D''(P)] + \kappa(\hat{P} - c)D''(P)\} \end{aligned}$$

$\Psi''(P) < 0$ if $\kappa < 1$ and $D' \leq 0$. For P^* to exceed $\hat{P}$, one needs

$$\text{(5)} \qquad \Psi'(\hat{P}) = (1/n)\{(\hat{P} - c)D'(\hat{P}) + (1-\kappa)D(\hat{P})\} > 0$$

Since $(\hat{P} - c)D'(\hat{P}) + D(\hat{P}) > 0$, as $\hat{P}$ is associated with the noncollusive outcome, then $\Psi'(\hat{P}) > 0$ if κ is sufficiently close to zero, which holds, for example, if either the probability of detection or the damage multiple is sufficiently small. P^* is then defined by

$$\text{(6)} \qquad (1-\kappa)[(P^* - c)D'(P^*) + D(P^*)] + \kappa(\hat{P} - c)D'(P^*) = 0$$

Taking the total derivative of (6) with respect to κ,

$$\text{(7)} \qquad \frac{\partial P^*}{\partial \kappa} = \frac{[(P^* - c)D'(P^*) + D(P^*)] - (\hat{P} - c)D'(P^*)}{(1-\kappa)[2D'(P^*) + (P^* - c)D''(P^*)] + \kappa(\hat{P} - c)D''(P^*)} < 0$$

It is straightforward to show that κ is increasing in γ, $\hat{\phi}(0)$, β, and δ. The following results are then immediate.

RESULT 2. The steady-state cartel price is reduced when (i) the damage multiple, γ, is increased; (ii) the probability of detection, $\hat{\phi}(\cdot)$, is increased; (iii) the rate at which damages persist over time, β, is increased; and (iv) the discount factor, δ, is increased.

Numerical analysis reveals that when a change in a parameter causes the long-run cartel price to fall (rise), the entire price path declines (rises); see Harrington (2001). The first three results are quite immediate. To explain the last one, note that the cartel faces an intertemporal trade-off in that a higher price in the current period raises current profit but lowers the future payoff by both increasing the likelihood of detection and the penalty. As cartel members become more patient, they then prefer lower cartel prices. By comparison, standard repeated game models of collusion lacking detection considerations find that more patient firms price higher because it loosens up incentive compatibility constraints.

A final interesting comparative static exercise is to consider the influence of the but for price, $\hat{P}$, on the steady-state cartel price. Recall that the but for price is the price used in calculating damages. To better understand the ensuing result, it will be useful to generalize the damage function to

$$x(P) = (P - \hat{P})[\alpha(D(P)/n) + (1 - \alpha)(D(\hat{P})/n)] \tag{8}$$

where $\alpha \in [0, 1]$. U.S. antitrust practice is captured by $\alpha = 1$ whereas if damages were specified to equal the loss in consumer surplus then $\alpha = 0.5$, using a linear approximation. It is straightforward to derive

$$\frac{\partial P^*}{\partial \hat{P}} = \frac{\kappa[(1 - \alpha)D'(\hat{P}) - \alpha D'(P^*)]}{(1 - \kappa\alpha)[2D'(P^*) + (P^* - c)D''(P^*)] + \kappa\alpha(\hat{P} - c)D''(P^*)} \tag{9}$$

As before, the denominator is negative and it is immediate that the numerator is positive when $\alpha = 1$. Thus, if the cartel anticipates that a more competitive standard will be applied in calculating damages, this induces the cartel to set a *higher* price in the long run.

RESULT 3. The steady-state cartel price is decreasing in the but for price, $\partial P^*/\partial \hat{P} < 0$.

To understand this intriguing finding, first note that the numerator is positive (negative) when α is sufficiently close to one (zero). Next note that as α rises, the cartel's price has more of an influence on the level of demand used for calculating damages. Thus, the cartel price is decreasing in the but for price when the number of units upon which damages are assessed is sufficiently sensitive to the cartel's price. We can now explain this result. Lowering $\hat{P}$ raises the total amount of damages by increasing the overcharge, which is the amount of damages assigned per unit of damage demand. One response is to lower the cartel price so as to bring back down the overcharge. Alternatively, firms could raise the cartel price so as to reduce the number of units upon which damages are assessed. The latter effect dominates when the number of units used for the damage calculation is sufficiently sensitive to the collusive price.

4.3. *Robustness.* As mentioned at the beginning of Section 4, the current model generates counterfactual results when detection depends only on the price level in that the optimal cartel price path is characterized by an initial price spike and then a declining path thereafter. Though this suggests that detection is not exclusively driven by the price level, it is quite possible that detection depends on both factors. One then wonders to what extent the results of this study are robust to allowing detection to also depend on the price level. Though the complexity of allowing for both price changes and levels makes the model intractable, numerical analysis and a bit of intuition can shed light on this issue.

Let us begin with the dynamics of the price path. When only price changes matter, we learned price is increasing. Now consider the following probability of detection function:

$$\phi(P^t, P^{t-1}) = \min\left\{\phi_0 + \lambda\phi_1(P^t - \hat{P})^2 + (1-\lambda)\phi_2(P^t - P^{t-1})^2, 1\right\}$$

When $\lambda = 0$ then detection is only sensitive to price movements, whereas it depends only on the price level when $\lambda = 1$. Set $\phi_0 = 0.01$, $\phi_1 = 0.00000324$ (which implies that when $\lambda = 1$ then setting the monopoly price results in a 10% chance of detection), and $\phi_2 = 0.00003204$ (so that, when $\lambda = 0$, raising price from the noncollusive to the monopoly price in a single period results in a 90% chance of detection). The optimal price path was calculated for $\lambda \in \{0, 0.01, \ldots, 0.99, 1\}$.[13] All of the resulting price paths can be found at www.econ.jhu.edu/People/Harrington/cartelpricing.avi, where, by clicking the image, an animated movie shows how the price path changes when λ is raised from 0 to 1 so that the importance of the price level with regards to detection is increased relative to that of price changes. The smooth movement of the price path suggests that the price path is continuous with respect to λ.

