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When is an antitrust authority not aggressive enough in fighting cartels?

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If an antitrust authority chooses an enforcement policy to maximize the number of successfully prosecuted cartels, when does that policy minimize the number of cartels that form? When the detection and prosecution of cartels is inherently difficult, it is found that an antitrust authority's policy minimizes the number of cartels, as is socially desirable. However, when the detection and prosecution of cartels is not too difficult, an antitrust authority is not aggressive enough in that it prosecutes too few cartel cases.

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1 Introduction

When it comes to cartels, the socially desirable objectives of an antitrust authority (AA) are desistance, causing existing cartels to shut down, and deterrence, preventing cartels from forming. Desistance is achieved by discovering and successfully prosecuting cartels. The act of desistance along with the critical role of penalization serve the goal of deterrence. Putting aside heterogeneity in cartels, one simple characterization of what we want an AA to do is to minimize the number of cartelized industries, which I will refer to as the cartel rate.

Although an AA should minimize the cartel rate, do their interests coincide with such an objective? Whatever manner in which a member of an AA is rewarded (e.g. internal promotion, bonuses, status, personal satisfaction or career advancement) it is reasonable to suppose that these rewards are tied to some observable measure of performance. Unfortunately, the cartel rate is not observed. We can document how many suspected cartels

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By the time I came out of graduate school in 1984, almost all of what I knew about oligopoly theory came from Jim Friedman's masterpiece *Oligopoly and the Theory of Games*. Few books offer such clarity, rigor and seriousness about the enterprise of theoretical research. Then, as an eager but immature industrial organization theorist, I visited Jim Friedman at the University of North Carolina in Spring 1987. During my semester-long stay, we discussed many research topics but, most importantly, Jim patiently conveyed the distinction between cranking out papers and performing meaningful research. I am forever grateful to Jim for pointing my research career in the right direction. I appreciate the thoughtful and constructive comments of the referee, and gratefully acknowledge the support of the National Science Foundation (SES-0516943).

there are and how many are convicted, but that does not tell us how many cartels are active. As an AA can presumably only be rewarded based on *observable* measures of performance, its actions are presumably not intended to minimize the *unobservable* cartel rate.

This observation suggests that there might be an incongruity between what society wants an AA to do and what an AA actually does. In the present paper, this issue is explored. The performance of an AA is assumed to be measured by the number of successfully prosecuted cartels, and the extent to which the actions of an AA coincide with those that minimize the number of active cartels is investigated. When the detection and prosecution of cartels is inherently difficult, an AA is shown to choose the policy that minimizes the number of cartels, as is socially desirable. However, when the detection and prosecution of cartels is not difficult, an AA is not aggressive enough in that it prosecutes too few cartel cases.

2 Model

In the simple model described here, it is implicitly assumed that cartels are homogeneous so all that matters is the cartel rate, which is the fraction of industries that are cartelized. Denoted $C(\sigma)$, the cartel rate is assumed to depend on σ, which is the probability that a cartel is caught, prosecuted and convicted; in other words, σ is the probability that a cartel will be penalized. It is natural to assume that the higher is the probability that prospective cartels attach to paying penalties, the fewer cartels there are.[1]

Assumption 1 $C(\sigma)\colon [0, 1] \to [0, 1]$ *is a twice differentiable decreasing function.*

$\sigma = q \times r \times s$, where q is the probability that a cartel is discovered, r is the fraction of discovered cartels that the AA prosecutes and s is the probability that the AA gets a conviction for a cartel it is prosecuting. The discovery of a cartel is presumed to be exogenous and to come from customers, uninvolved employees, the accidental discovery of evidence through a proposed merger, and other sources; $q \in (0, 1]$ is then a parameter. What the AA controls is how many cases to take on, the fraction of reported cases that it chooses to prosecute, which is denoted r and referred to as the AA's enforcement policy. Finally, of those cases discovered and prosecuted, the likelihood of the AA being successful, denoted s, depends on the AA's caseload. Implicit is that the AA has limited resources so that a bigger caseload means fewer resources per case and, therefore, a lower probability of winning any case. Let $s = p(R)$, where $R = qrC(qrs)$ is the mass of cases handled by the AA, and assume that p is decreasing.[2]

[1] The relationship between the probability of paying penalties and the cartel rate is endogenized in Harrington and Chang (2009) by modeling the equlibrium formation of cartels. Here, it is taken as exogenous in order to explore other issues.

