INTERNATIONAL ECONOMIC REVIEW
Vol. 56, No. 1, Febuary 2015

COMPETITION POLICY AND CARTEL SIZE*

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This article examines endogenous cartel formation in the presence of a competition authority. Competition policy is shown to make the most inclusive stable cartels less inclusive. In particular, small firms that might have been cartel members in the absence of a competition authority are no longer members. Regarding the least inclusive stable cartels, competition policy can either decrease or increase their size and, in the latter case, the collusive price can rise.

1. INTRODUCTION

Research has extensively explored how competition policy affects whether collusion is stable and, when it is stable, the price set by cartel members and the cartel's duration. For example, there is a large and growing literature that examines the impact of corporate leniency programs on whether a cartel forms and what price it sets if it does form.[2] All that analysis, however, has made two restrictive assumptions about the cartel's composition. First, that the cartel is all-inclusive. Second, that the inclusiveness of the cartel is fixed with regards to competition policy. Practice runs contrary to the first assumption in that many cartels comprise some, but not all, firms in a market,[3] and, with regards to the second assumption, it is natural to expect that a tougher competition policy could influence which firms choose to join a cartel and how inclusive a cartel must be for the cartel to be stable.

The objective of this article is to explore the impact of competition policy on a cartel's size and composition. In earlier work (Bos and Harrington, 2010), the set of stable cartels was characterized but in the absence of antitrust enforcement. In this article, that model is amended to allow for a competition authority that can detect and convict cartels and, as a consequence, impose financial penalties and cause the cartel to shut down. We also allow for a corporate leniency program so that a cartel member can receive a reduced penalty in exchange for cooperating with the authorities.

Our main findings are as follows. First, we find that competition policy results in the most inclusive stable cartel being less inclusive. In particular, small firms are no longer cartel members when there are competition laws and an authority to enforce them. This is a benefit of competition policies—such as a leniency program—that has not previously been recognized. Second, antitrust enforcement has an ambiguous effect on the size of the least inclusive stable cartels. If market demand is highly inelastic, then the least inclusive stable cartels encompass more firms, but there are other market conditions such that the least inclusive stable cartels involve fewer firms. Combining these results, we find that antitrust enforcement either reduces the range of sizes of stable cartels—as it increases the size of the smallest cartels and decreases the size of the largest cartels—or it shifts the range of stable cartels down—making the most and least inclusive cartels less encompassing.

*Manuscript received June 2013; revised January 2014.

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[2] For references, see Spagnolo (2008) and Harrington and Chang (2012).

[3] Examples are in Harrington (2006) and Bos and Harrington (2010) and references cited therein.

In the next section, the model is introduced. Section 3 establishes equilibrium cartel behavior, and Section 4 provides the conditions for a cartel to be stable. The main results of the article are in Section 5, which characterizes the impact of antitrust enforcement on the set of stable cartels, and are complemented with an example in Section 6. Section 7 offers some implications for competition policy, and Section 8 concludes. All proofs are in the Appendix.

2. MODEL

To explore the impact of competition policy on the inclusiveness of cartels, the capacity-constrained price-setting repeated game in Bos and Harrington (2010) is modified to allow for a cartel to be convicted and penalized. Consider an industry with $n \geq 3$ firms producing a homogeneous good at common marginal cost $c \geq 0$.[4] Let $N \equiv \{1, \ldots, n\}$ denote the set of firms. Firm i has a fixed production capacity k_i and firms have a common discount factor $\delta \in (0, 1)$. The setting is one of perfect monitoring so, in any period, all past prices are common knowledge.

Market demand is given by $D(p)$, which is a twice continuously differentiable and decreasing function of price. Moreover, $D(c) > 0$ and monopoly profit, $(p - c)D(p)$, is strictly concave. The monopoly price p^m is defined by $D(p^m) + (p^m - c)D'(p^m) = 0$. In each period, firms simultaneously choose prices from $\{0, \varepsilon, \ldots, c - \varepsilon, c, c + \varepsilon, \ldots\}$ and produce to meet demand up to capacity. Results will be derived for $\varepsilon > 0$ and sufficiently small.[5] Demand of firm i is denoted $D_i(p_i, p_{-i})$, which depends on its own price p_i and the vector of rivals' prices, p_{-i}. As in Bos and Harrington (2010), three fairly general assumptions are made on firm demand and capacities. In stating these assumptions, let $\Phi(p) \equiv \{j : p_j = p\}$ denote the set of firms that price at p and define $p_{-i}^{\min} \equiv \min\{p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n\}$.

A1. $lim_{\eta \to 0^+} D_i(p_{-i}^{min} + \eta, p_{-i}) = max\{D(p_{-i}^{min}) - \sum_{j \in \Phi(p_{-i}^{min})} k_j, 0\}$.

A2. *If* $0 < \sum_{i \in \Phi(p)} D_i(p_i, p_{-i}) < \sum_{i \in \Phi(p)} k_i$, *then* $0 < D_i(p_i, p_{-i}) < k_i, \forall i \in \Phi(p)$.

A3. $k_i < D(p^m)$ *and* $\sum_{j \neq i} k_j \geq D(c), \forall i \in N$.

A1 holds for any well-behaved residual demand function, whereas A2 imposes some symmetry across firms. The first part of A3 imposes an upper bound on firm size. It has the implication that, for prices not exceeding the monopoly price, a firm that charges a price below all of its rivals is capacity-constrained. The second part of A3 states that any $n - 1$ firms have sufficient production capacity to meet competitive demand. This assumption ensures that the one-shot game has two symmetric Nash equilibria with prices of c and $c + \varepsilon$. Thus, for sufficiently small ε, static Nash equilibrium profit is approximately zero.

Firms can potentially enhance their profits through the formation of a price-fixing cartel. Consider a cartel $\Gamma \subseteq N$ with common cartel price $p > c + \varepsilon$. If $\Gamma \subset N$, then the cartel faces competition from at least one outsider.[6] As proven in Lemma 2 in Bos and Harrington (2010), noncolluding firms optimally set their prices slightly below the cartel price and produce up to capacity. Residual cartel demand is then given by $D(p) - (K - K_\Gamma)$, where $K = \sum_{i \in N} k_i$ and $K_\Gamma = \sum_{i \in \Gamma} k_i$ denote, respectively, industry and cartel capacity. Clearly, collusion is beneficial only when the cartel faces positive demand, which requires $D(p) - (K - K_\Gamma) > 0$ or $K_\Gamma > K - D(p)$. Thus, a necessary condition for a cartel to be successful is that it control a sufficiently

[4] Given that we are interested in exploring the inclusivity of cartels, it is necessary to assume there are at least three firms.

[5] Although there is no explicit reference made to ε in the results of this article, we draw upon results in Bos and Harrington (2010) that presume ε is sufficiently small.

[6] $\subset$ refers to strict set inclusion so that $\Gamma' \subset \Gamma''$ means Γ' is a strict subset of Γ''.

large part of industry capacity. Under the assumption that cartel profit is allocated in proportion to capacity, profit of firm $i \in \Gamma$ is[7]

$$(p - c)\left[D(p) - (K - K_{\Gamma})\right]\left(\frac{k_i}{K_{\Gamma}}\right).$$

Firms that take part in a cartel become subject to antitrust enforcement. In each period in which at least one firm sets the collusive price, the antitrust authority discovers cartel Γ with probability $\rho(\Gamma) \in [0, 1]$. Thus, $\rho(\cdot)$ maps from the set of subsets of N with at least two members into $[0, 1]$. The absence of antitrust enforcement is when $\rho(\cdot) = 0$. In the event of an investigation, conviction occurs for sure and results in the immediate and permanent breakdown of the cartel; hence, firms return to a static Nash equilibrium forever. With probability $1 - \rho(\Gamma)$ there is no investigation (and thus no chance of conviction) in the current period. Although firms outside of the cartel benefit from the higher prices—indeed, they price just below the collusive price—it is customary for them to be innocent of violating the law, and they are typically not liable for customer damages. It is then assumed that they are not subject to penalties though will be harmed with the subsequent fall in prices due to antitrust enforcement shutting down the cartel.

As stated in A4, cartels with more members are assumed to have a higher probability of investigation and conviction.

A4. *If* $\Gamma' \subset \Gamma''$ *then* $\rho(\Gamma') < \rho(\Gamma'')$, *and if* $i \notin \Gamma$ *then* $\lim_{k_i \to 0} \rho(\Gamma \cup \{i\}) > \rho(\Gamma)$.

Thus, the probability of being caught is higher when a cartel adds firms, which strikes us as a natural assumption. The chances that the competition authority receives a complaint from a buyer is more likely when more buyers are affected, which is the case when the cartel is more inclusive. If discovery comes from a cartel member inadvertently revealing information to an uninvolved employee within the firm, then again this is more likely when more people are engaged in collusion, which is true when there are more firms in the cartel. A4 also assumes that the increase in probability from adding a firm to the cartel is bounded above zero even if the additional cartel member is arbitrarily small in terms of capacity (and, as a result, market share). This condition seems reasonable given that much of the reason why more members makes detection more likely is that there are more people with knowledge of the cartel and thus more opportunities for information to leak out, which is nontrivial even when a firm is very small.

In case of discovery, a cartel member faces an antitrust penalty that is proportional to the profit it earned while colluding:

$$\gamma(p - c)\left[D(p) - (K - K_{\Gamma})\right]\left(\frac{k_i}{K_{\Gamma}}\right), \tag{1}$$

where $\gamma > 0$ is a penalty multiplier.[8] Thus, larger cartel members face a larger penalty, all else equal. Many jurisdictions have a leniency program that gives cartel participants the opportunity to turn themselves in in exchange for a reduction of their penalty. To encompass such a program, assume that if firm i is the first firm to receive leniency then it pays a penalty equal to the expression in (1) multiplied by $\theta \in [0, 1]$. θ is a policy parameter and includes the case of no leniency ($\theta = 1$) and full leniency ($\theta = 0$). Of course, in response to an investigation, all cartel members may simultaneously race for leniency, in which case it is natural to suppose that each

[7] A motivation and extensive discussion of this assumption is provided in Bos and Harrington (2010).