Typical of these price paths is Figure 1, which shows what occurs when $\lambda = 0.15$. Consistent with Theorem 2, price is gradually rising at first but, contrarily, eventually declines and approaches its steady-state value from above. The explanation is as follows. At the time of cartel formation, price is at its noncollusive level so that the task is to raise price but without triggering detection. This results in a gradual increase in price. As price tends towards its steady-state value, price changes are going to zero, which means there is no first-order effect of price changes on detection but, with price bounded above the noncollusive level, there *is* a first-order effect on detection from changing the price level. Hence, as price converges, the marginal impact of the price level on detection is becoming large

[13] This simulation assumes market demand is $1{,}000 - P$, constant marginal cost of zero, and parameter values of $n = 2$, $\delta = 0.75$, $\beta = 0.95$, $\gamma = 1$, and $F = 0$.

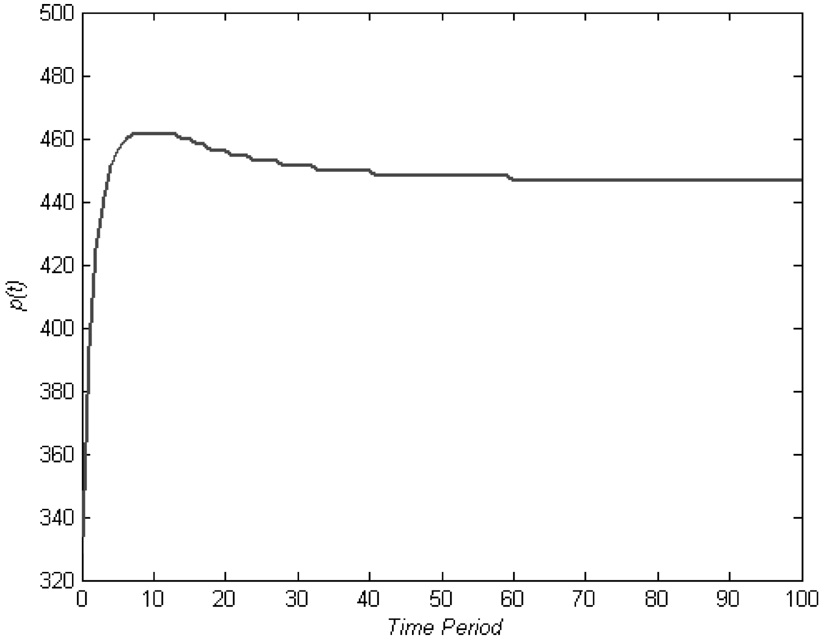

FIGURE 1

OPTIMAL CARTEL PRICE PATH

relative to the marginal impact of price changes. The price path is then declining as the cartel seeks to lower the probability of detection with a lower price level. While the initial gradual rise in price is robust to allowing detection to depend on price levels, the monotonicity throughout the path is not.

That the steady-state price is less than the simple monopoly price (when penalties include damages) and decreasing in the damage multiple and the probability of detection would seem robust as they do not appear to be due to the form of the detection technology. The driving forces seem to be the same as those that drive these results in the static models. Of course, the comparative static of the steady-state price with respect to the but for price is new to the literature though I do not believe it is tied to the dynamic structure. Indeed, let me show that the result holds for a static model. Let $\tilde{\phi} : \Omega \rightarrow [0, 1]$ denote the probability of detection function that depends only on the price level. The cartel's optimal price in the static problem is then

$$\tilde{P} = \arg\max (P - c)(D(P)/n) - \tilde{\phi}(P)[\gamma(P - \hat{P})(D(P)/n) + F]$$

Assuming this objective function is strictly concave, it is straightforward to derive:

$$\text{sign}\left\{\frac{\partial \tilde{P}}{\partial \hat{P}}\right\} = \text{sign}\{\tilde{\phi}'(\tilde{P})D(\tilde{P}) + \tilde{\phi}(\tilde{P})D'(\tilde{P})\}$$

Thus, if $\tilde{\phi}'(\tilde{P})$ is not too large then $\partial\tilde{P}/\partial\hat{P} < 0$, consistent with Result 3. The two forces identified in connection with Result 3 are still present here, but there is a third force as well. In response to a lower but for price (and thus higher penalties), there is an additional incentive to lower price as doing so reduces the probability of detection. As long as that effect is not too strong, the cartel price is decreasing in the but for price. I then believe Result 3 is quite robust.

The final result to consider is the neutrality of the steady-state price with respect to fixed penalties, a result not found in static models. As explained in Section 4.1, this result is closely tied to the detection technology. Allowing for detection to depend on price levels will cause us to lose pure neutrality but the result is not knife-edge, as a small role for price levels ought to imply the near-neutrality of the steady-state price to fines. Key to the argument for the neutrality result is that, in the steady state, there is no first-order effect of price on the probability of detection because $\partial\phi(P^*, P^*)/\partial P^t = \hat{\phi}'(0) = 0$. However, if detection depends on the price level then $\partial\phi(P^*, P^*)/\partial P^t > 0$, which, we will argue, results in the steady-state price being below the simple monopoly price even when penalties are fixed. But suppose not. By marginally lowering price below the simple monopoly price, there is a (favorable) first-order effect on the expected penalty through its effect on the probability of detection whereas there is only an (unfavorable) second-order effect on current profit. The conclusion to draw is that when detection is largely driven by price changes, the steady-state price is close to the simple monopoly price when penalties are fixed. As with many neutrality results, they describe an extreme case but, as long as one has robustness in the sense of continuity, they provide a useful benchmark.