[2] A richer model could endogenize the amount of resources devoted to prosecuting price-fixing by endowing the AA with a fixed amount of resources and then allowing them to allocate those resources across their various activities, which include prosecuting cartels, evaluating mergers and investigating monopolization practices. The cost of devoting more resources to fighting cartels is then the opportunity cost from having fewer resources to, for example, handle merger cases. However, even if resources were endogenized in this or some other manner, I conjecture that the paper's main results would persist. As long as the marginal

Assumption 2 $p(R)\colon [0,1] \to [0,1]$ *is a twice differentiable, decreasing and weakly concave function.*

The probability of a conviction depends on the caseload, $s = p(R)$, the caseload depends on the number of cartels, $R = qrC(qrs)$, and the number of cartels depends on the probability of conviction. Therefore, the equilibrium probability of conviction is a fixed point:

$$s^* = p(qrC(qrs^*)). \tag{1}$$

Define $\psi(s) \equiv p(qrC(qrs))$. $\psi(s)$ is an increasing function:

$$\psi'(s) = (qr)^2 p'(qrC(qrs))C'(qrs) > 0.$$

Next note that: $\psi(0) > 0$, $\psi(1) < 1$. Hence, by the continuity of ψ, there exists an interior fixed point. We will further assume that this fixed point is unique.

Assumption 3 *There exists unique* $s^* \in (0,1)$ *such that* $s^* = p(qrC(qrs^*))$.

To present sufficient conditions for Assumption 3 to hold, note that:

$$\psi''(s) = (qr)^3[qrp''(qrC(qrs))(C'(qrs))^2 + p'(qrC(qrs))C''(qrs)].$$

If $\psi''(s) \leq 0$ then ψ is a contraction mapping and, therefore, has a unique fixed point. If $C(\sigma)$ is linear then, along with Assumptions 1 and 2, $\psi''(s) \leq 0$. If $p(R)$ is strictly concave and $C(\sigma)$ is not too concave then again $\psi''(s) \leq 0$. We will sometimes use the expression $\sigma^*(r) \equiv qrs^*(r)$ for the equilibrium probability that a cartel pays penalties, given enforcement policy r.

A social planner (SP) is assumed to choose the enforcement policy that minimizes the cartel rate, which is equivalent to maximizing the probability of paying penalties:

$$\min_{r\in[0,1]} C(qrs^*(r)) \Leftrightarrow \max_{r\in[0,1]} qrs^*(r).$$

Denote r^{sp} as the social planner's optimal enforcement policy:

$$r^{sp} \in \arg\min C(qrs^*(r)).$$

Under Assumptions 1–3, r^{sp} exists.

The AA is rewarded according to some observable measure of performance. As the cartel rate is unobserved, the number of successful cases is used to evaluate the AA's performance. The AA is assumed to choose an enforcement policy that maximizes the number of convicted cartels:

$$\max_{r\in[0,1]} qrs^*(r)C(qrs^*(r)).$$

opportunity cost of resources is increasing with the amount of resources used, one would not expect the amount of resources to scale up with the caseload and instead for resources per case to decline with the number of cases, which is exactly the implicit assumption made with Assumption 2.

Denote r^{aa} as the AA's optimal enforcement policy,

$$r^{aa} \in \arg\max qrs^*(r)C(qrs^*(r)),$$

which exists by our assumptions. Defining $\Delta \equiv \{qrs^*(r)\colon r \in [0, 1]\}$, we can also cast the AA's problem as: $\max_{\sigma\in\Delta}\sigma C(\sigma)$. It will be useful to assume that the AA's objective function is well-behaved.

Assumption 4 *$\sigma C(\sigma)$ is strictly quasi-concave in σ.*

Finally, Assumption 5 is made, which holds if q is sufficiently small or p and C are not too sensitive.

Assumption 5 $p'(qrC(qrs))(qr)^2C'(qrs) < 1, \forall(r, s) \in [0, 1]^2$.

