RAND Journal of Economics
Vol. 35, No. 4, Winter 2004
pp. 651–673

Cartel pricing dynamics in the presence of an antitrust authority

Joseph E. Harrington, Jr.*

Cartel pricing is characterized when firms are concerned about creating suspicions that a cartel has formed. Balancing concerns about maintaining the stability of the cartel with those of avoiding detection, the cartel may either (i) gradually raise price to its steady-state level or (ii) gradually raise price and then have it decline to its steady-state level. Antitrust laws may have a perverse effect as they make cartel stability easier and thus allow for higher cartel prices.

1. Introduction

■ As evidenced by recent cases in lysine, graphite electrodes, and vitamins, price fixing remains a perennial problem. In reviewing the price paths of these recent cartels, one finds that, upon formation of the cartel, price does not immediately jump from the endowed noncollusive price to some new higher steady-state level; rather, there is a transitional path.[1] In some cases, price gradually rises and eventually stabilizes, though in other cases the pattern is more complex. In spite of the rich literature on collusive pricing that has developed in the last thirty years, to my knowledge there is not a single theory that predicts such a price path. In asking why theory has failed here, it is useful to reconsider the problem faced by firms that have just cartelized. First, they obviously want to raise price, for higher profit is the primary reason for creating a cartel. Second, they want to raise price in such a manner as to maintain the internal stability of the cartel; that is, all firms prefer to go along with the proposed collusive price path rather than cheat on the cartel and grab a bigger share of the market. Theoretical research has focused on those two factors, with the second one embodied in the usual equilibrium conditions (or incentive-compatibility constraints).[2] In doing so, it has developed a rich set of implications and a vastly improved understanding of cartel

* Johns Hopkins University; joe.harrington@jhu.edu.

I appreciate the comments of Susan Athey, Glenn Ellison, Rich Gilbert, and Peter Møllgaard and seminar participants at Cal Tech, UC-Berkeley, Stanford, Yale, Arizona, George Washington, Minnesota, Minneapolis Fed, Duke/UNC Theory Workshop, Harvard/MIT I.O. Seminar, Pompeu Fabra, UAB-IAE, Alicante, Tulane, EUI, and Bocconi and at the 2003 Winter Econometric Society Meetings (Washington, D.C.), 2003 International I.O. Conference (Boston), 2003 Summer Econometric Society Meetings (Evanston), 2003 Society for Computational Economics Meetings (Seattle), 2003 EARIE Meetings (Helsinki), and the 2003 National Meetings on Antitrust and Regulation (Siena, Italy). The superb research assistance of Joe Chen is gratefully acknowledged. This research is supported by the National Science Foundation under grant no. SES-0209486.

[1] These price series can be found in Connor (2001) and Levenstein and Suslow (2001).

[2] There is also some work concerned with the external stability of cartels, that is, avoiding entry in response to raising price. See, for example, Harrington (1989).

pricing. However, very little attention has been given to a third dimension of the cartel's problem. In that the explicit coordination of price and quantity is a violation of antitrust law, a cartel wants to raise price in such a manner as to avoid creating suspicions that a cartel has formed. Firms want to raise prices but not suspicions that they are coordinating their behavior. If high prices or rapidly increasing prices or, more generally, anomalous price movements may make customers and the antitrust authorities suspicious that a cartel is operating, one would expect this to have implications for how the cartel prices. This article is part of a research project whose objective is to enrich our models of cartel pricing by recognizing that firms want to avoid detection. A particular goal is to understand observed cartel pricing dynamics and, with this new theory, to explore the impact of antitrust policy.

In another paper (Harrington, forthcoming), I characterize the joint profit-maximizing price path under the constraint of possible detection and antitrust penalties. Issues of internal cartel stability were ignored, however, as the implicit assumption was made that incentive-compatibility constraints were not binding. Assuming that the probability of detection is sensitive to price changes, it was shown that the cartel gradually raises price, with price converging to a steady-state level. Comparative statics on the steady-state price reveal that it is decreasing in the damage multiple and the probability of detection but is independent of the level of fixed fines. Furthermore, if fines are the only penalty, the cartel's steady-state price is the same as in the absence of antitrust laws, though fines do affect the path to the steady state. Another intriguing result is that a more stringent standard for calculating damages *increases* the steady-state price.

The current article explores the interaction of internal cartel stability and detection avoidance by allowing incentive-compatibility constraints to bind. There are two main contributions. First, I show that, depending on the parameter values, there are two qualitatively distinct cartel price paths. One is the same as in Harrington (forthcoming)—the cartel gradually raises price and it converges to a steady-state level. This establishes that the monotonicity of the price path when incentive-compatibility constraints do not bind extends to when they do. The second price path has the cartel gradually raise price, but then price declines to the steady state. Though reducing price lowers profit and cannot make detection less likely, this occurs because of the need to maintain cartel stability. Thus, in some circumstances, it can be incentive compatible for a cartel to raise price to some level, but it is not incentive compatible for it to keep price there. The second main contribution concerns the impact of antitrust laws, where I find that they can have a perverse effect. Though making price-fixing illegal may induce a cartel to initially price lower, in some cases it may allow the cartel to eventually price *higher*. This result is due to how antitrust laws affect incentive-compatibility constraints. The risk of detection and penalties can serve to stabilize a cartel and thereby allow it to set higher prices.