5. CONCLUDING REMARKS

In choosing a price path, it is natural to expect a price-fixing cartel to try to avoid creating suspicions that collusion is afoot. This study is the first to explore how detection impacts cartel pricing in the context of a dynamic model when detection and penalties are endogenous. There are many directions that one can go from here. With this particular model, a natural next step is to take account of (binding) equilibrium conditions so as to ensure that, more generally, firms do not want to deviate from the cartel price path. Of particular interest is to explore how antitrust policy interacts with these conditions. To what extent do concerns about detection make cheating more or less desirable and what is the role of antitrust policy in destabilizing the internal stability of cartels? This is explored in Harrington (2003a). A second set of extensions is to encompass leniency programs. There have been a number of interesting papers exploring how leniency—in the form of allowing cartel members who provide evidence to receive reduced penalties—affects the degree of collusion and welfare. That work, however, does not take into account the endogeneity of detection and antitrust penalties. A third extension is related to the fact that detection has been assumed to depend only on movements in a common firm price. However, suspicions about collusion are also generated by firms' prices moving in tandem. If buyers may infer, rightly or wrongly, from parallel price movements that a cartel is present, this will also have implications for pricing behavior.

In conclusion, by taking into account issues of detection, theory may eventually be able to empirically distinguish between explicit and tacit collusion. Tacit collusion I define as when firms engage in a pricing arrangement that serves to raise price and is achieved without explicit communication. Although it is possible to prosecute tacitly colluding firms, it is very difficult. Explicit collusion is when firms engage in direct communication regarding the setting of prices (or some other form of collusion such as market allocation). Although antitrust case law makes a critical distinction between them, existing theory does not.[14] Since explicit collusion is vastly more prosecutable than tacit collusion, concerns about detection should have a bigger impact on pricing dynamics when firms have formed a "hard-core cartel." This suggests a promising avenue to empirically distinguish between the two modes of collusion.

APPENDIX

A.1. *Proof of Theorem 2.* There are several steps in the proof. First, it is shown that if it is optimal to form a cartel then it is optimal to collude forever. Second, the optimal price path is bounded above by P^*. Third, the optimal price path is nondecreasing over time. Fourth, the optimal price path converges to P^*.

- It is optimal to collude forever.

The strategy is to show that if it is optimal to collude in, say, period T then it must be optimal to collude in period $T+1$. Assume it is optimal to form a cartel. It is sufficient to show that it is optimal to collude forever when $\sigma(P^{t-1}, X^{t-1}) = 0\ \forall(P^{t-1}, X^{t-1})$ so that the terminal payoff from stopping collusion is $\hat{\pi}/(1-\delta)$. Suppose it is optimal to collude until period T where T is finite. For it to be optimal to collude in T, it must be true that

$$\pi(P^t) - \delta\phi(P^T, P^{T-1})[\beta X^{T-1} + \gamma x(P^t) + F] + \frac{\delta\hat{\pi}}{1-\delta} \geq \frac{\hat{\pi}}{1-\delta}$$

The LHS is the payoff from colluding in T and stopping collusion as of $T+1$ and the RHS is the payoff from stopping collusion in T. This expression is equivalent to

$$\pi(P^t) - \delta\phi(P^T, P^{T-1})[\beta X^{T-1} + \gamma x(P^t) + F] \geq \hat{\pi} \tag{A.1}$$

[14] There are a few exceptions. McCutcheon (1997) models meetings between firms. Athey et al. (2004) and Athey and Bagwell (2001) model the exchange of cost information by firms, which would seem more appropriate for explicit than tacit collusion.

For it to be optimal to dismantle the cartel in $T+1$, it is necessary that

$$\text{(A.2)} \quad \frac{\hat{\pi}}{1-\delta} > \pi(P^T) - \delta\hat{\phi}(0)[\beta(\beta X^{T-1} + \gamma x(P^T)) + \gamma x(P^T) + F] + \frac{\delta\hat{\pi}}{1-\delta}$$
$$\Leftrightarrow \hat{\pi} > \pi(P^T) - \delta\hat{\phi}(0)[\beta(\beta X^{T-1} + \gamma x(P^T)) + \gamma x(P^T) + F]$$

The RHS of the first line in (A.2) is the payoff from maintaining a price of P^T in $T+1$ and then stopping collusion as of $T+2$.[15] Note that $\phi(P^T, P^T) = \hat{\phi}(0)$. Combining (A.1) and (A.2):

$$\pi(P^T) - \delta\phi(P^T, P^{T-1})[\beta X^{T-1} + \gamma x(P^T) + F]$$
$$\geq \hat{\pi} > \pi(P^T) - \delta\hat{\phi}(0)[\beta(\beta X^{T-1} + \gamma x(P^T)) + \gamma x(P^T) + F]$$

A necessary condition for this to hold is

$$\pi(P^T) - \delta\phi(P^T, P^{T-1})[\beta X^{T-1} + \gamma x(P^T) + F]$$
$$> \pi(P^T) - \delta\hat{\phi}(0)[\beta(\beta X^{T-1} + \gamma x(P^T)) + \gamma x(P^T) + F]$$

or

$$\hat{\phi}(0)[\beta(\beta X^{T-1} + \gamma x(P^T)) + \gamma x(P^T) + F] > \phi(P^T, P^{T-1})[\beta X^{T-1} + \gamma x(P^T) + F]$$