[8] This is a natural specification in jurisdictions that have customer damages. Although customer damages are typically calculated in such a way that they do not generally equal the incremental profit from collusion, incremental profits are still a good approximation. In jurisdictions where the primary penalty is government fines this specification is more problematic because those fines are often based more on revenue than on profit.

has an equal chance of receiving it. If leniency is given only to one firm, then a firm can expect to pay the full penalty multiplied by $\frac{|\Gamma|-1+\theta}{|\Gamma|}$; each cartel member has probability $\frac{1}{|\Gamma|}$ of receiving leniency and probability $\frac{|\Gamma|-1}{|\Gamma|}$ of not receiving leniency and paying the full penalty. To allow for such discounts—as well as other programs that may impact the penalties actually paid—it is assumed that, in response to a conviction, a cartel member can expect to pay a penalty equal to (1) multiplied by $\psi(\Gamma) \in (0,1]$. Hence, the expected penalty that firm $i \in \Gamma$ faces prior to learning whether or not there is an investigation is[9]

$$\rho(\Gamma)\,\psi(\Gamma)\,\gamma(p-c)\,[D(p)-(K-K_\Gamma)]\left(\frac{k_i}{K_\Gamma}\right). \tag{2}$$

It is assumed $\psi(\Gamma)$ is weakly larger for cartels encompassing more members.

A5. *If* $\Gamma' \subset \Gamma'''$ *then* $\psi(\Gamma') \leq \psi(\Gamma'')$.

The special case of $\psi(\Gamma) = \frac{|\Gamma|-1+\theta}{|\Gamma|}$ obviously satisfies these conditions.[10]

3. CARTEL'S OBJECTIVE AND EQUILIBRIUM PRICE

Consider a cartel Γ with a common cartel price $p > c+\varepsilon$. The collusive value for member $i \in \Gamma$, denoted $V_i(p,\Gamma)$, is defined recursively by

$$V_i(p,\Gamma) = (p-c)\,[D(p)-(K-K_\Gamma)]\left(\frac{k_i}{K_\Gamma}\right)$$
$$-\rho(\Gamma)\,\psi(\Gamma)\,\gamma(p-c)\,[D(p)-(K-K_\Gamma)]\left(\frac{k_i}{K_\Gamma}\right)+\delta(1-\rho(\Gamma))\,V_i(p,\Gamma).$$

Solving it for $V_i(p,\Gamma)$ yields

$$V_i(p,\Gamma) = k_i\left(\frac{1-\rho(\Gamma)\,\psi(\Gamma)\,\gamma}{1-\delta(1-\rho(\Gamma))}\right)(p-c)\left[\frac{D(p)-(K-K_\Gamma)}{K_\Gamma}\right] = k_i V(p,\Gamma). \tag{3}$$

Observe that firm i's value equals a common value per unit of capacity, $V(p,\Gamma)$, multiplied by its capacity: $V_i(p,\Gamma) = k_i V(p,\Gamma)$. When the cartel is not all-inclusive, noncartel members optimally set their prices slightly below the cartel price and produce up to their capacity. Since $k_i < D(p^m)\ \forall i \in N$ (Assumption 3), this implies that a member that undercuts the collusive price optimally prices at $p-\varepsilon$ or $p-2\varepsilon$. Specifically, it will choose $p-\varepsilon$ when it is capacity constrained at that price; otherwise it sets $p-2\varepsilon$. As to the latter, the cheating firm would be charging the lowest price in the industry and is therefore capacity constrained by assumption. Consequently, cutting price further would be unprofitable. Thus, for ε sufficiently small, a cheating firm $i \in \Gamma$ earns approximately $(p-c)\,k_i$ in terms of current profit and zero future profit (as all firms revert to static Nash equilibrium pricing).[11] Finally, recall that there is still a chance of being caught in the period of defection so that cheating members remain subject to antitrust enforcement. A deviating firm therefore also has the possibility to simultaneously apply for leniency.

[9] Although the expression in (2) suggests that $\psi(\Gamma)$ is redundant because only $\rho(\Gamma)\,\psi(\Gamma)$ enters, we will soon present expressions that depend only on $\rho(\Gamma)$.

[10] One reason why leniency would be given in response to an investigation—even though conviction occurs for sure—is to save on resources in prosecuting the case. We could also specify $\rho(\Gamma)$ as the probability of an investigation and introduce ω as the probability of conviction when there is no leniency program. In that case, $\rho(\Gamma)\,\omega$ is the probability of paying penalties when firms do not seek leniency and $\rho(\Gamma)\,\psi(\Gamma)$ is the probability when they do.

[11] We conjecture that all results extend to when the cartel can be reformed in the future.

Given a cartel Γ, the incentive compatibility constraint (ICC) for firm $i \in \Gamma$ is then

$$\left(\frac{1-\rho(\Gamma)\,\psi(\Gamma)\,\gamma}{1-\delta(1-\rho(\Gamma))}\right)(p-c)\left[\frac{D(p)-(K-K_\Gamma)}{K_\Gamma}\right]k_i$$
$$\geq (p-c)\,k_i - \min\{\rho(\Gamma)\,\psi(\Gamma),\theta\}\gamma\,(p-c)\left[\frac{D(p)-(K-K_\Gamma)}{K_\Gamma}\right]k_i.$$

Rearranging, the ICC can be presented as

$$\Omega(\Gamma) \equiv \left(\frac{1-\rho(\Gamma)\,\psi(\Gamma)\,\gamma}{1-\delta(1-\rho(\Gamma))}\right) + \min\{\rho(\Gamma)\,\psi(\Gamma),\theta\}\gamma \geq \frac{K_\Gamma}{D(p)-(K-K_\Gamma)}. \tag{4}$$

Note that the ICC is the same for all cartel members. Whether a cheating firm finds it optimal to apply for leniency depends on the values of $\rho(\Gamma)\,\psi(\Gamma)$ and θ. A deviating member finds it optimal to turn itself in (prior to any investigation) only when leniency is sufficiently generous ($\theta < \rho(\Gamma)\,\psi(\Gamma)$) and otherwise prefers not to self-report ($\theta > \rho(\Gamma)\,\psi(\Gamma)$).[12]

The cartel's problem is to choose price to maximize cartel value per unit of capacity subject to the ICC:

$$p^*(\Gamma) = \arg\max \left(\frac{1-\rho(\Gamma)\,\psi(\Gamma)\,\gamma}{1-\delta(1-\rho(\Gamma))}\right)(p-c)\left[\frac{D(p)-(K-K_\Gamma)}{K_\Gamma}\right] \tag{5}$$

subject to

$$\Omega(\Gamma) \geq \frac{K_\Gamma}{D(p)-(K-K_\Gamma)}. \tag{6}$$

Let $\widehat{p}(\Gamma)$ be the maximum price that satisfies the ICC: $\widehat{p}(\Gamma) \equiv D^{-1}(K-K_\Gamma+(K_\Gamma/\Omega(\Gamma)))$. Hence, firms can only sustain a price above cost when

$$D^{-1}(K-K_\Gamma+(K_\Gamma/\Omega(\Gamma))) > c \Rightarrow \Omega(\Gamma) > \frac{K_\Gamma}{D(c)-(K-K_\Gamma)}. \tag{7}$$

This condition is guaranteed to hold when $\rho(\Gamma)\to 0$ and $\delta \to 1$, provided that a sufficient amount of industry capacity is under the control of the cartel. Since the objective function is strictly concave,

$$\frac{\partial^2 V(p,\Gamma)}{\partial p^2} = \left(\frac{1-\rho(\Gamma)\,\psi(\Gamma)\,\gamma}{1-\delta(1-\rho(\Gamma))}\right)\left(\frac{2D'(p)+(p-c)\,D''(p)}{K_\Gamma}\right) < 0,$$

the first-order condition is sufficient to determine the nonbinding solution. Let $p^o(\Gamma)$ denote the unconstrained optimal cartel price:

$$D(p^o(\Gamma)) - (K-K_\Gamma) + (p^o(\Gamma)-c)\,D'(p^o(\Gamma)) = 0. \tag{8}$$

[12] Note that it is assumed a deviating firm pays a penalty proportional to its profit when it set the collusive price. If deviation occurred in the first period of collusion, it would seem more reasonable to assume that it is proportional to the profit it received while deviating. If deviation occurred after many periods of collusion, then the penalty ought to be proportional to average profit during the time of the cartel, which will be a weighted average of collusive profit (for the many periods of collusion) and deviation profit (for the one period of deviation), which will be close to collusive profit. We chose the latter specification since it describes the steady state. However, we have no reason to think that our conclusions are sensitive to this assumption.

Observe that the unconstrained solution is independent of antitrust enforcement. Finally, since $V(p,\Gamma)$ is strictly concave in p, it follows that $p^*(\Gamma) = \min\{\widehat{p}(\Gamma), p^o(\Gamma)\}$. The equilibrium collusive value for a cartel Γ is then

$$(9)\quad V^*(\Gamma) \equiv V(p^*(\Gamma),\Gamma) = \left(\frac{1-\rho(\Gamma)\psi(\Gamma)\gamma}{1-\delta(1-\rho(\Gamma))}\right)(p^*(\Gamma)-c)\left[\frac{D(p^*(\Gamma))-(K-K_\Gamma)}{K_\Gamma}\right].$$

Consistent with previous work for all-inclusive cartels, more intense antitrust enforcement reduces the optimal cartel price by tightening the ICC. Thus, for a given cartel, a stricter antitrust regime leads to a weakly lower cartel price.

Theorem 1. *For cartel Γ, $p^*(\Gamma)$ is nonincreasing in $\rho(\Gamma)$, $\psi(\Gamma)$, and γ. If $p^*(\Gamma) < p^o(\Gamma)$, then $p^*(\Gamma)$ is decreasing in $\rho(\Gamma)$, $\psi(\Gamma)$, and γ.*

4. DEFINING THE SET OF STABLE CARTELS

Let us now direct our attention to cartel formation and identifying what coalition configurations are stable. D'Aspremont et al. (1983) were among the first to provide a clear and intuitive notion of cartel stability. A cartel is considered to be stable when (i) none of its members wants to leave the cartel (*internal stability*) and (ii) none of the noncartel members wants to join the cartel (*external stability*).

To derive the exact conditions in our model for a cartel to be both internally and externally stable, consider some candidate cartel Γ. The equilibrium value to firm i is $k_i V^*(\Gamma)$ when $i \in \Gamma$ and is $\frac{[p^*(\Gamma)-c]k_i}{1-\delta(1-\rho(\Gamma))}$ when $i \notin \Gamma$. Define $W_i(\Gamma)$ to be the equilibrium value to firm i when $i \in \Gamma$ and it does not join the cartel and when $i \notin \Gamma$ and it joins the cartel. $W_i(\Gamma)$ is then the payoff that firm i expects if it acts contrary to expectations about cartel membership, whether it means not joining the cartel when it should have or joining the cartel when it should not have.[13] With these equilibrium values, a cartel is stable when all cartel members strictly prefer to be a member (internal stability) and all nonmembers weakly prefer not to be a member (external stability). Internal stability requires a strict preference for cartel membership in order to rule out the trivial case in which firms are members of a cartel but the cartel prices the same as when there is no cartel (that is, at cost).