After laying out the model in Section 2, an optimal collusive price path is defined in Section 3. To characterize its intertemporal properties, Section 4 engages in a two-step analysis. First, I engage in numerical analysis to identify what types of properties the price path can have. There the two qualitatively distinct price paths mentioned above are identified. Second, so as to explain these price paths, the underlying forces of the model are derived by proving results for special cases of the model. Section 5 investigates the role of antitrust policy and, in particular, identifies some possible perverse effects from the prohibition of price-fixing. Conclusions are provided in Section 6.

2. Model

■ Consider an industry with n symmetric firms. $\overline{\pi}(P_i, P_{-i})$ denotes firm i's profit when its price is P_i and all other firms charge a common price of P_{-i}. Define $\pi(P) \equiv \overline{\pi}(P, P)$ to be a firm's profit and $D(P)$ a firm's demand when every firm charges P. The space of feasible prices is Ω, which is assumed to be a nonempty, compact, convex subset of $\Re_+$. An additional restriction will be placed on Ω later.

Assumption A1.[3] Either

(i) $\overline{\pi}(P_i, P_{-i})$ is continuous in P_i and P_{-i}, quasi-concave in P_i, and $\exists$ unique $\hat{P}$ such that $P \gtreqless \psi(P)$ as $P \lesseqgtr \hat{P}$, where $\psi(P_{-i}) \in \arg\max \overline{\pi}(P_i, P_{-i})$; or

(ii)

$$\overline{\pi}(P_i, P_{-i}) = \begin{cases} (P_i - c)nD(P_i) & \text{if } P_i < P_{-i} \\ (P_i - c)D(P_i) & \text{if } P_i = P_{-i} \\ 0 & \text{if } P_i > P_{-i} \end{cases}$$

and $D(c) > 0$.

Part (i) results in the stage game encompassing many differentiated-products models, while (ii) makes it inclusive of the Bertrand price game (homogeneous goods and constant marginal cost). Allowing for the latter is important for some existence results, though the characterization results hold much more generally. $\hat{P}$ will denote a symmetric Nash equilibrium price under either (i) or (ii) where, in the latter case, $\hat{P} = c$. Let $\hat{\pi} \equiv \pi(\hat{P})$ be the associated profit. As a convention, $\overline{\pi}(\psi(P), P) = (P - c)nD(P)$ under (ii). Assumption A2 defines the cartel profit function and the joint profit-maximizing price.

Assumption A2. $\pi(P)$ is differentiable and quasi-concave in P, if $\pi(P) > 0$ then it is strictly quasi-concave in P, and $\exists P^m > \hat{P}$ such that $\pi(P^m) > \pi(P) \forall P \neq P^m$.

Firms engage in this price game for an infinite number of periods. The setting is one of perfect monitoring so that firms' prices over the preceding $t - 1$ periods are common knowledge in period t. In this article, "detection" always refers to a third party, such as buyers, detecting the existence of a cartel. Assume a firm's payoff is the expected discounted sum of its income stream, where the common discount factor is $\delta \in (0, 1)$.

If firms form a cartel, they meet to determine price. Assume these meetings, and any associated documentation, provide the "smoking gun" if an investigation is pursued. The cartel is detected with some probability and incurs penalties in that event. Assume, for simplicity, that detection results in the discontinuance of collusion forever.[4] Detection in period t then generates a terminal payoff of $[\hat{\pi}/(1 - \delta)] - X^t - F$, where X^t is a firm's damages and F is any (fixed) fines (which may include the monetary equivalent of prison sentences). If not detected, collusion continues on to the next period. It is useful to think of $X^t + F$ as a "hidden liability" for a firm that is incurred only if the cartel is discovered.

Damages are assumed to evolve in the following manner:

$$X^t = \beta X^{t-1} + \gamma x(P^t) \qquad \text{where } \beta \in [0, 1) \text{ and } \gamma \geq 0.$$

As time progresses, damages incurred in previous periods become increasingly difficult to document, and $1 - \beta$ measures the rate of deterioration of the evidence. $x(P^t)$ is the level of damages incurred by each firm in the current period, where γ is the damage multiple applied. While U.S. antitrust law specifies treble damages, γ is often well less than three because of out-of-court settlements.

Assumption A3. $x : \Omega \rightarrow \Re_+$ is bounded and continuous and is nondecreasing over $[\hat{P}, P^m]$.

Current U.S. antitrust practice is $x(P^t) = (P^t - \hat{P})D(P^t)$, where $\hat{P}$ is referred to as the "but for price" and is the price that would have occurred but for collusion. By the boundedness of $x(\cdot)$, it follows that damages are bounded by $\overline{X} \equiv \gamma\overline{x}/(1 - \beta)$, where $\overline{x} \geq x(P)$ for all $P \in \Omega$. We then have $X^t \in [0, \overline{X}]$. Note that X^t is to be interpreted as the part of antitrust penalties that are sensitive to firms' prices and how long they've been colluding. Even though buyers cannot

[3] For clarity, assumptions are labelled according to the portion of the analysis to which they refer.

[4] I conjecture that results are robust to allowing firms to collude again after some finite length of time.

collect damages in, for example, the European Union, X^t is still relevant as long as EU penalties are sensitive to cartel behavior.