Note that Assumption 5 holds when r is sufficiently low because it holds when $r = 0$ as then

$$p'(qrC(qrs^*))(qr)^2C'(qrs^*) = 0,$$

given that p' and C' are bounded. In the Appendix, it is explained that Assumption 5 is required for the model to be well-behaved in that, if it does not hold, then $\partial s^*/\partial r$ is not bounded.

3 Results

The AA chooses an enforcement policy to maximize the number of successfully prosecuted cases, while the socially optimal policy is one that minimizes the cartel rate. In this section, it is demonstrated when these two distinct objectives lead to the same policy. As stated in Theorem 1, if the SP's optimal enforcement policy is not to prosecute all cases then the AA's optimal enforcement policy coincides with the socially optimal policy. Therefore, the policy that maximizes the number of successful cases also serves to minimize the cartel rate. The proof is in the Appendix.

Theorem 1 *If $r^{sp} \in (0, 1)$ then $r^{aa} = r^{sp}$.*

Let us provide some intuition as why Theorem 1 is true. Suppose the SP has an interior solution, $r^{sp} \in (0, 1)$. As the SP is choosing r to maximize the probability a cartel pays penalties, $qrs^*(r)$, an interior optimum means that

$$\frac{\partial\sigma^*(r)}{\partial r} = q\left[s^*(r^{sp}) + r^{sp}\left(\frac{\partial s^*(r^{sp})}{\partial r}\right)\right] = 0$$

and, therefore,

$$\frac{\partial s^*(r^{sp})}{\partial r} < 0.$$

Thus, the conviction rate is declining in the enforcement policy. The only reason it could be optimal not to prosecute all cases is that, by taking on more cases, the conviction rate is lowered due to a bigger caseload. Because $s = p(qrC(qrs))$, as r increases, there are two forces affecting the conviction rate. First, holding the cartel rate C fixed, more aggressive enforcement (i.e. higher r) means a bigger caseload qrC and a lower conviction rate. Second, more aggressive enforcement lowers the cartel rate, $C(qrs)$, which reduces the caseload and, therefore, raises the conviction rate. The former effect obviously dominates when $\partial s^*/\partial r < 0$. Thus, when the SP has an interior solution, an increase in the enforcement rate raises the caseload, $qrC(qrs)$, holding s fixed.

Now consider the marginal effect of enforcement on the AA's objective:

$$\begin{aligned}\frac{\partial \sigma^*(r)\, C(\sigma^*(r))}{\partial r} &= \left(\frac{\partial \sigma^*}{\partial r}\right)[C(\sigma^*) + \sigma^* C'(\sigma^*)] \\ &= q\left[s^* + r\left(\frac{\partial s^*}{\partial r}\right)\right][C(\sigma^*) + \sigma^* C'(\sigma^*)].\end{aligned} \tag{2}$$

Holding s fixed, the caseload $qrC(qrs)$ is increasing in r if and only if the number of successfully prosecuted cartels $qrsC(qrs)$ is increasing in r. The latter condition can be stated as

$$C(\sigma^*) + \sigma^* C'(\sigma^*) > 0,$$

and, using (2), we have

$$sign\left\{\frac{\partial \sigma^*(r)C(\sigma^*(r))}{\partial r}\right\} = sign\left\{q\left[s^* + r\left(\frac{\partial s^*}{\partial r}\right)\right]\right\} = sign\left\{\frac{\partial \sigma^*(r)}{\partial r}\right\}.$$

Hence, what maximizes $\sigma^*(r)C(\sigma^*(r))$ also maximizes $\sigma^*(r)$and, therefore, minimizes $C(\sigma^*(r))$.

Let us summarize the preceding explanation. That the conviction rate is declining in the enforcement policy means the caseload is increasing in the enforcement policy. Because the number of successfully prosecuted cartels is just the caseload multiplied by the conviction rate then, holding the conviction rate fixed, the number of successfully prosecuted cartels is increasing in the enforcement rate. This implies that the AA wants to increase σ because $\sigma C(\sigma)$ is increasing in σ. Of course, the SP wants to increase σ because its objective is to maximize σ. Hence, both the SP and the AA choose an enforcement policy that maximizes σ.