Since, by A8, $\phi(P^T, P^{T-1}) \geq \hat{\phi}(0)$, a necessary condition is

$$\beta(\beta X^{T-1} + \gamma x(P^T)) + \gamma x(P^T) > \beta X^{T-1} + \gamma x(P^T) \Leftrightarrow \frac{\gamma x(P^T)}{(1-\beta)} > X^{T-1}$$

Intuitively, if it is optimal to collude at a price of P^T in period T but it is not optimal to do so in $T+1$ then damages must be higher in $T+1$. For that to be the case, what is added to damages in T, $\gamma x(P^T)$, must exceed the amount of damages lost through depreciation, $(1-\beta)X^{T-1}$. This produces the above condition.

Next note that it is never optimal for the cartel price to exceed the simple monopoly price of P^m. Relative to a price of P^m, a higher price yields strictly lower current profit, weakly higher damages, and, as price initially starts below P^m, a weakly higher probability of detection. It is straightforward to show that a price path with prices above P^m yields a lower payoff to one in which all those prices exceeding P^m are replaced with P^m. Since then $P^T \leq P^m$, it follows from A10 that

$$\text{(A.3)} \qquad \pi(P^T) - \delta\hat{\phi}(0)\left[\left(\frac{\gamma x(P^T)}{1-\beta}\right) + F\right] > \hat{\pi}$$

[15] The assumption is used that a firm must strictly prefer not to collude for it to dissolve the cartel.

Given it has been shown that X^{T-1} is bounded above by $\gamma x(P^t)/(1-\beta)$, (A.3) contradicts (A.2). This contradiction establishes that the claim that collusion stops in finite time is false.

- The optimal price path is bounded above by P^*.

The proof strategy is to show that if the price path ever exceeds P^* that a higher payoff is realized by pricing at P^* forever, starting in the period with which price first exceeds P^*.

Assuming firms collude forever, the payoff starting from period t' for the collusive price path $\{\bar{P}^t\}_{t=1}^{\infty}$ can be represented as

(A.4)

$$\begin{aligned}&\left[\pi(\bar{P}^{t'})-\bar{\Delta}^{t'}\gamma x(\bar{P}^{t'})-(\hat{\pi}-(1-\delta)F)\right]-\bar{\Delta}^{t'}\beta X^{t'-1}\\&\quad+\sum_{t=t'+1}^{\infty}\delta^{t-t'}\left\{\prod_{j=t'}^{t-1}[1-\phi(\bar{P}^j,\bar{P}^{j-1})][\pi(\bar{P}^t)-\bar{\Delta}^t\gamma x(\bar{P}^t)-(\hat{\pi}-(1-\delta)F)]\right\}\\&\quad+[(\hat{\pi}/(1-\delta))-F]\end{aligned}$$

where

$$\bar{\Delta}^t\equiv\delta\sum_{\tau=t}^{\infty}(\delta\beta)^{\tau-t}\phi(\bar{P}^\tau,\bar{P}^{\tau-1})\prod_{j=t}^{\tau-1}[1-\phi(\bar{P}^j,\bar{P}^{j-1})]$$

In considering (A.4), it is as if a colluding firm receives net income in each period equal to $\pi(\bar{P}^t)-\bar{\Delta}^t\gamma x(\bar{P}^t)$ where $\pi(\bar{P}^t)$ is gross profit and $\bar{\Delta}^t\gamma x(\bar{P}^t)$ is the expected present value of damages associated with colluding in that period.[16]

Suppose it is not true that price is bounded above by P^* so $\exists t'$ such that $\bar{P}^{t'}>P^*\geq\bar{P}^{t'-1}$. If this price path is optimal then, starting from period t', it must yield at least as high a payoff as a price path in which firms collude and price at P^* forever. This is true iff

(A.5)

$$\begin{aligned}&\left[\pi(\bar{P}^{t'})-\bar{\Delta}^{t'}\gamma x(\bar{P}^{t'})-(\hat{\pi}-(1-\delta)F)\right]-\bar{\Delta}^{t'}\beta X^{t'-1}\\&\quad+\sum_{t=t'+1}^{\infty}\delta^{t-t'}\left\{\prod_{j=t'}^{t-1}[1-\phi(\bar{P}^j,\bar{P}^{j-1})][\pi(\bar{P}^t)-\bar{\Delta}^t\gamma x(\bar{P}^t)-(\hat{\pi}-(1-\delta)F)]\right\}\\&\geq\left[\pi(P^*)-\tilde{\Delta}^{t'}\gamma x(P^*)-(\hat{\pi}-(1-\delta)F)\right]-\tilde{\Delta}^{t'}\beta X^{t'-1}\\&\quad+\sum_{t=t'+1}^{\infty}\delta^{t-t'}\left\{\left[1-\phi\left(P^*,\bar{P}^{t'-1}\right)\right][1-\hat{\phi}(0)]^{t-t'-1}\right.\\&\qquad\left.\times[\pi(P^*)-\tilde{\Delta}^t\gamma x(P^*)-(\hat{\pi}-(1-\delta)F)]\right\}\end{aligned}$$

[16] The proof is available from the author.