Successful prosecution of a cartel—by which is meant that penalties are ultimately levied—involves multiple stages. First, detection: the creation of suspicions that a cartel has formed. Some party—for example, buyers—must recognize that, among all the thousands of industries, this particular one may be plagued by collusion. Second, investigation: in response to a complaint, the antitrust authority must decide that it is worthwhile to pursue a case. Third, prosecution: after conducting such an investigation, the antitrust authority must choose to prosecute the firms (and/or the buyers must decide whether to pursue civil damages litigation). The focus of my modelling is on detection. Detection of a cartel can occur from many sources, some of which are related to price—such as customer complaints—and some of which are unrelated to price—such as internal whistleblowers. Hay and Kelley (1974) find that detection was attributed to a complaint by a customer or a local, state, or federal agency in 13 of 49 price-fixing cases. In the recent graphite electrodes case, it was reported that the investigation began with a complaint from a steel manufacturer that is a purchaser of graphite electrodes (Levenstein and Suslow, 2001). High prices or price increases or simply anomalous price movements may cause customers to become suspicious and pursue legal action or share their suspicions with the antitrust authorities.[5] Though it isn't important for my model, I do imagine that buyers (in many price-fixing cases, they are industrial buyers) are the ones who may become suspicious about collusion.[6]

To capture these ideas in a tractable manner, I specify an exogenous probability-of-detection function that depends on the current and previous periods' price vectors. $\phi(\underline{P}^t, \underline{P}^{t-1})$ is the probability of detection when the cartel is active, where $\underline{P}^t \equiv (P_1^t, \ldots, P_n^t)$.[7] It is assumed that, in the event of detection, successful prosecution occurs for sure, so $\phi(\underline{P}^t, \underline{P}^{t-1})$ also serves as the probability of paying penalties.[8] As a notational convention, the vector of prices will be replaced with a scalar when firms charge a common price. This specification can capture how high prices and big price changes can create suspicions among buyers that firms may not be competing.[9] The impact of the properties of this detection technology on the joint profit-maximizing price path was explored in Harrington (forthcoming). There it was found that cartel pricing dynamics are empirically plausible when detection is driven by price changes rather than price levels. As a result, in this article I will largely focus on when detection is sensitive to price changes. Assumption A4 specifies that the probability of detection is minimized when prices don't change and is weakly higher with respect to price increases. These seem plausible in the context of a stationary environment so that buyers don't expect to see much in terms of price fluctuations.

Assumption A4. $\phi : \Omega^{2n} \to [0, 1]$ is continuous and

(i) $\phi(P^0, P^0) \le \phi(P', P^0)$ and $\phi(P^0, P^0) \le \phi(P^0, P')$, for all $P', P^0 \in \Omega$, and

(ii) if $\underline{P}'' \ge \underline{P}' \ge \underline{P}^0$ (component-wise), then $\phi(\underline{P}'', \underline{P}^0) \ge \phi(\underline{P}', \underline{P}^0)$.

To derive various properties of the cartel price path, additional restrictions will later be placed on ϕ. For purposes of generality, I have sought to impose the minimal restrictions for a particular

[5] The Nasdaq case is one in which truly anomalous pricing resulted in suspicions about collusion. It was scholars rather than market participants who observed that dealers avoided odd-eighth quotes and ultimately explained it as a form of collusive behavior (Christie and Schultz, 1994). The market makers paid an out-of-court settlement of almost $1 billion.

[6] "As a general rule, the [Antitrust] Division follows leads generated by disgruntled employees, unhappy customers, or witnesses from ongoing investigations. As such, it is very much a reactive agency with respect to the search for criminal antitrust violations. . . . Customers, especially federal, state, and local procurement agencies, play a role in identifying suspicious pricing, bid, or shipment patterns" (McAnney, 1991, pp. 529, 530).

[7] In much of the analysis, it is unnecessary to specify the exact form of detection when the cartel collapses. At this point, it is sufficient to suppose that detection of past collusion can occur during the post-cartel phase, but it need not be as likely as when the cartel was active.

[8] Alternatively, one can allow the probability of successful prosecution, given detection, to lie between zero and one, where this probability is embedded in $\phi(\underline{P}^t, \underline{P}^{t-1})$.

[9] While customers are implicitly assumed to be forgetful in that their likelihood of becoming suspicious depends only on recent prices, the inclusion of a more comprehensive price history would significantly complicate the analysis by greatly expanding the state space, without any apparent gain in insight.