Following from Theorem 1, there are three possible cases:

Case 1 Minimizing the cartel rate requires not prosecuting all cases and, by Theorem 1, this means that the AA chooses the socially optimal policy: $r^{sp} = r^{aa} \in (0, 1)$.

Case 2 It is socially optimal to prosecute all cases and the AA does so: $r^{sp} = r^{aa} = 1$.

Case 3 It is socially optimal to prosecute all cases and the AA does not do so: $r^{aa} < r^{sp} = 1$.

In the remainder of this section, sufficient conditions are provided for each of these cases to occur.

Starting with the Case 1, a sufficient condition for $r^{sp} \in (0, 1)$ is:

$$\left[\frac{\partial \sigma^*(r)}{\partial r}\right]_{r=1} < 0 \Rightarrow s^*(1) + \left(\frac{\partial s^*(1)}{\partial r}\right) < 0.$$

Substituting the expression for $\partial s^*(1)/\partial r$ (a derivation of which is in the Appendix), we have:

$$s^*(1) + \frac{qp'(qC(qs^*(1)))[C(qs^*(1)) + qs^*(1)C'(qs^*(1))]}{1 - p'(qC(qs^*(1)))\,q^2C'(qs^*(1))} < 0.$$

If we let the probability of conviction (with full enforcement) go to zero, $s^*(1) \to 0$, the preceding expression converges to

$$\frac{qp'(qC(0))\,C(0)}{1 - p'(qC(0))\,q^2C'(0)} < 0,$$

which does indeed hold because $p'(qC(0)) < 0$ and $C(0) > 0$. Thus, if the probability of conviction is sufficiently low when all cartel cases are prosecuted ($s^*(1) \simeq 0$) then the policy that minimizes the cartel rate involves prosecuting some but not all cases, and (by Theorem 1) this policy is also chosen by the AA, even though its interest lies in maximizing the number of convicted cartels.

Next, consider Case 2: $r^{aa} = r^{sp} = 1$. A sufficient condition for $r^{sp} = 1$ is:

$$\frac{\partial \sigma^*(r)}{\partial r} = q\left[s^*(r) + r\left(\frac{\partial s^*(r)}{\partial r}\right)\right] > 0,\ \forall r. \tag{3}$$

Equation(3) holds if $\partial s^*/\partial r$ is sufficiently small relative to $s^*(r)$. Because

$$\frac{\partial s^*}{\partial r} = \frac{qp'(qrC(\sigma^*))\left[C(\sigma^*) + \sigma^*C'(\sigma^*)\right]}{1 - p'(qrC(\sigma^*))\,(qr)^2\,C'(\sigma^*)} \tag{4}$$

then

$$\lim_{q\to 0} \frac{\partial s^*}{\partial r} = 0,$$

and, furthermore,

$$\lim_{q\to 0} s^*(r) = p(0) > 0.$$

Therefore, (3) holds when q is sufficiently small, from which we conclude:

$$\lim_{q\to 0} r^{sp} = 1.$$

Next, consider the AA's optimal policy. A sufficient condition for $r^{aa} = 1$ is:

$$\frac{\partial \sigma^*(r)\,C(\sigma^*(r))}{\partial r} = \left(\frac{\partial \sigma^*(r)}{\partial r}\right)\left[1 - 2\sigma^*(r)\right] > 0,\ \forall r. \tag{5}$$

As we just showed that $\partial\sigma^*(r)/\partial r > 0$ when q is sufficiently small, (5) holds when

$$1 - 2\sigma^*(r) > 0, \ \forall r. \tag{6}$$

Given that

$$\lim_{q\to 0} \sigma^*(r) = 0,$$

it follows from (6) that (5) is true and, therefore,

$$\lim_{q\to 0} r^{aa} = 1.$$

To conclude, if detection is weak, that is, q is sufficiently low, then an AA implements the socially optimal policy of prosecuting all cartel cases.