Definition 1. A cartel Γ is **stable** if (i) $k_i V^*(\Gamma) > W_i(\Gamma)$ for all $i \in \Gamma$ and (ii) $\frac{[p^*(\Gamma)-c]k_i}{1-\delta(1-\rho(\Gamma))} \geq W_i(\Gamma)$ for all $i \notin \Gamma$.

In specifying $W_i(\Gamma)$, it is standard in the literature (including our earlier paper) to assume that, regardless of firms' decisions as to whether or not to join the cartel, the resulting cartel acts according to the equilibrium yielding the highest collusive value. Although that is a natural specification when firms act according to expectations with respect to the cartel membership decision, there could be other reasonable responses when firms do not act according to expectations, either by not joining a cartel for which it was supposed to be a member or joining a cartel for which it was not supposed to be a member. A novel feature of our approach is to consider various equilibria in response to such events. One equilibrium is the standard specification: The cartel accommodates the disequilibrium membership decision by achieving the maximal

[13] In thinking about a firm joining a cartel for which it was not expected to be a member, the issue is not whether a firm can force its way into a cartel but rather whether it desires to join a cartel for which it was not expected to be a member. As it turns out, if the existing members of a cartel find it unprofitable for an outsider to join, then that outsider will find it unprofitable as well; thus, a firm will never want to try and force itself on a cartel. The problematic situation is when a firm prefers not to join a cartel and existing cartel members would like for it to join.

level of collusion given whichever firms are in the cartel. We refer to this as the *accommodative equilibrium* and it implies

$$\text{if } i \in \Gamma, \text{ then } W_i(\Gamma) = \frac{[p^*(\Gamma \backslash \{i\}) - c]\, k_i}{1 - \delta(1 - \rho(\Gamma \backslash \{i\}))},$$

and

$$\text{if } i \notin \Gamma, \text{ then } W_i(\Gamma) = k_i V^*(\Gamma \cup \{i\}).$$

An alternative equilibrium is that the cartel responds in a punishing manner by disbanding so that all firms receive the static Nash equilibrium payoff. Referring to it as the *aggressive equilibrium*, $W_i(\Gamma) = 0 \; \forall i$. There are other equilibria one could consider.

In performing an equilibrium selection, we are guided by the objective of this article, which is to assess the effect of antitrust enforcement on the range of stable cartels. Hence, we will consider the *most expansive set* of cartels. Given that the aggressive equilibrium is the equilibrium with the lowest payoffs, if a cartel is not stable with the aggressive equilibrium, then it is not stable with any other equilibrium. This then argues to specifying the aggressive equilibrium. However, one modification to that specification is appropriate on plausibility grounds. It would seem nonsensical for a cartel to punish a cartel member for departing or a noncartel member for joining when such an action actually improves the payoffs of cartel members. Thus, when entry into (exit from) the cartel enhances the value of the original (remaining) members of the cartel, it is assumed that the accommodative equilibrium ensues. This assumption is embodied in the following two conditions used in evaluating the stability of cartel Γ. First,

$$\text{(10)} \qquad \text{if } i \notin \Gamma \text{ and } V^*(\Gamma \cup \{i\}) > V^*(\Gamma), \text{ then } W_i(\Gamma) = k_i V^*(\Gamma \cup \{i\});$$

that is, if an outsider joining the cartel raises each original cartel member's payoff under the supposition of the accommodative equilibrium, then cartel members do in fact respond with the accommodative equilibrium. Second,

$$\text{(11)} \qquad \text{if } i \in \Gamma \text{ and } V^*(\Gamma \backslash \{i\}) > V^*(\Gamma), \text{ then } W_i(\Gamma) = \frac{[p^*(\Gamma \backslash \{i\}) - c]\, k_i}{1 - \delta(1 - \rho(\Gamma \backslash \{i\}))};$$

that is, if an insider leaving the cartel raises each remaining cartel member's payoff under the supposition of the accommodative equilibrium then cartel members do in fact respond with the accommodative equilibrium. For all other cases, cartel members respond with the aggressive equilibrium so $W_i(\Gamma) = 0$.

5. IMPACT OF A COMPETITION AUTHORITY ON THE SET OF STABLE CARTELS

First note that if antitrust enforcement is sufficiently strong, then no cartels are stable because collusion is ineffective at sustaining prices above the noncollusive price. That is not the scenario examined here. Instead, we are considering when stable cartels still exist and asking whether they tend to be larger or smaller compared to the absence of antitrust enforcement. Given that there can be many stable cartels, the analysis will focus on the impact on the range of cartel size.[14] For this purpose, we define:

DEFINITION 2. Γ' is a **minimal stable cartel** if Γ' is stable and Γ is not a stable cartel for all $\Gamma \subset \Gamma'$.

[14] Multiplicity of stable cartels is common in these types of models; see, for example, Donsimoni (1985), Donsimoni et al. (1986), and Diamantoudi (2005).

DEFINITION 3. Γ' is a **maximal stable cartel** if Γ' is stable and Γ is not a stable cartel for all $\Gamma \supset \Gamma'$.

A minimal stable cartel is a stable cartel for which there is no subcoalition of that cartel that is stable, whereas a maximal stable cartel is a stable cartel for which there is no supercoalition containing that cartel that is stable. There can be multiple minimal and maximal stable cartels.

The impact of antitrust enforcement on the size of maximal stable cartels is examined in Section 5.1 and on the size of minimal stable cartels in Section 5.2. It will be shown that competition policy can reduce the size of the largest cartels but, depending on the circumstances, can either increase or decrease the size of the smallest cartels.

5.1. *Maximal Cartel Size.* The task is to compare the size of maximal stable cartels with and without antitrust enforcement. As a benchmark, Lemma 1 shows that if collusion is sustainable in the absence of a competition authority—for which $\delta > \frac{K-D(c)}{K}$ is a necessary and sufficient condition—then there is a unique maximal cartel and it is the all-inclusive cartel.[15]

LEMMA 1. *Assume $\delta > \frac{K-D(c)}{K}$. In the absence of antitrust enforcement, the maximal stable cartel is the all-inclusive cartel.*

It is an immediate corollary that antitrust enforcement cannot cause the largest stable cartel to be more inclusive. Thus, the issue is whether it can cause the largest stable cartel to contract. Although the all-inclusive cartel generates the highest profits without antitrust enforcement, this may no longer be true with an antitrust authority because expected penalties are increasing in cartel size. Although each additional member to the cartel adds value for the original cartel members by restricting its supply below its capacity, it also creates a cost to those original members by increasing the probability of discovery and conviction (and perhaps reducing the chances of an original member receiving leniency since now there will be more firms striving for it). Given that the impact on the collusive price from a firm joining a cartel is positively related to firm size—a larger firm brings more capacity under the control of the cartel, which means output is restricted more and price rises more—a sufficiently small firm may not raise the collusive price enough to offset having increased expected penalties. As a result, a small firm earns higher profit outside of the cartel—which is the case whether or not there is antitrust enforcement—and, in addition, the remaining cartel members also earn higher profit when a small firm remains outside of the cartel—which is only true when there is antitrust enforcement and is because expected penalties are lower. This leads us to Theorem 2, which shows that antitrust enforcement results in small firms not being members of stable cartels.

THEOREM 2. *In the presence of antitrust enforcement, $\exists \underline{k} > 0$ such that if $k_i < \underline{k}$, then firm i is not a member of a stable cartel.*

A corollary of this result is that if a market has sufficiently small firms, then the cartel is not all-inclusive.

COROLLARY 1. *In the presence of antitrust enforcement, $\exists \underline{k} > 0$ such that if $k_i < \underline{k}$ for some $i \in N$, then a maximal stable cartel is not all-inclusive.*

Without a competition authority, the all-inclusive cartel is the most profitable—in terms of value per unit of cartel capacity—and it is stable because cartel members can threaten to

[15] We did not necessarily find the all-inclusive cartel to be stable in Bos and Harrington (2010) because the accommodative equilibrium was used in assessing internal stability. Here, we assume the aggressive equilibrium except where previously noted.

dismantle the cartel if any member leaves. However, when there is a competition authority, the most profitable cartel can be less than all-inclusive because additional members raise expected penalties and this can exceed the benefits from controlling more capacity. In particular, the most inclusive stable cartel will exclude small firms. Thus, competition policy not only reduces price for a given cartel (Theorem 1) but also deters small firms from joining the cartel and, therefore, cartels are no longer all-inclusive.[16]

5.2. *Minimal Cartel Size.* Next we turn to considering the smallest stable cartels. A cartel can be "not inclusive enough" with respect to stability for either of two reasons. First, it may not control enough capacity to sustain any collusion; that is, the collusive price is just the static Nash equilibrium price. In that case, the cartel is not internally stable. Second, it may not be externally stable in that a nonmember prefers to join because doing so sufficiently raises the collusive price and the rise in the new member's price–cost margin is enough to offset it having lower output and becoming liable for penalties. Using these two conditions, the analysis in this section shows that antitrust enforcement can make the smallest cartels either more inclusive or less inclusive.

The next result provides conditions whereby the smallest cartels are at least as large when there is antitrust enforcement. Specifically, when some cartels are unstable without antitrust enforcement, then they are unstable with antitrust enforcement. Hence, if Γ' is a minimal stable cartel without a competition authority—which means that subcoalitions are unstable—then those subcoalitions are still unstable when there is a competition authority. If Γ' is still stable in an environment with antitrust enforcement, then it remains a minimal stable cartel and, if it is no longer stable, then a minimal stable cartel either strictly contains Γ' or there is no minimal stable cartel containing Γ'.