property to be true; hence, the form of those restrictions will vary with the result. For the reader who prefers to have one unifying set of assumptions, it can be shown that the later assumptions made on ϕ (denoted as Assumptions B1 and D3) hold for the following two classes of functions.[10] For the first class, suppose the probability-of-detection function is additively separable in the individual price changes:

$$\phi(\underline{P}^t, \underline{P}^{t-1}) = \sum_{j=1}^{n} \omega_j(\underline{P}^t)\widetilde{\phi}(P_j^t - P_j^{t-1}),$$

where $\omega_j : \Omega^n \to [0, 1]$ and $\sum_{j=1}^{n} \omega_j = 1$. Assume $\widetilde{\phi} : \Re \to [0, 1]$ is differentiable, $\widetilde{\phi}'(\varepsilon) \geq (\leq) 0$ when $\varepsilon \geq (\leq) 0$, and $\widetilde{\phi}''(\varepsilon) \geq 0$ when $\varepsilon \geq 0$. Thus, when the price change is negative, the probability of detection is nonincreasing in the price change and, when the price change is positive, is a weakly convex nondecreasing function of the price change. The second class has detection depend on movements in a summary statistic of firms' prices. Suppose

$$\phi(\underline{P}^t, \underline{P}^{t-1}) = \widetilde{\phi}(f(\underline{P}^t) - f(\underline{P}^{t-1})),$$

where $f : \Omega^n \to \Omega$ and (i) $f(P, \ldots, P) = P$, and (ii) if $P^0 \leq P$, then $f(P, \ldots, P^0, \ldots, P) \leq P$. $\widetilde{\phi}$ has the properties specified above. Examples for f include the average price—either unweighted or weighted by market share—and the median price.

This modelling of detection warrants further discussion, since it does not model those agents who might engage in detection. The first point to make concerns tractability. With two distinct sources of structural dynamics—detection and antitrust penalties—in addition to the usual (repeated game–style) behavioral dynamics, this model is rich enough to provide new insight into cartel pricing dynamics even with a reduced-form modelling of the detection process, and its complexity already pushes the boundaries of formal analysis. It would seem prudent to understand the workings of this model before moving on to the much more difficult problem of endogenizing the probability-of-detection function. Tractability issues aside, there is another motivation that makes the analysis of intrinsic interest. The objective of this article is to develop insight and testable hypotheses about cartel pricing. A good model of the detection process is then one that is a plausible description of how cartel members *perceive* the detection process. It strikes me as quite plausible that firms might simply postulate that higher prices or bigger price changes result in a greater likelihood of creating suspicions without having derived that relationship from first principles about buyers. Thus, even if this modelling of the detection process is wrong, the resulting statements about cartel pricing may be right if that model is a reasonable representation of firms' perceptions.

In period 1, firms have the choice of forming a cartel (and risking detection and penalties) or earning noncollusive profit of $\hat{\pi}$. If they choose the former, they can, at any time, choose to discontinue colluding. However, a finitely lived cartel will cause collusion to unravel so that, in equilibrium, firms either collude forever or not at all (subject to the cartel being exogenously terminated because of detection). Firms are then not allowed to form and dissolve a cartel more than once. While the possibility of temporarily shutting down the cartel is not unreasonable (firms may want to "lie low" for a bit of time), the analysis is complicated enough without allowing for this. Exploration of that strategic option is left for future research.

□ **Related work.** Though no previous work allows for the rich set of dynamics of this model, there have been articles that take account of detection considerations in the context of cartel pricing. Block, Nold, and Sidak (1981) offer a static model in which the probability of detection is increasing in the price level. Spiller (1986), Salant (1987), and Baker (1988) allow buyers to adjust their purchases under the anticipation that they may be able to collect multiple damages if sellers were shown to have been colluding. Also within a static setting, Besanko and Spulber

[10] The proof is available on request.

(1989, 1990), LaCasse (1995), Souam (2001), and Schinkel and Tuinstra (2002) explore a context in which firms have private information, which influences whether or not they collude, and either the government or buyers must decide whether to pursue costly legal action. Three articles consider a dynamic setting. Cyrenne (1999) modifies Green and Porter (1984) by assuming that a price war, and the ensuing raising of price after the war, results in detection for sure and with it a fixed fine. Spagnolo (2000) and Motta and Polo (2003) consider the effects of leniency programs on the incentives to collude in a repeated game of perfect monitoring.[11] Though considering collusive behavior in a dynamic setting with antitrust laws, these articles exclude the sources of dynamics that are the foci of the current analysis, specifically, that the probability of detection and penalties are sensitive to firms' pricing behavior. It is this sensitivity that will generate predictions about cartel pricing dynamics.

3. Optimal symmetric subgame-perfect equilibrium

■ The cartel's problem is to choose an infinite price path so as to maximize the expected sum of discounted income subject to the price path being incentive compatible (IC). In determining the set of IC price paths, the assumption is made that deviation from the collusive path results in the cartel being dissolved and firms behaving according to a Markov-perfect equilibrium (MPE).

Suppose a firm deviates and the cartel collapses. Since cartel meetings are no longer taking place, the damage variable simply depreciates at the exogenous rate of $1 - \beta$: $X^t = \beta X^{t-1}$.[12] This is still a dynamic problem, however, in that price movements can create suspicions and, while firms are no longer colluding, an investigation could reveal evidence of past collusion. The state variables at t are the vector of lagged prices, $\underline{P}^{t-1}$, and (common) damages, X^{t-1}. A MPE is then defined by a stationary policy function that maps $\Omega^n \times \Re_+$ into Ω. Let $V_i^{mpe}(\underline{P}^{t-1}, X^{t-1})$ denote firm i's payoff at a MPE. When there is a symmetric state and a symmetric MPE, the payoff is denoted $V^{mpe}(P^{t-1}, X^{t-1})$.