where

$$\tilde{\Delta}^{t'} \equiv \delta \left\{ \phi(P^*, \bar{P}^{t'-1}) + \sum_{\tau=t'+1}^{\infty} (\delta\beta)^{\tau-t'} [1 - \phi(P^*, \bar{P}^{t'-1})][1 - \hat{\phi}(0)]^{\tau-t'-1} \hat{\phi}(0) \right\}$$

$$\tilde{\Delta}^{t} \equiv \delta \sum_{\tau=t}^{\infty} (\delta\beta)^{\tau-t} [1 - \hat{\phi}(0)]^{\tau-t} \hat{\phi}(0), \quad t \geq t' + 1$$

and recall that $\phi(P^*, P^*) = \hat{\phi}(0)$. Since there are more price changes associated with $\{\bar{P}^t\}_{t=1}^{\infty}$, it is straightforward to show that $\bar{\Delta}^t \geq \tilde{\Delta}^t \; \forall t \geq t'$. Consider the LHS expression in (A.5). Since it is nonincreasing in $\bar{\Delta}^t$ and $\bar{\Delta}^t \geq \tilde{\Delta}^t \; \forall t \geq t'$, the expression is weakly increased if $\tilde{\Delta}^t$ replaces $\bar{\Delta}^t \; \forall t \geq t'$. It follows that if (A.5) holds then it must be true that

(A.6)

$$\begin{aligned} &\left[\pi(\bar{P}^{t'}) - \tilde{\Delta}^{t'} \gamma x(\bar{P}^{t'}) - (\hat{\pi} - (1-\delta)F)\right] - \tilde{\Delta}^{t'} \beta X^{t'-1} \\ &\quad + \sum_{t=t'+1}^{\infty} \delta^{t-t'} \left\{ \prod_{j=t'}^{t-1} [1 - \phi(\bar{P}^j, \bar{P}^{j-1})][\pi(\bar{P}^t) - \tilde{\Delta}^t \gamma x(\bar{P}^t) - (\hat{\pi} - (1-\delta)F)] \right\} \\ &\geq \left[\pi(P^*) - \tilde{\Delta}^{t'} \gamma x(P^*) - (\hat{\pi} - (1-\delta)F)\right] - \tilde{\Delta}^{t'} \beta X^{t'-1} \\ &\quad + \sum_{t=t'+1}^{\infty} \delta^{t-t'} \{[1 - \phi(P^*, \bar{P}^{t'-1})][1 - \hat{\phi}(0)]^{t-t'-1} \\ &\qquad\qquad \times [\pi(P^*) - \tilde{\Delta}^t \gamma x(P^*) - (\hat{\pi} - (1-\delta)F)]\} \end{aligned}$$

The objective is to establish that a contradiction follows from (A.6). The summation term on the RHS is at least as great as the summation term on the LHS because the product of the probability terms is larger on the RHS, since there are fewer price changes,

$$\pi(P^*) - \tilde{\Delta}^t \gamma x(P^*) \geq \pi(\bar{P}^t) - \tilde{\Delta}^t \gamma x(\bar{P}^t), \quad t \geq t' + 1$$

by A11, and $\pi(P^*) - \tilde{\Delta}^t \gamma x(P^*) - (\hat{\pi} - (1-\delta)F) > 0$ can be shown to follow from A10. Thus, a necessary condition for (A.6) to be true is

$$\text{(A.7)} \qquad \pi(\bar{P}^{t'}) - \tilde{\Delta}^{t'} \gamma x(\bar{P}^{t'}) \geq \pi(P^*) - \tilde{\Delta}^{t'} \gamma x(P^*)$$

Since $\gamma x(\bar{P}^{t'}) \geq \gamma x(P^*)$ (so that the LHS is decreasing in $\tilde{\Delta}^{t'}$ at a faster rate than the RHS), it follows from $\tilde{\Delta}^{t'} \geq \tilde{\Delta}^t$ that (A.7) implies

$$\pi(\bar{P}^{t'}) - \tilde{\Delta}^t \gamma x(\bar{P}^{t'}) \geq \pi(P^*) - \tilde{\Delta}^t \gamma x(P^*)$$

Since $\tilde{\Delta}^t = \delta\hat{\phi}(0)/[1 - \delta\beta(1 - \hat{\phi}(0))]$ and $\bar{P}^{t'} > P^*$, this cannot be true by A11. This proves that the price path is bounded above by P^*.

- The optimal price path is nondecreasing over time.

The proof strategy involves two parts. First, suppose that price falls from $t'-1$ to t' and furthermore that price never exceeds its level prior to the decline, that is, $P^{t'-1} \geq P^t \; \forall t \geq t'$. It is shown that a higher payoff is realized when price is kept constant at $P^{t'-1} \; \forall t \geq t'$. Second, suppose that price falls from $t'-1$ to t' and remains at or below $P^{t'-1}$ over periods $t'+1, \ldots, t''$. It is then shown that a higher payoff is realized by skipping the price path over periods $t'+1, \ldots, t''$ and jumping to a price of $P^{t''+1}$ in period t', $P^{t''+2}$ in period $t'+1$, and so forth.