THEOREM 3. *In the absence of antitrust enforcement, assume Γ' is a minimal stable cartel and $p^*(\Gamma) = c$ for all $\Gamma \subset \Gamma'$. Then, in the presence of antitrust enforcement, Γ is not stable for all $\Gamma \subset \Gamma'$.*

In establishing that minimal stable cartel size is weakly higher with antitrust enforcement, Theorem 3 presumed that, without antitrust enforcement, $p^*(\Gamma) = c$ for all subsets of the minimal stable cartel. We will now derive a sufficient condition for that property to hold. Given that (7) is necessary and sufficient for a cartel to support a collusive price (that is, a price exceeding cost), rearranging (7) it follows that Γ' is a smallest cartel that can support a collusive price if and only if

$$K_{\Gamma'} > \frac{K - D(c)}{\delta} \geq K_{\Gamma'} - k_i\,, \quad \text{for all } i \in \Gamma'. \tag{12}$$

The left-hand side (LHS) inequality ensures that Γ' can sustain a price above cost—and thus it is internally stable—and the right-hand side (RHS) inequality ensures that any subset of Γ'

[16] Although this intuitive result is not surprising, it is worth noting that its derivation does require properly defining the conditions for a stable cartel so that it does not rely on nonsensical responses to leaving or joining a cartel (see the discussion surrounding (10)–(11)). For suppose instead no restrictions are placed on how equilibria are used. In that case, a cartel is defined to be stable when (1) there exists an equilibrium E^* such that if a cartel member were to leave the cartel and E^* were to result, then the firm prefers to remain in the cartel, and (2) there exists an equilibrium E^{**} such that if a noncartel member were to join the cartel and E^{**} were to result, then the firm prefers to remain outside the cartel. With that specification then, even under antitrust enforcement, the all-inclusive cartel is the maximal stable cartel (as long as the all-inclusive cartel is able to sustain a price above cost); in other words, an analogue to Lemma 1 is true. Thus, for antitrust enforcement to cause the maximal stable cartel to exclude sufficiently small firms requires properly defining what it means to be a stable cartel.

cannot.[17] Γ' is externally stable if and only if

$$\left(\frac{1}{1-\delta}\right)(p^*(\Gamma')-c)\,k_j \geq \left(\frac{1}{1-\delta}\right)(p^*(\Gamma'\cup\{j\})-c)\left(\frac{D(p^*(\Gamma'\cup\{j\}))-K+K_{\Gamma'}+k_j}{K_{\Gamma'}+k_j}\right)k_j,$$
$$\text{for all } j\notin\Gamma',$$

where the LHS is the payoff to firm $j \notin \Gamma'$ being outside of cartel Γ' and the RHS is the payoff to it joining cartel Γ'. Rearranging this condition yields[18]

$$(13)\qquad [p^*(\Gamma'\cup\{j\})-c]\,[K-D(p^*(\Gamma'\cup\{j\}))] \geq [p^*(\Gamma'\cup\{j\})-p^*(\Gamma')]\,(K_{\Gamma'}+k_j), \quad \text{for all } j\notin\Gamma'.$$

Intuitively, given that a firm that joins the cartel goes from producing at capacity to producing below capacity, a necessary condition for it to find it profitable to join a cartel is that, by bringing more capacity under the control of the cartel, the collusive price is sufficiently higher. (Recall that both insiders and outsiders charge, approximately, the same price.)

Examining (13), if the rise in price from joining the cartel, $p^*(\Gamma'\cup\{j\})-p^*(\Gamma')$, is sufficiently small, then Γ' is externally stable. Here are two examples for which $p^*(\Gamma'\cup\{j\})-p^*(\Gamma')$ will be small. First, k_j is small, in which case all noncartel members have little effect on price from joining the cartel. Second, market demand is perfectly inelastic with a choke price: $D(p)=Q'$ for $p\leq\overline{p}$ (with $\overline{p}>c$) and $D(p)=0$ for $p>\overline{p}$. In that case, it can be shown that the collusive price is the choke price $\overline{p}$ and thus is unaffected by firm j joining the cartel. That result is robust to allowing market demand to be highly inelastic as long as it still implies that the collusive price is the choke price.[19] Section 6 also provides an example with linear demand and symmetric firms for which (12) and (13) hold.

Now let us presume the condition in Theorem 3 does not hold. In that case, it is possible that antitrust enforcement reduces the size of a minimal stable cartel. Specifically, consider a cartel that was not externally stable in the absence of antitrust enforcement because a firm outside of the cartel found it profitable to join because it would significantly increase the collusive price. With antitrust enforcement, that same cartel may now be externally stable when the addition of that firm sufficiently increases the likelihood of detection and as a result it is no longer profitable to join. As long as the cartel can still sustain a collusive price, it is also internally stable.

THEOREM 4. *In the absence of antitrust enforcement, assume* Γ' *is a minimal stable cartel and* $p^*(\Gamma'')>c$ *for some* $\Gamma''\subset\Gamma'$. *Then, in the presence of antitrust enforcement, there exist* $\rho(\cdot)$ *such that* Γ'' *is stable; hence, it is a subset of* Γ' *that is a minimal stable cartel.*

For the condition in Theorem 4 to hold, one needs to show that, in the absence of antitrust enforcement, there exists Γ' such that (1) Γ' is stable; (2) $\forall\Gamma\subset\Gamma'$, Γ is unstable (so that Γ' is a minimal stable cartel); and (3) $\exists\Gamma\subset\Gamma'$ such that $p^*(\Gamma)>c$. There must then be cartels

[17] As long as δ is close enough to 1—so that a large enough cartel can collude and thus the LHS inequality holds—there exist Γ' such that (12) is true because the RHS is sure to hold when $|\Gamma'|=2$, as then a subset of Γ' is the degenerate cartel of one firm that, by our assumptions, cannot sustain price above cost (because the static Nash equilibrium has price equal to cost).

[18] To be clear, the condition in Theorem 3—Γ' is a minimal stable cartel and $p^*(\Gamma)=c$ for all $\Gamma\subset\Gamma'$—holds if and only if (12) and (13) hold.

[19] Highly inelastic market demand may plausibly hold for cartels that sell an input to industrial buyers where the input makes up a small part of the cost of producing the industrial buyer's product. For example, consider the lysine cartel (Connor, 2001). Lysine is used to build tissue in hogs and is a very small part of the cost of producing hogs. Thus, the price of lysine could significantly increase without much of a change in the demand for hogs and thus without much of a change in the derived market demand for lysine. Furthermore, the choke price could be the price at which customers switch over to the next best alternative.

that are internally stable (that is, able to sustain a collusive price) but are not externally stable because a firm would want to join for the purpose of sufficiently raising price. Here we offer some sufficient conditions (details are in the Online Appendix). Assume $\delta \simeq 1$ so that, without antitrust enforcement, the collusive price is the unconstrained price. Assuming $D(p) = 1 - p$, it can be shown that cartel Γ is internally stable (that is, $p^*(\Gamma) > c$) if and only if $K_\Gamma > K - 1 + c$, and is externally stable (that is, an outsider does not increase its profit by joining) if and only if $k_j \leq \sqrt{K_\Gamma^2 - (K - 1 + c)^2}, \forall j \notin \Gamma$.[20] Firm j does not want to join if it is sufficiently small, which makes sense since joining will not have much of an impact on the collusive price—given that noncartel capacity has declined by only a small amount—but will have a proportionally large effect on firm j's supply. Consider cartels comprising the largest firms: $\Gamma(m) \equiv \{1, ..., m\}$, where $k_1 \geq \cdots \geq k_n$. There exist cartels satisfying the property in Theorem 4 if there exists h such that

$$k_1 + \cdots + k_{h-1} \leq K - 1 + c < k_1 + \cdots + k_h < \sqrt{k_{h+1}^2 + (K - 1 + c)^2}. \tag{14}$$

The first two inequalities imply that cartel $\Gamma(h)$ is the smallest cartel that is able to sustain a collusive price, and the third inequality implies that the cartel is not externally stable because firm $h + 1$ wants to join it.[21] Given that $K - 1 + c < \sqrt{k_{h+1}^2 + (K - 1 + c)^2}$ then there exist capacity vectors satisfying (14).

5.3. *Discussion.* Based upon the preceding analysis, let us draw out some general insight regarding how the introduction of a competition authority affects the set of stable cartels. Without antitrust enforcement, the rationale for joining a cartel is that it raises the collusive price—by bringing more capacity under the control of the cartel—but at the cost that the firm must then constrain its supply below capacity. With antitrust enforcement, there is an additional cost of joining, which is that the firm now becomes liable for penalties. Given the associated reduction in profit from becoming a cartel member, it is more likely that any cartel satisfies external stability. As a result, cartels that were externally stable without antitrust enforcement are probably externally stable with antitrust enforcement. The impact on cartel size is then in terms of making previously (externally) unstable cartels now (externally) stable. Specifically, in the absence of antitrust enforcement, a relatively small cartel may have been unstable because a firm wanted to join in order to expand cartel capacity and raise the collusive price. Now that there is antitrust enforcement, the prospect of becoming liable for penalties may discourage that firm from joining, in which case that small cartel is now externally stable. It would then seem that antitrust enforcement augments external stability—firms are less inclined to join a cartel—and this may result in smaller cartels now being stable.

Turning to internal stability, it requires, first, that the cartel has enough capacity so that a collusive price can be sustained, and, second, that the cartel value does not rise in response to a member leaving. If the latter condition did not hold, then the remaining cartel members would accommodate a firm exiting and, given that accommodation, the exiting firm would earn more profit outside of the cartel. The preceding analysis suggests that a competition authority makes it less likely that a cartel will satisfy internal stability. We know that enforcement (weakly) reduces the collusive price by tightening the ICC. Thus, it could cause a cartel to lose the ability to sustain a collusive price and thereby cause the cartel to be internally unstable. To successfully collude, the cartel may then need to be more inclusive in order to control more capacity. Hence, antitrust enforcement may result in small cartels becoming larger in order to sustain a collusive price. However, for the most inclusive cartels, antitrust enforcement may undermine internal

[20] As shown in the Online Appendix, this last inequality is a rearranging of the condition that firm j's profit outside of cartel Γ is at least as great as its profit from joining Γ.

[21] Using the condition mentioned above, cartel $\Gamma(h)$ is not externally stable if $k_{h+1} > \sqrt{K_{\Gamma(h)}^2 - (K - 1 + c)^2}$. Rearranging this condition delivers the third inequality in (14).

stability for a very different reason. A firm contributes to cartel value by having the cartel control more capacity, which then allows it to raise the collusive price, but, at the same time, it detracts from cartel value by increasing the probability of discovery. Thus, antitrust enforcement could result in cartel value being enhanced by a firm leaving the cartel (which also implies the firm finds it profitable to leave), which makes the cartel internally unstable. This force means that antitrust enforcement will tend to reduce the size of large cartels. In sum, antitrust enforcement undermines internal stability, which can cause the smallest cartels to be larger and the largest cartels to be smaller.

6. EXAMPLE

In this section, a linear demand symmetric quadropoly is investigated for the purpose of making three points. First, we show that the conditions under which antitrust enforcement expands minimal cartel size (Theorem 3) are quite plausible. Second, antitrust enforcement—by expanding the minimal stable cartel—is shown to increase price. Third, it is shown that more intense antitrust enforcement can cause the maximal cartel size to expand and the associated collusive price to rise. These results serve to flesh out the subtle and potentially counterproductive effects of antitrust enforcement when the composition of a cartel is endogenized.