For the characterization of cartel pricing, it is not necessary to characterize a MPE, it being sufficient that the MPE payoff satisfy the following condition:

$$\frac{\hat{\pi}}{1-\delta} \geq V_i^{mpe}(\underline{P}^{t-1}, X^{t-1}) \geq \frac{\hat{\pi}}{1-\delta} - \beta X^{t-1} - F, \qquad \forall\, (\underline{P}^{t-1}, X^{t-1}) \in \Omega^n \times [0, \overline{X}], \quad (1)$$

that is, a MPE results in a payoff weakly lower than the static Nash equilibrium payoff but weakly higher than the static Nash equilibrium payoff less the cost of incurring the penalties for sure. The issue then is under what conditions a MPE exists that satisfies (1). Note that it holds if the post-cartel price path is sufficiently close to pricing at $\hat{P}$ and the probability of incurring penalties during the post-cartel phase is sufficiently great. For example, (1) holds when infinite repetition of the static Nash equilibrium is a MPE, a sufficient condition for which is that the stage game is the Bertrand price game.[13] In the ensuing analysis, (1) is assumed in some cases and in others occurs for free, being implied by other assumptions. Though this property need not always hold (for an example where it doesn't, see Harrington, 2003), it is useful to limit our attention to when it does so as to be able to provide a coherent set of results. Let me emphasize that I could have done away with (1) by simply focusing on the Bertrand price game. The route I have taken is

[11] Rey (2003) offers a nice review of some of this work along with other theoretical analyses pertinent to optimal antitrust policy.

[12] Implicit in this specification is that damages stop accumulating once the cartel is dismantled. Though this is a useful approximation, if the post-cartel price exceeds $\hat{P}$, it is because of past collusion, so one could argue that additional damages should be assessed. Whether they are, in practice, is another matter.

[13] The proof is available on request, but I will sketch the argument: Suppose, to the contrary, a (symmetric) MPE has all firms pricing at $P' > \hat{P}$ $(= c)$ in the current period. By the usual argument, a firm could produce an n-fold increase in current profit by pricing just below P'. As the change in the current price vector is arbitrarily small, there is almost no effect on the firm's future payoff, since the change in the probability of detection is small and the change in the state variable is small.

more general as, by assuming a MPE payoff satisfies (1), it includes the Bertrand price game as a special case.

It is natural to assume that, at the start of the cartel, damages are zero and firms are charging the noncollusive price: $(P^0, X^0) = (\hat{P}, 0)$. While many of the ensuing results are robust to these initial conditions, they will be assumed throughout the article so as to simplify some of the proofs. Before providing the conditions defining the cartel solution, the assumption is made that damages are assessed only in those periods for which the cartel has been functioning properly and, more specifically, are not assessed when a firm deviates from the cartel price. Thus, when a firm considers cheating on the agreement, it assumes that the act of deviation negates damages for that period.[14]

As the focus is on symmetric collusive solutions, it is sufficient to define the state variables as $(P^{t-1}, X^{t-1}) \in \Omega \times [0, \overline{X}]$. The firms' problem is either to not form a cartel—and price at $\hat{P}$ in every period with each firm receiving a payoff of $\hat{\pi}/(1-\delta)$—or form a cartel and choose a price path so as to

$$\max_{\{P^t\}_{t=1}^{\infty} \in \Gamma} \sum_{t=1}^{\infty} \delta^{t-1} \prod_{j=1}^{t-1} [1 - \phi(P^j, P^{j-1})] \pi(P^t)$$
$$+ \sum_{t=1}^{\infty} \delta^t \phi(P^t, P^{t-1}) \prod_{j=1}^{t-1} [1 - \phi(P^j, P^{j-1})] \left[\frac{\hat{\pi}}{1-\delta} - \sum_{j=1}^{t} \beta^{t-j} \gamma x(P^j) - F \right], \qquad (2)$$

where

$$\Gamma \equiv \Bigg\{ \{P^t\}_{t=1}^{\infty} \in \Omega^{\infty} : \sum_{\tau=t}^{\infty} \delta^{t-\tau} \prod_{j=t}^{\tau-1} [1 - \phi(P^j, P^{j-1})] \pi(P^\tau)$$
$$+ \sum_{\tau=t}^{\infty} \delta^{\tau-t+1} \phi(P^\tau, P^{\tau-1})$$
$$\times \prod_{j=t}^{\tau-1} [1 - \phi(P^j, P^{j-1})] \left[\frac{\hat{\pi}}{1-\delta} - \sum_{j=1}^{\tau} \beta^{\tau-j} \gamma x(P^j) - F \right]$$
$$\geq \max_{P_i} \overline{\pi}\left(P_i, P^t\right) + \delta\phi\left(\left(P^t, \ldots, P_i, \ldots, P^t\right), P^{t-1}\right)$$
$$\times \left[\frac{\hat{\pi}}{1-\delta} - \sum_{j=1}^{t-1} \beta^{t-j} \gamma x(P^j) - F \right]$$
$$+ \delta \left[1 - \phi\left(\left(P^t, \ldots, P_i, \ldots, P^t\right), P^{t-1}\right)\right]$$
$$\times V_i^{mpe} \left((P^t, \ldots, P_i, \ldots, P^t), \sum_{j=1}^{t-1} \beta^{t-j} \gamma x(P^j) \right), \quad \forall\, t \geq 1 \Bigg\}.$$

In (2), $\prod_{j=1}^{t-1}[1 - \phi(P^j, P^{j-1})]$ is the probability that the cartel has not been detected as of period t. Γ is the set of price paths that satisfy the incentive-compatibility constraints (ICCs). A solution to (2) is referred to as an optimal symmetric subgame-perfect equilibrium (OSSPE) price path.