Finally, consider Case 3 so that the AA is not sufficiently aggressive: $0 < r^{aa} < r^{sp} = 1$. It has already been stated that a sufficient condition for $r^{sp} = 1$ is (3), which holds when $\partial s^*(r)/\partial r$ is sufficiently small. By (4), $\partial s^*(r)/\partial r$ is close to zero when $p'(R)$ is close to zero. Thus, if the probability of conviction is not too sensitive to the caseload then prosecuting all cases minimizes the cartel rate, $r^{sp} = 1$. A necessary and sufficient condition for $r^{aa} < 1$ is:

$$\left[\frac{\partial\sigma^*(r)\, C(\sigma^*(r))}{\partial r}\right]_{r=1} = \left(\frac{\partial\sigma^*(1)}{\partial r}\right)[1 - 2\sigma^*(1)] < 0. \tag{7}$$

Because we already have that $\partial\sigma^*(1)/\partial r > 0 \forall r$, (7) holds if:

$$\sigma^*(1) = qs^*(1) > \frac{1}{2},$$

which is true when q and $s^*(1)$ are sufficiently close to one.

Summing up Case 3, if it is not difficult to detect ($q \simeq 1$) and to convict ($s^*(1) \simeq 1$) cartels and the AA is relatively unconstrained in terms of prosecutorial resources (so that $p'(R) \simeq 0$ and, therefore, the probability of conviction is not very sensitive to the caseload), then the policy that minimizes the cartel rate is to prosecute all cases. However, an AA that is interested in maximizing the number of convicted cartels will instead choose not to prosecute all cases. The AA might prosecute fewer cases than is socially optimal in order to reduce deterrence and to raise the cartel rate. From the perspective of the AA, an enforcement policy can be too aggressive in that deterrence induces a decline in the cartel rate, which results in fewer convicted cartels.

To summarize these three cases, if conditions are tough for fighting cartels (either detection is difficult or conviction is difficult), then the AA implements the policy that minimizes the cartel rate. If conditions are sufficiently conducive to fighting cartels then enforcement by the AA is less aggressive than is socially optimal.

4 Concluding remarks

When it comes to fighting cartels, previous research has typically modeled an AA as a social welfare-maximizer when deriving its behavior; examples include Besanko and Spulber (1989), Motta and Polo (2003) and Harrington (2008). Although the specification of that objective is appropriate for identifying the socially desirable policy, it is not necessarily correct when it comes to describing the actual policy choices of an antitrust authority. In this paper, a more reasonable alternative is considered, which is that an antitrust authority acts to maximize the number of convicted cartels, which is an *observable* measure of performance.

Although maximizing the number of successfully prosecuted cartels is distinct from minimizing the number of cartels, conditions are derived whereby the resulting behavior is equivalent. In particular, if the socially optimal policy is not to prosecute all suspected cartels then a self-serving antitrust authority will choose it. However, if the socially optimal policy is to prosecute all suspected cartels then an AA which is interested in maximizing the number of convicted cartels might choose not to prosecute all cartel cases.

Reflective of the possible policy challenges that might arise when an AA is not driven to maximize social welfare is Chang and Harrington (2009). In that paper, the implications of an AA that maximizes the number of convicted cartels are explored with respect to evaluating the impact of a corporate leniency program. Contrary to the current model, the effect of antitrust policy on the cartel rate is endogenized through an equilibrium process of cartel birth and death, and the enforcement policy is the fraction of non-leniency cases that the AA pursues. When enforcement is chosen to minimize the cartel rate, a leniency program is shown to lower the cartel rate. However, this need not be the case when the antitrust authority is motivated by the number of convicted cartels. The additional caseload provided by the leniency program induces the antitrust authority to prosecute a smaller fraction of cartel cases identified outside of the program and, because of this less aggressive enforcement policy, it is possible that the cartel rate is higher when there is a leniency program.

Understanding the behavior of an antitrust authority and then designing policy in light of any behavioral biases are large issues, while this small paper barely scratches the surface. Clearly, more attention to these matters is warranted.