Assume four identical firms, each with x units of capacity, and linear demand, $D(p) = a - bp$. Assumption A3 requires

$$\frac{a-bc}{3} < x < \frac{a-bc}{2}, \tag{15}$$

so that an individual firm has insufficient capacity to supply the monopoly quantity and, after excluding one of the firms, the remaining firms have sufficient capacity to supply the competitive quantity. Assume $\delta \cong 1$ so that, without antitrust enforcement, the ICC is not binding. This implies that if $a - bc - (K - K_\Gamma) > 0$ (so cartel Γ has some residual demand if it were to price at cost), then the collusive price is

$$p^*(m) = \frac{a-(K-K_\Gamma)+bc}{2b} = \frac{a-(n-m)x+bc}{2b}, \tag{16}$$

where $m = |\Gamma|$ (and recall that firms have identical capacities so only the number of firms in the cartel matters).

To begin, let us derive necessary and sufficient conditions for a two-firm cartel to be stable in the absence of antitrust enforcement. A two-firm cartel is internally stable if $p^*(2) > c$, which is indeed true because $x < (a - bc)/2$ (from (15)) implies there is residual demand for a two-firm cartel: $a - bc - 2x > 0$. A two-firm cartel is then stable if it is also externally stable, the condition for which is

$$(p^*(2) - c)\,x \geq (p^*(3) - c)\,[a - x - bp^*(3)]\,(1/3). \tag{17}$$

The LHS is the profit to a noncartel member when there is a two-firm cartel, and the RHS is the profit to a member of a three-firm cartel. Inserting the expression for the collusive price from (16), (17) is

$$\left(\frac{a-2x+bc}{2b} - c\right)x \geq \left(\frac{a-x+bc}{2b} - c\right)\left[a - x - b\left(\frac{a-x+bc}{2b}\right)\right](1/3),$$

and simplifying yields: $-13x^2 + 8(a - bc)x - (a - bc)^2 \geq 0$, which implies

$$\frac{x}{a - bc} \in \left[\frac{8 - \sqrt{12}}{26}, \frac{8 + \sqrt{12}}{26}\right] \cong [.175, .440] \tag{18}$$

Combining restrictions in (15) and (18):

$$\frac{x}{a - bc} \in \left(\frac{1}{3}, \frac{8 + \sqrt{12}}{26}\right] \tag{19}$$

Thus, if (19) is true, then the conditions of Theorem 3 hold in that the minimal stable cartel has two firms and any smaller cartel (which necessarily has a single firm) has price equal to cost.

Next let us derive sufficient conditions for antitrust enforcement to raise the minimal stable cartel from two firms to either three or four firms. Assume $\psi(\Gamma) = 1\ \forall\Gamma$, $\theta = 1$, and $\gamma = 0$ so there is no leniency program and penalties are zero.[22] Thus, antitrust enforcement in this example operates by discovering and convicting cartels and thereby shutting them down. Under these assumptions—and recalling that $\delta \cong 1$—the ICC from (4) takes the form

$$\frac{1}{\rho(m)} \geq \frac{mx}{D(p) - (K - mx)} \tag{20}$$

where $\rho(m)$ refers to the probability of cartel Γ paying penalties and $m = |\Gamma|$. With antitrust enforcement, a two-firm cartel is not stable if (20) does not hold strictly for $p = c$

$$\frac{2x}{a - bc - 2x} \geq \frac{1}{\rho(2)} \Rightarrow \frac{x}{a - bc} \geq \frac{1}{2(1 + \rho(2))}. \tag{21}$$

For (21) to be consistent with (19), we need

$$\frac{8 + \sqrt{12}}{26} \geq \frac{1}{2(1 + \rho(2))} \Rightarrow \rho(2) \geq \frac{5 - \sqrt{12}}{8 + \sqrt{12}} \cong .134.$$

With antitrust enforcement, a three firm-cartel is internally stable when (20) holds strictly for $m = 3$ and $p = c$

$$\frac{1}{\rho(3)} > \frac{3x}{a - bc - x} \Rightarrow \frac{x}{a - bc} < \frac{1}{\rho(3)3 + 1}. \tag{22}$$

If, in addition, the three-firm cartel is externally stable, then it is the minimal stable cartel. If it is not externally stable, then it means that the remaining noncartel member prefers to join the cartel and, therefore, the four-firm cartel is internally stable (and it is trivially externally stable). Combining (21) and (22),

$$\frac{1}{2(1 + \rho(2))} \leq \frac{x}{a - bc} < \frac{1}{\rho(3)3 + 1}. \tag{23}$$

If (23) holds, then, under antitrust enforcement, the minimal stable cartel has either three or four firms.

[22] Results are continuous with respect to γ and thus hold as long as penalties are not too strong. Although we have no reason to think that our conclusions are tied to assuming penalties are weak, allowing $\gamma > 0$ complicates the expressions.

Combining the conditions in (19) and (23), the minimal stable cartel has two firms in the absence of antitrust enforcement and has either three or four firms with antitrust enforcement when

$$\max\left\{\frac{1}{3}, \frac{1}{2\,(1+\rho\,(2))}\right\} < \frac{x}{a-bc} < \min\left\{\frac{8+\sqrt{12}}{26}, \frac{1}{\rho\,(3)\,3+1}\right\}. \tag{24}$$

Examining the LHS inequality,

$$\frac{1}{3} < \frac{1}{2\,(1+\rho\,(2))} \Leftrightarrow \rho\,(2) < \frac{1}{2}.$$

Turning to the RHS inequality,

$$\frac{8+\sqrt{12}}{26} < \frac{1}{\rho\,(3)\,3+1} \Leftrightarrow \rho\,(3) < \frac{18-\sqrt{12}}{3(8+\sqrt{12})} \cong 0.422.$$

Thus, if $\rho\,(2) < \rho\,(3) < 0.422$ then (24) becomes

$$\frac{1}{2\,(1+\rho\,(2))} < \frac{x}{a-bc} < \frac{8+\sqrt{12}}{26}, \tag{25}$$

and we already showed that this interval is nonempty when $\rho\,(2) \geq 0.134$.

In sum, (24) is satisfied when $0.134 < \rho\,(2) < \rho\,(3) < 0.422$ and (25) holds. For example, if $\rho\,(2) = 0.3 < \rho\,(3) < 0.422$, then (25) is $x/\,(a-bc) \in (0.385, 0.441)$. Under those parameter conditions, antitrust enforcement causes the minimal stable cartel to expand from two firms to either three or four firms.

The preceding analysis showed that antitrust enforcement can cause a cartel to be more encompassing when firms settle on the minimally stable cartel. Our second objective is to show that price can be higher. As cartel size increases from two firms to three or four firms, the collusive price will definitely rise if cartel members are able to sustain the unconstrained optimal collusive price in (16) (so the ICC is not binding). Though it is assumed $\delta = 1$, it is not immediate that the ICC is not binding because the probability that the cartel is caught and shut down is positive. Nevertheless, conditions will be derived on those probabilities such that antitrust enforcement increases both the size of the minimal stable cartel and price.

For a three-firm cartel, the ICC is satisfied at the unconstrained optimal collusive price if and only if

$$\frac{1}{\rho\,(3)} \geq \frac{3x}{a-b\left(\frac{a-x+bc}{2b}\right)-x} \Rightarrow \frac{x}{a-bc} \leq \frac{1}{1+\rho\,(3)\,6}.$$

The analogous property for a four-firm cartel is

$$\frac{1}{\rho\,(4)} \geq \frac{4x}{a-b\left(\frac{a+bc}{2b}\right)} \Rightarrow \frac{x}{a-bc} \leq \frac{1}{8\rho\,(4)}.$$

Combining these two conditions yields

$$\frac{x}{a-bc} \leq \min\left\{\frac{1}{1+\rho\,(3)\,6}, \frac{1}{8\rho\,(4)}\right\}. \tag{26}$$

Given that $\frac{1}{1+\rho(3)6} \leq \frac{1}{1+\rho(3)3}$, then, combining (26) with (24), we have

$$\max\left\{\frac{1}{3}, \frac{1}{2(1+\rho(2))}\right\} < \frac{x}{a-bc} < \min\left\{\frac{8+\sqrt{12}}{26}, \frac{1}{\rho(3)6+1}, \frac{1}{8\rho(4)}\right\}.$$

It is straightforward to show that if $0.134 < \rho(2) < \rho(3) < \rho(4) < 0.25$, then[23]

$$\max\left\{\frac{1}{3}, \frac{1}{2+\rho(2)2}\right\} < \min\left\{\frac{8+\sqrt{12}}{26}, \frac{1}{\rho(3)6+1}, \frac{1}{8\rho(4)}\right\}$$

and therefore there exist values for $x/(a-bc)$ such that (1) without antitrust enforcement, the minimal stable cartel has two firms; (2) with antitrust enforcement, the minimal stable cartel has either three or four firms; and (3) for the minimal stable cartel, the collusive price is higher with antitrust enforcement than without antitrust enforcement.[24]

Continuing with the preceding structure, we will now consider an initial level of antitrust enforcement such that a maximal stable cartel has three firms and then show how intensifying antitrust enforcement can expand the maximal stable cartel to be all-inclusive and also raise the collusive price. If $p^*(\Gamma) > c$, then a cartel is internally stable if and only if the exit of a cartel member does not raise the expected profit of the remaining firms.[25] The condition for all cartel members $i \in \Gamma$ to earn at least as high profit when firm $j \in \Gamma$ is part of the cartel as opposed to being outside the cartel is

$$\left(\frac{1-\rho(\Gamma)\psi(\Gamma)\gamma}{1-\delta(1-\rho(\Gamma))}\right)(p^*(\Gamma)-c)\left[\frac{D(p^*(\Gamma))-(K-K_\Gamma)}{K_\Gamma}\right]k_i$$
$$\geq \left(\frac{1-\rho(\Gamma\backslash\{j\})\psi(\Gamma\backslash\{j\})\gamma}{1-\delta(1-\rho(\Gamma\backslash\{j\}))}\right)(p^*(\Gamma\backslash\{j\})-c)\left[\frac{D(p^*(\Gamma\backslash\{j\}))-(K-K_\Gamma+k_j)}{K_\Gamma-k_j}\right]k_i.$$

Rearranging, using symmetry, and setting $\psi(\Gamma)=1, \gamma=0, \delta=1$, this condition takes the form

$$\left(\frac{1}{\rho(m)}\right)(p^*(m)-c)\left[\frac{D(p^*(m))-(n-m)x}{mx}\right] \geq \left(\frac{1}{\rho(m-1)}\right)(p^*(m-1)-c)\left[\frac{D(p^*(m-1))-(n-m+1)x}{(m-1)x}\right]. \tag{27}$$

A four-firm (all-inclusive) cartel is then not internally stable when (27) does not hold:

$$4\rho(4)(p^*(3)-c)[D(p^*(3))-x] > 3\rho(3)(p^*(4)-c)D(p^*(4)). \tag{28}$$

Suppose the collusive price is the unconstrained price,

$$p^*(m) = \frac{a-(n-m)x+bc}{2b}, \tag{29}$$

[23] One method of derivation is to assume $\rho(2)=\rho(3)=\rho(4)=\rho$ and show that $\max\left\{\frac{1}{3}, \frac{1}{2+\rho 2}\right\} < \frac{x}{a-bc} < \min\left\{\frac{8+\sqrt{12}}{26}, \frac{1}{\rho 6+1}, \frac{1}{8\rho}\right\}$ is nonempty for all $\rho \in (.134, .25)$.