As I do not have a general proof of existence for a pure-strategy MPE, it is necessary to make the following assumption so as to provide a general proof of the existence of an OSSPE price path.[15] Recall that if the stage game is the Bertrand price game, then infinite repetition of

[14] In practice, it is not clear when damages are no longer assessed, and this assumption is probably as good as any other. Furthermore, it has a nice property that is useful for both analytical and numerical work. If damages were assigned in the period that a firm deviated, then, entering the post-deviation phase, firms would have different levels of damages and this would expand the state space.

[15] To my knowledge, there is no general existence theorem for MPE, even in mixed strategies, when the state space is uncountable; see Fudenberg and Tirole (1991).

the static Nash equilibrium is a MPE. This satisfies both the existence and continuity specified in Assumption A5.

Assumption A5. $\forall\,(\underline{P}^{t-1}, X^{t-1}) \in \Omega^n \times [0, \overline{X}]$, $\exists$ a Markov-perfect equilibrium and, furthermore, $\exists$ a continuous function $V_i^{mpe} : \Omega^n \times [0, \overline{X}] \rightarrow \Re$ such that $V_i^{mpe}(\underline{P}^{t-1}, X^{t-1})$ is the payoff associated with a Markov-perfect equilibrium.

Define the firms' choice set as $\{No\ Cartel\} \cup \Gamma$, where it is understood that choosing an element from Γ implies forming a cartel, whereas choosing $No\ Cartel$ implies that all firms price at $\hat{P}$ in all periods. An OSSPE price path is a selection from $\{No\ Cartel\} \cup \Gamma$ that maximizes each firm's payoff. All proofs are available in the web Appdenix.[16]

Theorem 1. If Assumptions A1–A5 hold, then an OSSPE price path exists.

Proof. See the web Appendix.

$V(P^{t-1}, X^{t-1})$ denotes the payoff associated with an OSSPE path. When something is stated to be a property of an OSSPE path, it is meant to refer to an OSSPE path that involves cartel formation.

To simplify the proofs, the assumption is made from here on that $\Omega = [0, P^m]$ so that the cartel does not set price above the simple monopoly price. While I don't believe this assumption is essential for any result, I cannot dismiss the possibility that an OSSPE path would have price exceed the simple monopoly price in some periods. I will later elaborate on this point and note in the proofs where this assumption is used. However, I also conjecture that the most relevant part of the parameter space is where an OSSPE path lies below P^m.

4. Dynamic properties of the collusive price path

■ When ICCs are not binding, an OSSPE price path is nondecreasing over time as the cartel gradually raises price to reduce the probability of detection while achieving higher profit (Harrington, forthcoming). When instead cartel stability is a concern, the analysis is more subtle. Critical is how these ICCs evolve, in response to the state variables, and whether collusion is becoming more or less difficult. My approach to this problem has three steps. First, a numerical analysis is conducted to identify what types of price paths may occur. Second, an analytical characterization of the price path is conducted for special cases of the model to identify the key forces at work. Third, the analytical and numerical results are pulled together to draw some general conclusions about the properties of the collusive price path.

□ **Numerical analysis.** Consider an industry with symmetrically differentiated products. Assuming a representative utility function of

$$U(q_1, \ldots, q_n) = a\sum_{i=1}^{n} q_i - \left(\frac{1}{2}\right)\left(b\sum_{i=1}^{n} q_i^2 + e\sum_{i=1}^{n}\sum_{j\neq i} q_i q_j\right),$$

where q_i is the amount consumed of firm i's product, if all firm demands are nonnegative, then firm i's demand is

$$D(P_i, P_{-i}) = \left(\frac{a}{b+(n-1)e}\right) - \left(\frac{b+(n-2)e}{(b+(n-1)e)(b-e)}\right)P_i + \left(\frac{e(n-1)}{(b+(n-1)e)(b-e)}\right)P_{-i},$$

where $a > 0$ and $b > e > 0$. (For details, see Vives (1999).) The firm cost function is $C(q) = cq$,

[16] Available at www.rje.org/main/sup-mat.html. All references to the web Appendix may be found at this URL.

where $a > c \geq 0$. Assume the damage function is $x(P) = (P - \hat{P})D(P, P)$ and that detection depends on movement in the average transaction price,[17]

$$f(P_1, \ldots, P_n) = \sum_{i=1}^{n} \left(\frac{D(P_i, P_{-i})}{\sum_{j=1}^{n} D(P_j, P_{-j})} \right) P_i .$$

Letting $f^t \equiv f(P_1^t, \ldots, P_n^t)$, the probability of detection when the cartel is active is specified to be

$$\phi(f^t, f^{t-1}) = \begin{cases} \min\left\{\alpha_0 + \alpha_1^u \left(f^t - f^{t-1}\right)^2, 1\right\} & \text{if } f^t \geq f^{t-1} \\ \min\left\{\alpha_0 + \alpha_1^d \left(f^t - f^{t-1}\right)^2, 1\right\} & \text{if } f^t < f^{t-1} \end{cases}$$

and when the cartel is inactive it is the same, though with a zero constant. I then allow for an asymmetric response to price increases and price decreases and consider parameter values such that $0 \leq \alpha_1^d \leq \alpha_1^u$. When the cartel is active, α_0 captures sources of detection that are independent of price movements.