Suppose $\{\bar{P}^t\}_{t=1}^{\infty}$ is an optimal price path and it is not nondecreasing over time. Hence, $\exists t' > 1$ such that $P^0 < \bar{P}^1 \leq \cdots \leq \bar{P}^{t'-1} > \bar{P}^{t'}$. A necessary condition for optimality is that the payoff, starting in t', from $\{\bar{P}^t\}_{t=1}^{\infty}$ is at least as great as maintaining price at $\bar{P}^{t'-1}$ forever:

(A.8)

$$\begin{aligned}
&\left[\pi(\bar{P}^{t'}) - \bar{\Delta}^{t'}\gamma x(\bar{P}^{t'}) - (\hat{\pi} - (1-\delta)F)\right] - \bar{\Delta}^{t'}\beta X^{t'-1} \\
&\quad + \sum_{t=t'+1}^{\infty} \delta^{t-t'} \prod_{j=t'}^{t-1} [1 - \phi(\bar{P}^j, \bar{P}^{j-1})][\pi(\bar{P}^t) - \bar{\Delta}^t \gamma x(\bar{P}^t) - (\hat{\pi} - (1-\delta)F)] \\
&\quad + [\hat{\pi}/(1-\delta) - F] \\
&\geq [\pi(\bar{P}^{t'-1}) - \tilde{\Delta}\gamma x(\bar{P}^{t'-1}) - (\hat{\pi} - (1-\delta)F)] - \tilde{\Delta}\beta X^{t'-1} \\
&\quad + \sum_{t=t'+1}^{\infty} \delta^{t-t'} \left[1 - \hat{\phi}(0)\right]^{t-t'} [\pi(\bar{P}^{t'-1}) - \tilde{\Delta}\gamma x(\bar{P}^{t'-1}) - (\hat{\pi} - (1-\delta)F)] \\
&\quad + [\hat{\pi}/(1-\delta) - F]
\end{aligned}$$

where

$$\tilde{\Delta} \equiv \delta \sum_{\tau=t}^{T} (\delta\beta)^{\tau-t} \left[1 - \hat{\phi}(0)\right]^{\tau-t} \hat{\phi}(0)$$

The first step is to show that if $\bar{P}^{t'-1} > \bar{P}^{t'}$ and $\bar{P}^{t'-1} \geq \bar{P}^t \; \forall t \geq t'+1$ then (A.8) cannot be true; maintaining price at $\bar{P}^{t'-1}$ forever is superior. Recall that price is bounded above by P^* so that $\bar{P}^{t'-1} \leq P^*$. Since $\tilde{\Delta} \leq \bar{\Delta}^t \; \forall t$ then, replacing $\bar{\Delta}^t$ with $\tilde{\Delta}$, a necessary condition for (A.8) to be true is

(A.9)

$$\begin{aligned}
&\left[\pi(\bar{P}^{t'}) - \tilde{\Delta}\gamma x(\bar{P}^{t'}) - (\hat{\pi} - (1-\delta)F)\right] \\
&\quad + \sum_{t=t'+1}^{\infty} \delta^{t-t'} \prod_{j=t'}^{t-1} [1 - \phi(\bar{P}^j, \bar{P}^{j-1})][\pi(\bar{P}^t) - \tilde{\Delta}\gamma x(\bar{P}^t) - (\hat{\pi} - (1-\delta)F)] \\
&\geq [\pi(\bar{P}^{t'-1}) - \tilde{\Delta}\gamma x(\bar{P}^{t'-1}) - (\hat{\pi} - (1-\delta)F)] \\
&\quad + \sum_{t=t'+1}^{\infty} \delta^{t-t'} \left[1 - \hat{\phi}(0)\right]^{t-t'} [\pi(\bar{P}^{t'-1}) - \tilde{\Delta}\gamma x(\bar{P}^{t'-1}) - (\hat{\pi} - (1-\delta)F)]
\end{aligned}$$

To show that the summation term on the RHS is at least as great as that on the LHS, first note that A11 implies

$$\begin{aligned}&\pi(\bar{P}^{t'-1}) - \tilde{\Delta}\gamma x(\bar{P}^{t'-1}) - (\hat{\pi} - (1-\delta)F)\\ &\quad \geq \pi(\bar{P}^{t}) - \tilde{\Delta}\gamma x(\bar{P}^{t}) - (\hat{\pi} - (1-\delta)F), \quad t \geq t'+1\end{aligned}$$

as, by supposition, $\bar{P}^{t'-1} \geq \bar{P}^{t}\ \forall t \geq t'+1$ and it has already been proven that $P^* \geq \bar{P}^{t'-1}$. Next note that A10 implies

$$\pi(\bar{P}^{t'-1}) - \tilde{\Delta}\gamma x(\bar{P}^{t'-1}) - (\hat{\pi} - (1-\delta)F) > 0$$

because $\bar{P}^{t'-1} \leq P^m$ and $\tilde{\Delta} \leq \delta\hat{\phi}(0)/(1-\beta)$. Finally,

$$\left[1-\hat{\phi}(0)\right]^{t-t'} \geq \prod_{j=t'}^{t-1}[1-\phi(\bar{P}^{j}, \bar{P}^{j-1})], \quad t \geq t'+1$$

It is concluded that the summation term on the RHS of (A.9) is at least as great as the summation term on the LHS of (A.9). Therefore, for (A.9) (and hence, (A.8)) to be true, it is necessary that

$$\pi(\bar{P}^{t'}) - \tilde{\Delta}\gamma x(\bar{P}^{t'}) \geq \pi(\bar{P}^{t'-1}) - \tilde{\Delta}\gamma x(\bar{P}^{t'-1})$$

However, by $\bar{P}^{t'} < \bar{P}^{t'-1} \leq P^*$, this contradicts A11. It is concluded that the price path cannot be bounded above by $\bar{P}^{t'-1}$ for $t \geq t'$.