[24] Note that these are sufficient but not necessary conditions since they presume $\rho(m)$ is sufficiently low that an m-firm cartel can sustain the unconstrained optimal collusive price.

[25] The reason is that if a firm's exit lowers a remaining cartel member's profit, then the aggressive equilibrium ensues and, given $p^*(\Gamma) > c$, that implies lower profit for the exiting firm; hence, it would not want to exit.

which is the case when

$$\frac{x}{a-bc} \leq \min\left\{\frac{1}{1+\rho(3)6}, \frac{1}{8\rho(4)}\right\}.$$

Substituting (29) into (28), internal stability is violated if

$$4\rho(4)\left(\frac{a-x+bc}{2b}-c\right)\left[a-b\left(\frac{a-x+bc}{2b}\right)-x\right] > 3\rho(3)\left(\frac{a+bc}{2b}-c\right)\left[a-b\left(\frac{a+bc}{2b}\right)\right],$$

which, after simplifying, is

$$\frac{\rho(4)}{\rho(3)} > \left(\frac{3}{4}\right)\left(\frac{a-bc}{a-bc-x}\right)^2. \tag{30}$$

Suppose (30) is true so that a four-firm cartel is not internally stable.

Now suppose antitrust enforcement is changed so that $\rho(\cdot)$ is replaced with $\widehat{\rho}(\cdot)$. Assume ICCs hold at unconstrained collusive prices for both three-firm and four-firm cartels under both antitrust regimes:

$$\frac{x}{a-bc} \leq \min\left\{\frac{1}{1+\rho(3)6}, \frac{1}{8\rho(4)}, \frac{1}{1+\widehat{\rho}(3)6}, \frac{1}{8\widehat{\rho}(4)}\right\}. \tag{31}$$

A four-firm cartel is now internally stable if the converse to (30) holds. Thus, if

$$\frac{\rho(4)}{\rho(3)} > \left(\frac{3}{4}\right)\left(\frac{a-bc}{a-bc-x}\right)^2 \geq \frac{\widehat{\rho}(4)}{\widehat{\rho}(3)}, \tag{32}$$

then the maximal stable cartel is a three-firm cartel under $\rho(\cdot)$, and, under the new antitrust regime of $\widehat{\rho}(\cdot)$, the maximal stable cartel expands to four firms. The collusive price rises as well since the unconstrained collusive price is increasing with cartel size.

Sufficient conditions for (31) and (32) to hold are $\rho(3), \rho(4), \widehat{\rho}(3), \widehat{\rho}(4)$ are small, $\rho(4)/\rho(3)$ is large, and $\widehat{\rho}(4)/\widehat{\rho}(3)$ is small. Note that antitrust enforcement could have intensified so $\widehat{\rho}(\cdot) > \rho(\cdot)$. For example, suppose, under the original antitrust regime, the probability of discovery is very low except when the cartel is all-inclusive, $0 \simeq \rho(3) < \rho(4)$; and, under the new antitrust regime, the probability of discovery is increased to $\rho(4)$ for all cartel sizes: $\widehat{\rho}(3) \simeq \widehat{\rho}(4) \simeq \rho(4)$. In that case (for some values of x), intensifying enforcement results in the maximal cartel size expanding and the associated collusive price rising.

7. SOME IMPLICATIONS FOR COMPETITION POLICY

If the composition of the cartel is assumed to be fixed with respect to competition policy, then the introduction of antitrust enforcement or a rise in antitrust enforcement will cause the collusive price to decline (Theorem 1). When cartel membership is allowed to respond to competition policy, then its impact is more subtle and nuanced. Although antitrust enforcement could cause cartel size to contract and collusive price to fall, we also find that the opposite can be true. The introduction of antitrust enforcement can cause minimal cartel size to expand and the collusive price to rise, whereas more aggressive antitrust enforcement could expand maximal cartel size and raise the collusive price.

Though the relationship between antitrust enforcement and cartel size is too complex for us to provide specific guidance for enforcement policies, there is still some general advice that the theory can deliver. The pathological effects of antitrust enforcement—whereby cartels are bigger and the collusive price is higher—is attributable to the following force. Antitrust enforcement can make collusion unsustainable for small cartels, which causes more firms to join the cartel in order to sustain a supracompetitive price, and this can ultimately result in a higher collusive price, or a rise in antitrust enforcement that is not proportionally more intensive for larger cartels makes it more attractive for firms to form a more inclusive cartel. This would then suggest the adoption of policies that are not simply more aggressive but are *progressively more aggressive for more inclusive cartels*. With such a policy, a rise in enforcement that destabilizes the smallest stable cartel and inclines firms to expand the cartel may no longer be a viable response if enforcement is more severe for larger cartels. In other words, the level of antitrust enforcement does not just matter but also its gradient with respect to cartel inclusiveness.

To make this insight more concrete in terms of recommendations, consider two policies that have been proposed for enhancing the discovery of cartels: screening and whistleblowing. Screening involves the use of market data (such as prices and market shares) in order to identify suspicious patterns that may suggest the presence of a cartel and thus warrant investigation (Harrington, 2007). Screening will raise $\rho(\cdot)$ but probably not affect its sensitivity to cartel size since its efficacy is not related to how many firms are involved in the cartel. Whistleblowing is the provision of financial rewards to individuals uninvolved in a cartel who provide information leading to a conviction. Like screening, whistleblowing will raise $\rho(\cdot)$ but, in contrast to screening, it will make $\rho(\cdot)$ more sensitive to the number of cartel members; the more firms involved in the cartel, the more uninvolved employees there are who could uncover evidence of a cartel and report it to the competition authority. The analysis of this article suggests that whistleblowing is a more attractive avenue than screening because it is more likely to constrain cartel size. This is a point worth making since whistleblowing has been discussed for many years (as early as Kovacic, 2000) but has only been adopted in three jurisdictions: Hungary, Korea, and the United Kingdom.

An analogous line of argument delivers a previously unidentified benefit from a leniency program. For simplicity, consider a leniency program in which only the first firm to apply receives amnesty and all fines are waived. In the context of our model, this means $\psi(\Gamma) = (|\Gamma| - 1) / |\Gamma|$ and, conditional on an investigation, the penalty to firm i is $((|\Gamma| - 1) / |\Gamma|) \gamma \pi_i$ where π_i is firm i's profit as a cartel member. Thus, for any level of cartel profit, the penalty is higher when the number of cartel members, $|\Gamma|$, is higher. The intuition is simple enough. When there is a leniency program, a more inclusive cartel is less attractive because there are more firms with which to compete for leniency, if it should come down to wanting it.

Finally, note that if competition policies are such that $\rho(\Gamma)$ and $\psi(\Gamma)$ are independent of Γ, then the maximal stable cartel is the all-inclusive cartel (assuming it is able to sustain a supracompetitive price). By instead making $\rho(\Gamma)$ and/or $\psi(\Gamma)$ higher when Γ is more inclusive—even if the maximal values for $\rho(\Gamma)$ and/or $\psi(\Gamma)$ are unchanged or even a little lower—maximal cartel size can shrink and with it the collusive price.

8. CONCLUDING REMARKS

This article is an initial foray into how competition policy impacts the inclusiveness of cartels. We found that the presence of antitrust enforcement causes the most inclusive stable cartels to be less inclusive and, in particular, small firms that might have been cartel members in the absence of a competition authority are no longer members. Regarding the least inclusive stable cartels, the presence of antitrust enforcement can either increase or decrease their inclusiveness, depending on market conditions.

A potentially interesting extension is to allow cartels to use exclusionary tactics to restrain the supply of noncartel members. The current analysis presumes that noncartel members do not restrain their supply in that they price just below the cartel's price and produce up to capacity (see Bos and Harrington, 2010). This is obviously detrimental to the cartel, and, in practice, some cartels have augmented their collusive price-setting with exclusionary activities intended to constrain the supply of noncartel members. For example, in the district heating pipes cartel, the Swedish firm Powerpipe did not join the cartel and eventually complained to the European

Commission that there was a cartel that was acting in a predatory manner against it.[26] The use of exclusionary activities also explains why noncartel members are a common source of discovery of cartels. Hay and Kelley (1974) found that 12 out of 49 U.S. Department of Justice cases were discovered by means of a "complaint by a competitor" and was the second most common source of detection. This raises an interesting trade-off. A partially inclusive cartel could be made more profitable by engaging in exclusionary activities against noncartel members, but doing so runs the risk of those noncartel members complaining and the cartel being discovered. Encompassing those factors in our model could produce some new insight into both the structure of cartels and the properties of collusive practices.

APPENDIX: PROOFS

PROOF OF THEOREM 1. If the ICC is not binding, then $p^*(\Gamma)$ is independent of $\rho(\Gamma)$, $\psi(\Gamma)$, and γ. If the ICC is binding, then $p^*(\Gamma)$ satisfies

$$\Omega(\Gamma) = \frac{K_\Gamma}{D(p^*(\Gamma)) - (K - K_\Gamma)}, \tag{A.1}$$

where $\Omega(\Gamma)$ is defined in (4). If $\theta < \rho(\Gamma)\psi(\Gamma)$, then

$$\frac{\partial\Omega(\Gamma)}{\partial\rho(\Gamma)} = -\left(\frac{\psi(\Gamma)\gamma(1-\delta)+\delta}{[1-\delta(1-\rho(\Gamma))]^2}\right) < 0,\ \frac{\partial\Omega(\Gamma)}{\partial\psi(\Gamma)} = -\frac{\rho(\Gamma)\gamma}{1-\delta(1-\rho(\Gamma))} < 0,$$

$$\frac{\partial\Omega(\Gamma)}{\partial\gamma} = -\left(\frac{\rho(\Gamma)\psi(\Gamma)-\theta+\delta\theta(1-\rho(\Gamma))}{1-\delta(1-\rho(\Gamma))}\right) < 0.$$

If $\theta > \rho(\Gamma)\psi(\Gamma)$, then

$$\frac{\partial\Omega(\Gamma)}{\partial\rho(\Gamma)} = -\left(\frac{\psi(\Gamma)\gamma\delta(1-\rho(\Gamma))[1-\delta(1-\rho(\Gamma))]+\delta(1-\rho(\Gamma)\psi(\Gamma)\gamma)}{[1-\delta(1-\rho(\Gamma))]^2}\right) < 0,$$

$$\frac{\partial\Omega(\Gamma)}{\partial\psi(\Gamma)} = -\frac{\delta(1-\rho(\Gamma))\rho(\Gamma)\gamma}{1-\delta(1-\rho(\Gamma))} < 0,\ \frac{\partial\Omega(\Gamma)}{\partial\gamma} = -\frac{\delta(1-\rho(\Gamma))\rho(\Gamma)\psi(\Gamma)}{1-\delta(1-\rho(\Gamma))} < 0.$$

Given that $\Omega(\Gamma)$ is decreasing in $\rho(\Gamma)$, $\psi(\Gamma)$, and γ, then, at higher values for those parameters, (A.1) implies

$$\Omega(\Gamma) < \frac{K_\Gamma}{D(p^*(\Gamma)) - (K - K_\Gamma)}.$$

It follows that $p^*(\Gamma)$ must decline so that $D(p^*(\Gamma))$ is increased and $\frac{K_\Gamma}{D(p^*(\Gamma))-(K-K_\Gamma)}$ is decreased in order to satisfy the ICC.