Appendix

Totally differentiating (1), one derives:

$$\begin{aligned}&\left(\frac{\partial s^*}{\partial r}\right)\left[1 - p'\left(qrC\left(qrs^*\right)\right)\left(qr\right)^2 C'\left(qrs^*\right)\right] \\ &\quad = qp'\left(qrC\left(qrs^*\right)\right)\left[C\left(qrs^*\right) + qrs^*C'\left(qrs^*\right)\right].\end{aligned} \tag{8}$$

In justifying Assumption 5, note that if it does not hold then $\exists r' > 0$, such that:

$$\lim_{r \uparrow r'} 1 - p'(qrC(qrs^*))(qr)^2 C'(qrs^*) = 0. \tag{9}$$

Because the right-hand side of (8) is bounded, it follows from (9) that $\partial s^*/\partial r$ is not bounded, which seems implausible.

Solving (8), an explicit expression for how the enforcement policy affects the probability of a conviction is derived:

$$\frac{\partial s^*}{\partial r} = \frac{qp'\left(qrC\left(\sigma^*\right)\right)\left[C\left(\sigma^*\right) + \sigma^* C'\left(\sigma^*\right)\right]}{1 - p'\left(qrC\left(\sigma^*\right)\right)(qr)^2 C'\left(\sigma^*\right)}. \tag{10}$$

The next result derives a sufficient condition for $\sigma^*(r) \equiv qrs^*(r)$, which is what the SP is maximizing, to be strictly quasi-concave in r.

Lemma 2 *If $C(qrs^*(r)) + qrs^*(r)C'(qrs^*(r)) > 0$ $\forall r \in [0, r']$ then $rs^*(r)$ is strictly quasi-concave for $r \in [0, r']$.*

Proof**:** Using (10), first note that:

$$\begin{aligned}
\frac{\partial qrs^*(r)}{\partial r} &= q\left[s^*(r) + r\left(\frac{\partial s^*(r)}{\partial r}\right)\right] \\
&= q\left[s^* + r\left(\frac{qp'\left(qrC\left(\sigma^*\right)\right)\left[C\left(\sigma^*\right) + \sigma^* C'\left(\sigma^*\right)\right]}{1 - p'\left(qrC\left(\sigma^*\right)\right)(qr)^2 C'\left(\sigma^*\right)}\right)\right] \\
&= q\left[\frac{s^* + qrp'\left(qrC\left(\sigma^*\right)\right)C\left(\sigma^*\right)}{1 - p'\left(qrC\left(\sigma^*\right)\right)(qr)^2 C'\left(\sigma^*\right)}\right].
\end{aligned}$$

By the preceding expression and Assumption 5,

$$sign\left\{\frac{\partial qrs^*(r)}{\partial r}\right\} = sign\{s^* + qrp'(qrC(\sigma^*))C(\sigma^*)\},$$

$qrs^*(r)$ is strictly quasi-concave over $[0, r']$ if either: (i) $s^* + qrp'(qrC(\sigma^*))C(\sigma^*) > 0, \forall r \in [0, r']$; or (ii) $\exists r'' \in (0, r')$ such that $s^* + qrp'(qrC(\sigma^*))C(\sigma^*) \gtreqless 0$, as $r \lesseqgtr r''$, $\forall r \in [0, r']$. Evaluating $s^* + qrp'(qrC(\sigma^*))C(\sigma^*)$ at $r = 0$, we know that it is positive. Therefore, for (i) or (ii) to hold, it is sufficient that $s^* + qrp'(qrC(\sigma^*))C(\sigma^*)$ is monotonic in r. After some manipulation, we derive