There are 13 parameters: demand and cost parameters—a, b, e, c; detection parameters—$\alpha_0, \alpha_1^u, \alpha_1^d$; penalty parameters—$\beta, \gamma, F, T$; discount factor, δ; and number of firms, n. The benchmark parameter configuration is

$$a = 100, \quad b = 2, \quad e = 1, \quad c = 0, \quad n = 4, \quad \delta = .5, \quad \beta = .75, \quad \gamma = 1$$
$$\alpha_0 = .025, \quad \alpha_1^u = \frac{16}{(P^m - \hat{P})^2}, \quad \alpha_1^d = \frac{.8}{(P^m - \hat{P})^2}, \quad F = 0, \quad T = 8.$$

Note that if $\alpha_0 = 0$ and $\alpha_1^u = \omega/(P^m - \hat{P})^2$, then a price increase of $(P^m - \hat{P})/\sqrt{\omega}$ results in detection for sure. Throughout the analysis, a, e, c, F, and T do not vary. The model was run for 32 parameter configurations, 26 of which involved cartel formation.[18] These configurations involved various modifications to the benchmark configuration and included the following values:

$$\delta \in \{.3, .4, .5, .6, .9\}, \quad \beta \in \{.5, .75, .9\}, \quad b \in \{2, 3\}, \quad n \in \{2, 4, 6, 8\}$$
$$\alpha_1^u \in \left\{ \frac{1}{(P^m - \hat{P})^2}, \frac{16}{(P^m - \hat{P})^2}, \frac{32}{(P^m - \hat{P})^2} \right\}$$
$$\alpha_1^d \in \left\{0, \frac{.8}{(P^m - \hat{P})^2}\right\}, \quad \alpha_0 \in \{.025, .05\}, \quad \gamma \in \{1, 2\}.$$

The output produced is (i) the MPE value and policy functions, (ii) the constrained dynamic programming value and policy functions, and (iii) the unconstrained dynamic programming value and policy functions. A description of the numerical methods is provided in the Appendix.

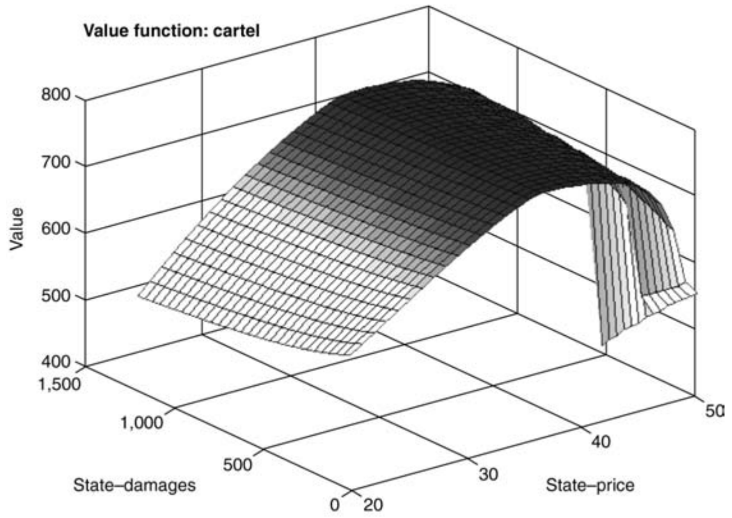
For the benchmark case, Figures 1 and 2 show the value and policy functions for the cartel case and the noncollusive case, respectively. The resulting cartel path is shown in Figure 3. The ICCs bind as the cartel raises price to 37, which is below the unconstrained steady-state price of 46. Figure 4 shows the MPE price path starting at the price and damages associated with the

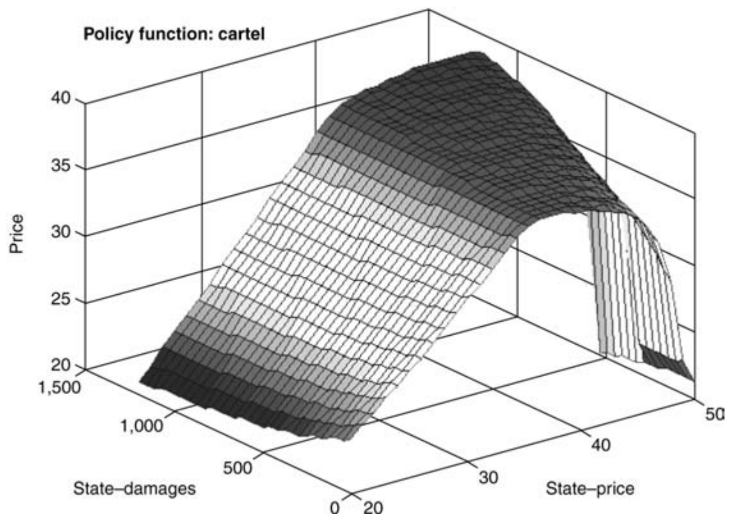
[17] Numerically, it is essential to have a summary statistic of the lagged price vector so as to limit the dimensionality of the state space.