PROOF OF LEMMA 1. Theorem 4 in Bos and Harrington (2010) shows that the cartel value without antitrust enforcement, $V^*(\Gamma)$, is greater when the cartel is more inclusive: If $\Gamma' \subset \Gamma'''$ then $V^*(\Gamma'') > V^*(\Gamma')$. Consider the all-inclusive cartel: $\Gamma = N$. As there are no outsiders, it is trivially externally stable. By assuming $W_i(\Gamma) = 0$—that is, an insider who leaves the cartel can expect the static Nash equilibrium—it follows from $V^*(N) > 0$ that it is internally stable. Therefore, N is stable in the absence of antitrust enforcement.

[26] This example, and other ones, can be found in Harrington (2006) and Marshall et al. (2011).

PROOF OF THEOREM 2. Consider some cartel Γ'. The collusive value per unit of capacity for cartel Γ' is

$$V^*(\Gamma') = \left(\frac{1-\rho(\Gamma')\psi(\Gamma')\gamma}{1-\delta(1-\rho(\Gamma'))}\right)(p^*(\Gamma')-c)\left(\frac{D(p^*(\Gamma'))-(K-K_{\Gamma'})}{K_{\Gamma'}}\right) \tag{A.2}$$

and recall that the value to firm $j \in \Gamma'$ is $k_j V^*(\Gamma')$. Let us contrast this with the collusive value per unit of capacity for cartel $\Gamma'' \equiv \Gamma' \backslash \{i\}$ that excludes firm $i \in \Gamma'$:

$$V^*(\Gamma'') = \left(\frac{1-\rho(\Gamma'')\psi(\Gamma'')\gamma}{1-\delta(1-\rho(\Gamma''))}\right)(p^*(\Gamma'')-c)\left(\frac{D(p^*(\Gamma''))-(K-K_{\Gamma'}+k_i)}{K_{\Gamma'}-k_i}\right). \tag{A.3}$$

Comparing (A.3) with (A.2), cartel Γ'' generates more value per unit of capacity than cartel Γ' when

$$\left(\frac{1-\rho(\Gamma'')\psi(\Gamma'')\gamma}{1-\delta(1-\rho(\Gamma''))}\right)(p^*(\Gamma'')-c)\left(\frac{D(p^*(\Gamma''))-(K-K_{\Gamma'}+k_i)}{K_{\Gamma'}-k_i}\right)$$
$$> \left(\frac{1-\rho(\Gamma')\psi(\Gamma')\gamma}{1-\delta(1-\rho(\Gamma'))}\right)(p^*(\Gamma')-c)\left(\frac{D(p^*(\Gamma'))-(K-K_{\Gamma'})}{K_{\Gamma'}}\right).$$

Rearranging this inequality yields

$$\frac{(p^*(\Gamma'')-c)\left(\frac{D(p^*(\Gamma''))-(K-K_{\Gamma'}+k_i)}{K_{\Gamma'}-k_i}\right)}{(p^*(\Gamma')-c)\left(\frac{D(p^*(\Gamma'))-(K-K_{\Gamma'})}{K_{\Gamma'}}\right)} > \frac{(1-\rho(\Gamma')\psi(\Gamma')\gamma)(1-\delta(1-\rho(\Gamma'')))}{(1-\rho(\Gamma'')\psi(\Gamma'')\gamma)(1-\delta(1-\rho(\Gamma')))}. \tag{A.4}$$

Consider the LHS of (A.4) as $k_i \to 0$,

$$\lim_{k_i\to 0}\frac{(p^*(\Gamma'')-c)\left(\frac{D(p^*(\Gamma''))-(K-K_{\Gamma'}+k_i)}{K_{\Gamma'}-k_i}\right)}{(p^*(\Gamma')-c)\left(\frac{D(p^*(\Gamma'))-(K-K_{\Gamma'})}{K_{\Gamma'}}\right)} = \lim_{k_i\to 0}\frac{(p^*(\Gamma'')-c)\,[D(p^*(\Gamma''))-(K-K_{\Gamma'})]}{(p^*(\Gamma')-c)\,[D(p^*(\Gamma'))-(K-K_{\Gamma'})]}.$$

When $\rho(\cdot)=0$, $\lim_{k_i\to 0} p^*(\Gamma'') = \lim_{k_i\to 0} p^*(\Gamma')$. By Theorem 1, $\rho(\cdot)>0$ implies $\lim_{k_i\to 0} p^*(\Gamma'') \geq \lim_{k_i\to 0} p^*(\Gamma')$ because, as $k_i \to 0$, cartel Γ' is equivalent to cartel Γ'' but with a higher probability of discovery $\rho = \rho(\Gamma')$. Therefore, $\lim_{k_i\to 0}(p^*(\Gamma'')-c)\,[D(p^*(\Gamma''))-(K-K_{\Gamma'})] \geq \lim_{k_i\to 0}(p^*(\Gamma')-c)\,[D(p^*(\Gamma'))-(K-K_{\Gamma'})]$, which implies the LHS of (A.4) is at least 1. Next consider the RHS of (A.4). Rearranging the expression, it is strictly less than one iff

$$\delta\,[\rho(\Gamma')-\rho(\Gamma'')] + (1-\delta)\,\gamma\,[\rho(\Gamma')\psi(\Gamma')-\rho(\Gamma'')\psi(\Gamma'')] + \delta\rho(\Gamma')\rho(\Gamma'')\gamma\,[\psi(\Gamma')-\psi(\Gamma'')] > 0.$$

This condition holds because of A4 and A5: $\lim_{k_i\to 0}\rho(\Gamma') > \lim_{k_i\to 0}\rho(\Gamma'')$ and $\psi(\Gamma') \geq \psi(\Gamma'')$. We have then shown that the firms in Γ'' earn a higher payoff when firm i is not a member of the cartel compared to when it is a member. By our specification, if firm i departs from cartel Γ'' then the remaining cartel members respond with the accommodative equilibrium.

The second step is to show that firm i prefers to depart from cartel Γ' when the accommodative equilibrium ensues, in which case Γ' is not internally stable. Firm i's payoff in the cartel is

$$\left(\frac{1-\rho(\Gamma')\psi(\Gamma')\gamma}{1-\delta(1-\rho(\Gamma'))}\right)(p^*(\Gamma')-c)\left(\frac{D(p^*(\Gamma'))-(K-K_{\Gamma'})}{K_{\Gamma'}}\right)k_i, \tag{A.5}$$

and outside of the cartel is

$$\frac{(p^*(\Gamma'') - c)k_i}{1 - \delta(1 - \rho(\Gamma''))}. \tag{A.6}$$

Given that

$$\frac{1}{1 - \delta(1 - \rho(\Gamma''))} > \frac{1 - \rho(\Gamma')\psi(\Gamma')\gamma}{1 - \delta(1 - \rho(\Gamma'))},$$

then (A.6) exceeds (A.5) if

$$p^*(\Gamma'') - c \geq (p^*(\Gamma') - c)\left(\frac{D(p^*(\Gamma')) - (K - K_{\Gamma'})}{K_{\Gamma'}}\right).$$

Given $\lim_{k_i \to 0} p^*(\Gamma'') \geq \lim_{k_i \to 0} p^*(\Gamma')$, then this condition holds.

In sum, as $k_i \to 0$, cartel Γ' is not stable because firm i finds it more profitable to be outside of the cartel assuming the cartel accommodates its departure and, given the remaining cartel members are better off with the departure, they do indeed accommodate it. Therefore, if a firm's capacity is sufficiently small, it is not a member of a stable cartel.

PROOF OF THEOREM 3. Recall that, without antitrust enforcement, a necessary and sufficient condition for a cartel Γ to sustain a collusive price is

$$\frac{1}{1 - \delta} > \frac{K_\Gamma}{D(c) - (K - K_\Gamma)}.$$

Hence, if

$$\frac{1}{1 - \delta} \leq \frac{K_\Gamma}{D(c) - (K - K_\Gamma)}, \tag{A.7}$$

then $p^*(\Gamma) = c$ and cartel Γ is (trivially) internally unstable. As postulated for when there is no antitrust enforcement, suppose Γ' is a minimal stable cartel and $p^*(\Gamma) = c$ for all $\Gamma \subset \Gamma$, which means (A.7) holds. We want to show that, in the presence of antitrust enforcement, the equivalent condition to (A.7) holds for all $\Gamma \subset \Gamma$, which means the minimal stable cartel is not a subset of Γ'.

With antitrust enforcement, the analogue to (A.7) is

$$\Omega(\Gamma) \equiv \left(\frac{1 - \rho(\Gamma)\psi(\Gamma)\gamma}{1 - \delta(1 - \rho(\Gamma))}\right) + \min\{\rho(\Gamma)\psi(\Gamma), \theta\}\gamma \leq \frac{K_\Gamma}{D(c) - (K - K_\Gamma)}. \tag{A.8}$$

To see that (A.7) implies (A.8), note that the RHSs are the same, whereas the LHS of (A.8) is lower. As to the latter, if $\rho(\Gamma)\psi(\Gamma) \geq \theta$, then we need

$$\frac{1 - \gamma(\rho(\Gamma)\psi(\Gamma) - \theta) - \theta\gamma\delta(1 - \rho(\Gamma))}{1 - \delta(1 - \rho(\Gamma))} \leq \frac{1}{1 - \delta},$$

which holds. If $\rho(\Gamma)\psi(\Gamma) < \theta$, then we need

$$\frac{1 - \delta(1 - \rho(\Gamma))\rho(\Gamma)\psi(\Gamma)\gamma}{1 - \delta(1 - \rho(\Gamma))} \leq \frac{1}{1 - \delta},$$

which also holds. Hence, with antitrust enforcement, $p^*(\Gamma) = c$ and therefore Γ is internally unstable for all $\Gamma \subset \Gamma'$.