Therefore, if $\bar{P}^{t'-1} > \bar{P}^{t'}$ then $\exists t'' \geq t'$ such that $\bar{P}^{t'-1} \geq \bar{P}^{t'+1}, \ldots, \bar{P}^{t''}$ and $\bar{P}^{t'-1} < \bar{P}^{t''+1}$. Once again compare this price path with one in which price is kept constant at $\bar{P}^{t'-1}$. By the arguments just given, one can show that the income from $\{\bar{P}^{t}\}_{t=1}^{\infty}$ is strictly lower at t' and is weakly lower at periods $t'+1, \ldots, t''$. Hence, a necessary condition for optimality is that the sum of the discounted terms for periods $t \geq t''+1$ be strictly higher:

(A.10)

$$\begin{aligned}&\sum_{t=t''+1}^{\infty} \delta^{t-t'} \prod_{j=t'}^{t-1}[1-\phi(\bar{P}^{j}, \bar{P}^{j-1})][\pi(\bar{P}^{t}) - \bar{\Delta}^{t}\gamma x(\bar{P}^{t}) - (\hat{\pi} - (1-\delta)F)]\\ &\quad > \sum_{t=t''+1}^{\infty} \delta^{t-t'}\left[1-\hat{\phi}(0)\right]^{t-t'}[\pi(\bar{P}^{t'-1}) - \tilde{\Delta}\gamma x(\bar{P}^{t'-1}) - (\hat{\pi} - (1-\delta)F)]\end{aligned}$$

or

$$\delta^{t''-t'+1}\prod_{j=t'}^{t''}[1-\phi(\bar{P}^j,\bar{P}^{j-1})]$$
$$\times\sum_{t=t''+1}^{\infty}\delta^{t-t''-1}\prod_{j=t''+1}^{t-1}[1-\phi(\bar{P}^j,\bar{P}^{j-1})][\pi(\bar{P}^t)-\bar{\Delta}^t\gamma x(\bar{P}^t)-(\hat{\pi}-(1-\delta)F)]$$
$$>\delta^{t''-t'+1}\left[1-\hat{\phi}(0)\right]^{t''-t'+1}$$
$$\times\sum_{t=t''+1}^{\infty}\delta^{t-t''-1}\left[1-\hat{\phi}(0)\right]^{t-t''-1}[\pi(\bar{P}^{t'-1})-\tilde{\Delta}\gamma x(\bar{P}^{t'-1})-(\hat{\pi}-(1-\delta)F)]$$

Since

$$\theta\equiv\delta^{t''-t'+1}\left[1-\hat{\phi}(0)\right]^{t''-t'+1}\geq\delta^{t''-t'+1}\prod_{j=t'}^{t''}[1-\phi(\bar{P}^j,\bar{P}^{j-1})]\equiv\xi$$

then a necessary condition for (A.10) is

$$Y\equiv\sum_{t=t''+1}^{\infty}\delta^{t-t''-1}\prod_{j=t''+1}^{t-1}[1-\phi(\bar{P}^j,\bar{P}^{j-1})][\pi(\bar{P}^t)-\bar{\Delta}^t\gamma x(\bar{P}^t)-(\hat{\pi}-(1-\delta)F)]$$
$$>\sum_{t=t''+1}^{\infty}\delta^{t-t''-1}\left[1-\hat{\phi}(0)\right]^{t-t''-1}[\pi(\bar{P}^{t'-1})-\tilde{\Delta}\gamma x(\bar{P}^{t'-1})-(\hat{\pi}-(1-\delta)F)]\equiv Z$$

From this condition it will be argued that a strictly superior price path to $\{\bar{P}^t\}_{t=t'}^{\infty}$ is to set $P^t=\bar{P}^{t+t''-t'+1}$, $t\geq t'$. The reason is simple. It has been shown that $\{\bar{P}^t\}_{t=t'}^{t''}$ does worse than a constant price of $\bar{P}^{t'-1}$ over periods $t',\ldots,t''$. The optimality of $\{\bar{P}^t\}_{t=t'}^{\infty}$ then requires that a strictly higher payoff be received after t''. Beginning from t', a higher payoff to $\{\bar{P}^t\}_{t=t'}^{\infty}$ can then be earned by skipping the prices over $t',\ldots,t''$ and pricing in t' according to the price path as of $t''+1$.

Define y and z as the payoff over $t',\ldots,t''$ from the price path $\{\bar{P}^t\}_{t=t'}^{\infty}$ and a constant price of $\bar{P}^{t'-1}$, respectively,

$$y\equiv\sum_{t=t'}^{t''}\delta^{t-t'}\prod_{j=t'}^{t-1}[1-\phi(\bar{P}^j,\bar{P}^{j-1})][\pi(\bar{P}^t)-\bar{\Delta}^t\gamma x(\bar{P}^t)-(\hat{\pi}-(1-\delta)F)]$$
$$z\equiv\sum_{t=t'}^{t''}\delta^{t-t'}\left[1-\hat{\phi}(0)\right]^{t-t'}[\pi(\bar{P}^{t'-1})-\tilde{\Delta}\gamma x(\bar{P}^{t'-1})-(\hat{\pi}-(1-\delta)F)]$$

Note that $Z=z/(1-\theta)$. In this notation, (A.8) takes the form

$$y+\xi Y-\bar{\Delta}^{t'}\beta X^{t'-1}+[\hat{\pi}/(1-\delta)-F]\geq z+\theta Z-\tilde{\Delta}\beta X^{t'-1}+[\hat{\pi}/(1-\delta)-F]$$

Consider

$$Y - (y + \xi Y) = (1 - \xi)Y - y > (1 - \xi)Y - z = (1 - \xi)Y - (1 - \theta)Z > 0$$