$$\frac{\partial\left[s^* + qrp'\left(qrC\left(\sigma^*\right)\right)C\left(\sigma^*\right)\right]}{\partial r}$$
$$= \frac{\partial s^*(r)}{\partial r} + qp'\left(qrC\left(\sigma^*\right)\right)C\left(\sigma^*\right) + q^2rp'\left(qrC\left(\sigma^*\right)\right)C'\left(\sigma^*\right)\left[s^*(r) + r\left(\frac{\partial s^*(r)}{\partial r}\right)\right]$$
$$+q^2rp''\left(qrC\left(\sigma^*\right)\right)C\left(\sigma^*\right)\left[C\left(\sigma^*\right) + qrC'\left(\sigma^*\right)\left(s^*(r) + r\left(\frac{\partial s^*(r)}{\partial r}\right)\right)\right]$$
$$= q\left[C\left(\sigma^*\right) + \sigma^*C'\left(\sigma^*\right)\right]\left\{\left(\frac{p'\left(qrC\left(\sigma^*\right)\right)}{1 - p'\left(qrC\left(\sigma^*\right)\right)\left(qr\right)^2 C'\left(\sigma^*\right)}\right)\times\right.$$
$$\left[1 + \left(qr\right)^2 p'\left(qrC\left(\sigma^*\right)\right)C'\left(\sigma^*\right) + \left(qr\right)^3 p''\left(qrC\left(\sigma^*\right)\right)C\left(\sigma^*\right)C'\left(\sigma^*\right)\right]$$
$$\left.+p'\left(qrC\left(\sigma^*\right)\right) + qrp''\left(qrC\left(\sigma^*\right)\right)C\left(\sigma^*\right)\right\}.$$

Because $p' < 0$, $1 - p'(qr)^2C' > 0$, $C' < 0$, and $p'' \leq 0$ then:

$$sign\left\{\frac{\partial[s^* + qrp'(qrC(\sigma^*))C(\sigma^*)]}{\partial r}\right\} = -sign\{C(\sigma^*) + \sigma^*C'(\sigma^*)\},$$

which proves the lemma. □

Proof of Theorem 1: If $r^{sp} \in (0, 1)$ then there exists at least one local maximum. Let $\widetilde{r}$ be the lowest local maximum so that: $s^* + r\left(\frac{\partial s^*}{\partial r}\right) \geqq 0\ \forall r \leqq \widetilde{r}$, and, at $r = \widetilde{r}$,

$$s^* + r\left(\frac{\partial s^*}{\partial r}\right) = s^* + r\left(\frac{qp'\left(qrC\left(\sigma^*\right)\right)\left[C\left(\sigma^*\right) + \sigma^*C'\left(\sigma^*\right)\right]}{1 - p'\left(qrC\left(\sigma^*\right)\right)\left(qr\right)^2 C'\left(\sigma^*\right)}\right) = 0.$$

This implies

$$C\left(q\widetilde{r}s^*\left(\widetilde{r}\right)\right) + q\widetilde{r}s^*\left(\widetilde{r}\right)C'\left(q\widetilde{r}s^*\left(\widetilde{r}\right)\right) > 0. \tag{11}$$

Because $qrs^*(r)$ is non-decreasing in r $\forall r \leq \widetilde{r}$, it follows from the strict quasi-concavity of $\sigma C(\sigma)$ in σ and (11) that:

$$C\left(qrs^*(r)\right) + qrs^*(r)\,C'\left(qrs^*(r)\right) > 0,\ \ \forall r \in [0, \widetilde{r}]\,. \tag{12}$$

By the continuity of $qrs^*(r)$ with respect to r and of $C(\sigma)$ and $C'(\sigma)$ with respect to σ, it follows from (12) that $\exists\varepsilon > 0$ such that:

$$C\left(qrs^*(r)\right) + qrs^*(r)\,C'\left(qrs^*(r)\right) > 0,\ \ \forall r \in [0, \widetilde{r} + \varepsilon]\,. \tag{13}$$

By Lemma 2, it follows from (13) that $qrs^*(r)$ is strictly quasi-concave in r, for $r \in [0, \widetilde{r} + \varepsilon]$. Hence, $s^* + r\left(\frac{\partial s^*}{\partial r}\right) = 0$ for $r = \widetilde{r}$ implies:

$$s^* + r\left(\frac{\partial s^*}{\partial r}\right) < 0,\ \ \forall r \in (\widetilde{r}, \widetilde{r} + \varepsilon)\,.$$

We next want to show that $qrs^*(r)$ is strictly quasi-concave in r, for $r \in [0, 1]$. Suppose not, in which case $\exists r' > \widetilde{r}$ and $\varepsilon > 0$ such that:

$$s^* + r\left(\frac{\partial s^*}{\partial r}\right) \leq 0 \; \forall r \in \left(\widetilde{r}, r'\right)$$

and

$$s^* + r\left(\frac{\partial s^*}{\partial r}\right) \geq 0, \; \forall r \in \left[r', r' + \varepsilon\right).$$