PROOF OF THEOREM 4. In the absence of antitrust enforcement, suppose Γ' is a stable cartel, Γ'' is not a stable cartel where $\Gamma'' \subset \Gamma'$, and $p^*(\Gamma'') > c$. Note that, in the absence of antitrust enforcement, internal stability of Γ'' is satisfied by specifying the aggressive equilibrium when a cartel member fails to join. Thus, if Γ'' is not stable, it is because it is not externally stable. Now suppose there is antitrust enforcement and let $\rho(\Gamma'') = \underline{\rho}$ and $\min\left\{\rho(\Gamma'' \cup \{j\}) : j \notin \Gamma''\right\} = \overline{\rho}$; that is, $\overline{\rho}$ is the smallest probability of conviction that results from an outsider joining Γ''. By continuity with the case of no antitrust enforcement, if $\underline{\rho} \simeq 0$ then Γ'' is internally stable. By setting $\overline{\rho} \simeq 1$, Γ'' is externally stable because another firm joining the cartel is unprofitable due to the high rate of conviction, making it very likely the cartel will shut down.[27]

SUPPORTING INFORMATION

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Online Appendix: Proof of conditions for the property in Theorem 4 to hold.

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[27] That the penalty multiple, γ, does not matter is because cartel shutdown is forever. If instead the cartel was allowed to reform, then the same theorem would be true, but, in addition to $\overline{\rho} \simeq 1$, we would need γ to be sufficiently high so that expected penalties are sufficiently high.

10 Online Appendix: Proof of Conditions for the Property in Theorem 9 to Hold (Not for Publication)

Theorem 9 showed that antitrust enforcement can reduce minimal cartel size. A key requirement for this to occur is that, in the absence of antitrust enforcement, there exists some subset of a minimal cartel that is able to sustain a price above costs. If this requirement is not met, then we know by Theorem 8 that antitrust enforcement will not lead to smaller minimal cartels. This raises the issue of when without antitrust enforcement there exists a cartel Γ such that: 1) Γ is stable, 2) Γ' is unstable for all $\Gamma' \subset \Gamma$ and 3) there exists a cartel $\Gamma' \subset \Gamma$ for which $p^*(\Gamma') > c$. In the following, we present sufficient conditions for this property to hold.

- **Step 1: Derive conditions under which a cartel Γ is stable.**

To begin, assume that $D(p) = 1 - p$ and $\delta \simeq 1$. Thus, for a given cartel Γ, the ICC is not binding. In this case, the optimal cartel price is given by

$$p^*(\Gamma) = \frac{1 - K + K_\Gamma + c}{2}.$$

For Γ to be internally stable through the aggressive equilibrium it must hold that $p^*(\Gamma) > c$, which requires

$$K_\Gamma > \frac{K - D(c)}{\delta} \simeq K - 1 + c.$$

This cartel is externally stable when none of the outsiders finds it profitable to join, which is the case when

$$\left(\frac{1}{1-\delta}\right)\left[p^*\left(\Gamma\right)-c\right]k_j \geq \left(\frac{1}{1-\delta}\right)\left[p^*\left(\Gamma\cup\{j\}\right)-c\right]\left(\frac{1-p^*\left(\Gamma\cup\{j\}\right)-\left(K-K_\Gamma-k_j\right)}{K_\Gamma+k_j}\right)k_j, \forall j\notin\Gamma,$$

or

$$p^*\left(\Gamma\right)-c \geq \left[p^*\left(\Gamma\cup\{j\}\right)-c\right]\left(\frac{1-p^*\left(\Gamma\cup\{j\}\right)-\left(K-K_\Gamma-k_j\right)}{K_\Gamma+k_j}\right), \forall j\notin\Gamma.$$

By using

$$p^*\left(\Gamma\cup\{j\}\right)=\frac{1-K+K_\Gamma+k_j+c}{2},$$

the external stability condition reduces to

$$\left(K_\Gamma+k_j\right)\left[p^*\left(\Gamma\right)-c\right] \geq \left(p^*\left(\Gamma\cup\{j\}\right)-c\right)^2, \forall j\notin\Gamma.$$

Substituting $p^*\left(\Gamma\right)=\frac{1-K+K_\Gamma+c}{2}$ and $p^*\left(\Gamma\cup\{j\}\right)=\frac{1-K+K_\Gamma+k_j+c}{2}$ and rearranging gives

$$K_\Gamma \geq \sqrt{k_j^2+\left(K-1+c\right)^2} \Leftrightarrow \sqrt{K_\Gamma^2-\left(K-1+c\right)^2} \geq k_j, \forall j\notin\Gamma,$$

which implies $K_\Gamma > K-1+c$. We therefore conclude that a cartel Γ is stable when the following condition holds:

$$K_\Gamma \geq \sqrt{k_j^2+\left(K-1+c\right)^2}, \forall j\notin\Gamma.$$

- **Step 2: Derive conditions under which the cartel is minimally stable.**

Let us now show when Γ' is unstable for all $\Gamma' \subset \Gamma$. Towards that end, consider cartels that involve the largest firms: $\Gamma(m) \equiv \{1, ..., m\}$ and define h by

$$k_1 + \cdots + k_{h-1} \leq K - 1 + c < k_1 + \cdots + k_h.$$

Thus, $p^*(\Gamma(h)) > c$ and $p^*(\Gamma(m)) = c$, $\forall m < h$. Cartel $\Gamma(h)$ is therefore the smallest cartel for which a price above cost is sustainable. Observe that there always exists a h for which this condition is satisfied since for $h = 2$ we have $k_1 \leq K - 1 + c$, which holds by assumption and for $h = n$ we have $K - 1 + c < k_1 + \cdots + k_n \Leftrightarrow 1 - c > 0$, which again holds by assumption.

Next, suppose there exists r satisfying:

$$k_1 + \cdots + k_{r-1} < \sqrt{k_r^2 + (K - 1 + c)^2},$$

and

$$\sqrt{k_{r+1}^2 + (K - 1 + c)^2} \leq k_1 + \cdots + k_r.$$

We show below that such an r exists and is unique. Now consider a cartel $\Gamma(m)$ for which $m < r$. Such a cartel is unstable because: 1) If $m \in \{h, ..., r-1\}$, then it is internally stable through the aggressive equilibrium, but externally unstable as firm $m+1$ wants to join; and 2) If $m \in \{2, ..., h-1\}$, then it is internally unstable as $p^*(\Gamma(m)) = c$. By contrast, the cartel $\Gamma(r)$ is stable because: 1) it is internally stable since $k_1 + \cdots + k_r \geq \sqrt{k_{r+1}^2 + (K - 1 + c)^2}$ implies $k_1 + \cdots + k_r > K - 1 + c$ and thus

$p^*(\Gamma(r)) > c$; and 2) it is externally stable because firm $r+1$ prefers not to join. In turn, this implies that firm s prefers not to join $\forall s > r+1$. That is, if firm $r+1$ does not want to join the cartel containing the r largest firms, then all smaller firms do not want to join either.

As a final step, let us show that $\Gamma(r)$ is not only stable, but also minimally stable. We know that

$$k_1 + \cdots + k_{r-1} < \sqrt{k_r^2 + (K - 1 + c)^2}.$$

So, the cartel $\Gamma(r-1)$ is not externally stable. Next consider any $\Gamma \subset \Gamma(r)$. If $p^*(\Gamma) = c$, then Γ is not internally stable. If $p^*(\Gamma) > c$, then it is internally stable, but not externally stable because

$$k_1 + \cdots + k_{r-1} < \sqrt{k_r^2 + (K - 1 + c)^2} \tag{10.1}$$

implies

$$\sum_{j \in \Gamma} k_j < \sqrt{k_i^2 + (K - 1 + c)^2} \text{ for } i \notin \Gamma \text{ and } i \leq r. \tag{10.2}$$

This is true, because $k_1 + \cdots + k_{r-1} \geq \sum_{j \in \Gamma} k_j$. Therefore, the LHS of (10.2) is smaller than the LHS of (10.1) and the RHS of (10.2) is weakly larger than the RHS of (10.1). Hence, firm $i \notin \Gamma$ would choose to join Γ, which means that all $\Gamma \subset \Gamma(r)$ are unstable. The cartel $\Gamma(r)$ is therefore not only stable, but also minimally stable.

- **Step 3: Derive conditions under which a subset of $\Gamma(r)$ can sustain a collusive price.**

What remains is to find m such that $r > m \geq h$. That is, there are subsets of the minimal stable cartel $\Gamma(r)$ that have sufficient capacity to support a collusive price. For such a subset it must hold that

$$K - 1 + c < k_1 + \cdots + k_m < \sqrt{k_{m+1}^2 + (K - 1 + c)^2}.$$

Note that if this condition is not satisfied for $m = h$, so that

$$\sqrt{k_{h+1}^2 + (K - 1 + c)^2} \leq k_1 + \cdots + k_h,$$

then it does not hold for any $m > h$ (as the RHS is increasing in m and the LHS is non-increasing in m). Thus, the necessary and sufficient condition is that $\exists h$ such that:

$$k_1 + \cdots + k_{h-1} \leq K - 1 + c < k_1 + \cdots + k_h < \sqrt{k_{h+1}^2 + (K - 1 + c)^2}.$$

This means that the smallest cartel that is able to sustain a collusive price (cartel $\Gamma(h)$) needs to be externally unstable.

To conclude, let us show that r exists and is unique. Define r by

$$\phi(r - 1) < 0 \leq \ \phi(r),$$

and let

$$\phi(m) \equiv k_1 + \cdots + k_m - \sqrt{k_{m+1}^2 + (K - 1 + c)^2}.$$

Note that $\phi(m)$ is strictly increasing in m. Hence, if $\phi(1) < 0 < \phi(n)$, then there exists a unique r such that: $\phi(r - 1) < 0 \leq \phi(r)$. For $\phi(1) < 0$, it must hold that

$$k_1 - \sqrt{k_2^2 + (K - 1 + c)^2} < 0.$$

This inequality is satisfied when

$$\sqrt{(K-1+c)^2} > k_1 \Leftrightarrow K - k_1 > 1 - c,$$

which is true by assumption. For $\phi(n) > 0$, it must hold that

$$K - \sqrt{(K-1+c)^2} > 0 \Leftrightarrow 1 - c > 0,$$

which too holds by assumption. Thus, there exists a unique r such that $\phi(r-1) < 0 \leq \phi(r)$.