The last inequality follows from $\theta \geq \xi$ and $Y > Z$. It is then true that $Y > y + \xi Y$. Now consider the payoff starting from t' in which $P^t = \bar{P}^{t+t''-t'+1}$, $t \geq t'$. It will be shown that it is bounded below by $Y - \bar{\Delta}^{t'}\beta X^{t'-1} + [\hat{\pi}/(1-\delta) - F]$. As defined, Y is the payoff from $\{\bar{P}^t\}_{t=t'}^{\infty}$ starting in $t''+1$ and discounting back to $t''+1$ with an initial price of $\bar{P}^{t''}$. It is also the payoff from $P^t = \bar{P}^{t+t''-t'+1}$ for $t \geq t'$, starting in t' and discounting back to t' but with one caveat. The preceding price to $\bar{P}^{t''+1}$ is not $\bar{P}^{t''}$ but rather $\bar{P}^{t'-1}$. Since $\bar{P}^{t''+1} > \bar{P}^{t'-1} \geq \bar{P}^{t''}$ then

$$(\bar{P}^{t''+1} - \bar{P}^{t''})g(\bar{P}^{t''}) \geq (\bar{P}^{t''+1} - \bar{P}^{t'-1})g(\bar{P}^{t'-1}) > 0$$

so that, by A7, the probability of detection at t' from the price path $P^t = \bar{P}^{t+t''-t'+1}$ is no greater than that at $t''+1$ from $\{\bar{P}^t\}_{t=t'}^{\infty}$.[17] Thus, the associated payoff is weakly higher than $Y - \bar{\Delta}^{t'}\beta X^{t'-1} + [\hat{\pi}/(1-\delta) - F]$.

To summarize, it has been shown that a price path of $P^t = \bar{P}^{t+t''-t'+1}$ for $t \geq t'$ yields a payoff of at least $Y - \bar{\Delta}^{t'}\beta X^{t'-1} + [\hat{\pi}/(1-\delta) - F]$ whereas $\{\bar{P}^t\}_{t=t'}^{\infty}$ yields a payoff of $y + \xi Y - \bar{\Delta}^{t'}\beta X^{t'-1} + [\hat{\pi}/(1-\delta) - F]$. Since $Y > y + \xi Y$ then the former is larger, which contradicts the optimality of $\{\bar{P}^t\}_{t=t'}^{\infty}$. This contradiction shows the falsity of the supposition that $\exists t' > 1$ such that $P^0 < P^1 \leq \cdots \leq \bar{P}^{t'-1} > \bar{P}^{t'}$. It is concluded that the price path is nondecreasing.

- The optimal price path converges to P^*.

A variational approach is used to characterize the limiting price. If $\{\bar{P}^t\}_{t=1}^{\infty}$ is an optimal price path then it is nondecreasing and is bounded above by P^*. Therefore, $\lim_{t\to\infty} P^t$ exists and is denoted $\bar{P}$. Consider a price path in which $P^t = \bar{P}^t$ for $t < T$ and $P^t = \bar{P}^t + \varepsilon$ for $t \geq T$. Starting with period T, it yields a payoff of

$$\begin{aligned}
&\pi(\bar{P}^T + \varepsilon) - \left\{\delta\phi(\bar{P}^T + \varepsilon, \bar{P}^{T-1}) + \delta\beta\left[1 - \phi(\bar{P}^T + \varepsilon, \bar{P}^{T-1})\right]\bar{\Delta}^{T+1}\right\} \\
&\quad \times \left[\gamma x(\bar{P}^T + \varepsilon) + \beta X^{T-1}\right] - [\hat{\pi} - (1-\delta)F] \\
&\quad + \sum_{t=T+1}^{\infty} \delta^{t-T}\left[1 - \phi(\bar{P}^T + \varepsilon, \bar{P}^{T-1})\right] \prod_{j=T+1}^{t-1}\left[1 - \phi(\bar{P}^j + \varepsilon, \bar{P}^{j-1} + \varepsilon)\right] \\
&\quad \times [\pi(\bar{P}^t + \varepsilon) - \bar{\Delta}^t\gamma x(\bar{P}^t + \varepsilon) - (\hat{\pi} - (1-\delta)F)] + [\hat{\pi}/(1-\delta) - F]
\end{aligned}$$

where

$$\bar{\Delta}^t \equiv \delta\sum_{\tau=t}^{\infty}(\delta\beta)^{\tau-t}\phi(\bar{P}^\tau + \varepsilon, \bar{P}^{\tau-1} + \varepsilon)\prod_{j=t}^{\tau-1}\left[1 - \phi(\bar{P}^j + \varepsilon, \bar{P}^{j-1} + \varepsilon)\right]$$

[17] This is the only step in the proof that requires g to be a nonincreasing function.

This payoff is continuous in ε and equals the payoff from $\{\bar{P}\}_{t=T}^{\infty}$ when $\varepsilon = 0$. Optimality requires that if the derivative of the payoff with respect to ε is defined then it equals 0 at $\varepsilon = 0$. Taking this derivative and evaluating it at $\varepsilon = 0$ as $T \to \infty$, one finds that it is indeed defined because $\hat{\phi}'(0)$ exists. Furthermore, it is equal to

$$\frac{\pi'(\bar{P}) - \bar{\Delta}\gamma x'(\bar{P})}{1 - \delta\left(1 - \hat{\phi}(0)\right)} \quad \text{where} \quad \bar{\Delta} \equiv \frac{\delta\hat{\phi}(0)}{1 - \delta\beta(1 - \hat{\phi}(0))}$$

Optimality then requires that $\pi'(\bar{P}) - \bar{\Delta}\gamma x'(\bar{P}) = 0$, which, by A11, implies $\bar{P} = P^*$. This completes the proof of Theorem 2. ■

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