By Assumption 5 and (10),

$$sign\left\{\frac{\partial s^*}{\partial r}\right\} = -sign\left\{C\left(\sigma^*\right) + \sigma^* C'\left(\sigma^*\right)\right\}. \tag{14}$$

From (14), a necessary condition for $s^* + r\left(\frac{\partial s^*}{\partial r}\right) \leq 0$ is that $C(\sigma) + \sigma C'(\sigma) > 0$. By the same argument as above, this implies $\exists \eta > 0$ such that:

$$C\left(qrs^*(r)\right) + qrs^*(r)\, C'\left(qrs^*(r)\right) > 0, \; \forall r \in \left(r', r' + \eta\right),$$

and, furthermore,

$$C\left(qrs^*(r)\right) + qrs^*(r)\, C'\left(qrs^*(r)\right) > 0, \; \forall r \in \left[0, r' + \eta\right).$$

By Lemma 2, $rs^*(r)$ is strictly quasi-concave in r, for $r \in [0, r' + \eta)$. However, this contradicts $rs^*(r)$ being increasing over $[0, \widetilde{r})$, decreasing over $(\widetilde{r}, r')$, and non-decreasing over $(r', r' + \eta)$. We conclude that the supposition that $qrs^*(r)$ is not strictly quasi-concave in r for $r \in [0, 1]$ is false. Therefore, if $r^{sp} \in (0, 1)$ then $qrs^*(r)$ is strictly quasi-concave in r for $r \in [0, 1]$.

Consider the first-order condition for the AA:

$$\frac{\partial \sigma^*(r)\, C\left(\sigma^*(r)\right)}{\partial r} = \left(\frac{\partial \sigma^*}{\partial r}\right)\left[C\left(\sigma^*\right) + \sigma^* C'\left(\sigma^*\right)\right]$$

$$\frac{\partial \sigma^*(r)\, C\left(\sigma^*(r)\right)}{\partial r} = q\left[s^* + r\left(\frac{\partial s^*}{\partial r}\right)\right]\left[C\left(\sigma^*\right) + \sigma^* C'\left(\sigma^*\right)\right]$$

$$\frac{\partial \sigma^*(r)\, C\left(\sigma^*(r)\right)}{\partial r} = q\left[s^* + r\left(\frac{\partial s^*}{\partial r}\right)\right]\left[C(qrs^*(r)) + qrs^*(r)C'(qrs^*(r))\right]. \tag{15}$$

We have shown that

$$s^* + r\left(\frac{\partial s^*}{\partial r}\right) \gtreqqless 0 \text{ as } r \lesseqqgtr r^{sp}.$$

Because

$$s^* + r\left(\frac{\partial s^*}{\partial r}\right) = 0 \text{ at } r = r^{sp}$$

then

$$C\left(qrs^*(r)\right) + qrs^*(r)\, C'\left(qrs^*(r)\right) > 0 \text{ at } r = r^{sp},$$

which, by Assumption 4, implies

$$C\left(qrs^*(r)\right) + qrs^*(r)\, C'\left(qrs^*(r)\right) > 0 \;\forall\; r \leq r^{sp}. \tag{16}$$

In addition,

$$s^* + r\left(\frac{\partial s^*}{\partial r}\right) < 0 \text{ for } r > r^{sp}$$

implies

$$C\left(qrs^*(r)\right) + qrs^*(r)\, C'\left(qrs^*(r)\right) > 0 \text{ for } r > r^{sp}. \tag{17}$$

Combining (16) and (17), we have:

$$C\left(qrs^*(r)\right) + qrs^*(r)\, C'\left(qrs^*(r)\right) > 0,\ \ \forall r \in [0, 1]\,. \tag{18}$$

It follows from (15) and (18) that:

$$\frac{\partial \sigma^* C(\sigma^*)}{\partial r} \gtreqless 0 \text{ as } r \lesseqgtr r^{sp},$$

in which case $r^{sp} = \arg\max \sigma^* C(\sigma^*)$. Therefore, $r^{aa} = r^{sp}$. □

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