[18] A typical case took 3–5 hours on a Dell Workstation PWS 350 with a 1.8 GHz Intel Xeon processor. When β and/or δ are close to one, it can take much longer.

FIGURE 1

VALUE AND POLICY CARTEL FUNCTIONS (BENCHMARK CASE)





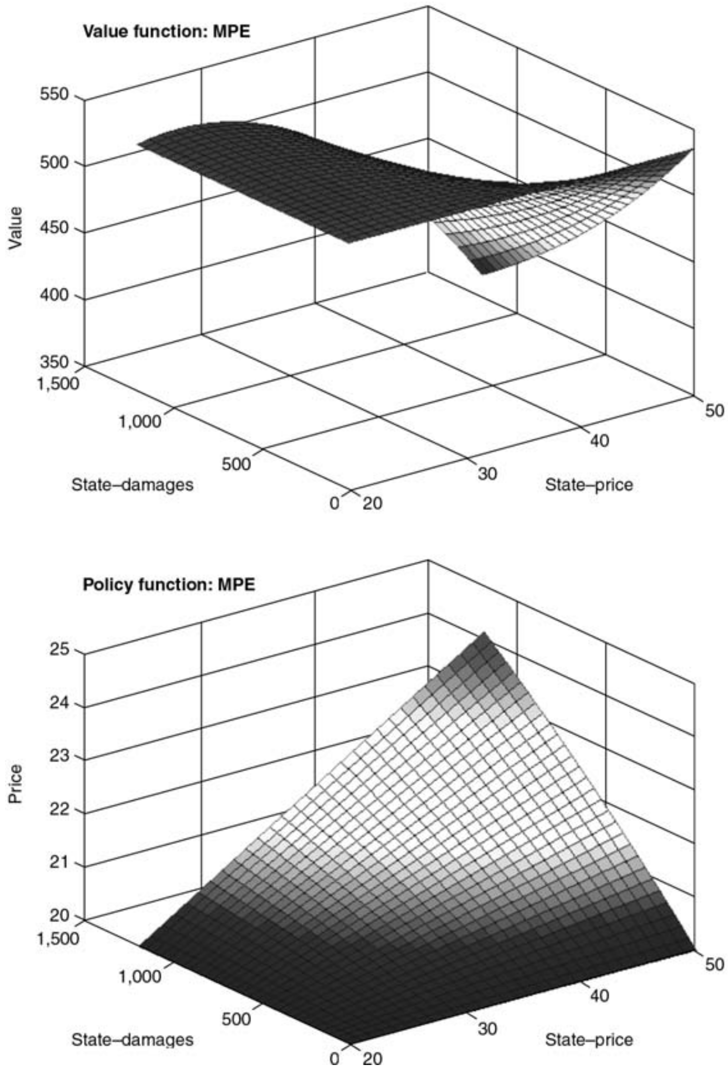
cartel steady state. Note that the MPE price doesn't immediately fall to the noncollusive level of 20, as firms mediate their price drops so as to make detection less likely.

Surveying the results for all of the parameter configurations, two qualitatively distinct cartel price paths emerge. First, the cartel price path is monotonically increasing, as represented in Figure 3. The cartel gradually raises price—so as to avoid detection—and price achieves some steady-state level that is typically below the monopoly price either because it isn't worth it for the cartel to risk detection by further raising price or it isn't feasible for the cartel to do so. This monotonicity of price—which is proved when ICCs do not bind in Harrington (forthcoming)—can then still occur when ICCs bind. Second, and more interestingly, the cartel price path initially increases and then declines, approaching its steady-state level from above. The value and policy functions for a representative example are shown in Figure 5, while Figure 6 shows the price path—price rises from 20 to over 45 during the first ten periods and then the cartel gives up about 10% of its price increase as it falls to its steady-state level.[19]

[19] The modifications to the benchmark case are $\beta = .9$, $\alpha_0 = .05$, $\alpha_1^u = 2/(P^m - \hat{P})^2$, $\alpha_1^d = 0$, and $\delta = .6$.

FIGURE 2

VALUE AND POLICY MARKOV-PERFECT EQUILIBRIUM FUNCTIONS (BENCHMARK CASE)



To understand these numerical findings, in the next two subsections I analytically derive properties of an OSSPE price path for special cases of the model. In particular, I consider each of the two dynamics—detection and penalties—in isolation. In the first subsection, penalties are fixed but the probability of detection remains endogenous. As in the case when ICCs do not bind, an OSSPE price path is shown to be increasing over time. In the second subsection, I allow penalties to evolve but fix the probability of detection. After price is raised in the first period, an OSSPE path is declining thereafter. In the final subsection, I draw general conclusions from the numerical and analytical results and perform comparative dynamics.

□ **Pricing dynamics with endogenous detection.** Assume there are only fines: $\gamma = 0$ and $F > 0$. The lone state variable for the cartel is lagged price and, in the event of a deviation, the vector of lagged prices. Though penalties are fixed, the probability of detection is sensitive to how the cartel prices, as specified in Assumption A4. Further structure is required to establish our main result.

FIGURE 3

PRICE AND DAMAGE PATH (BENCHMARK CASE)

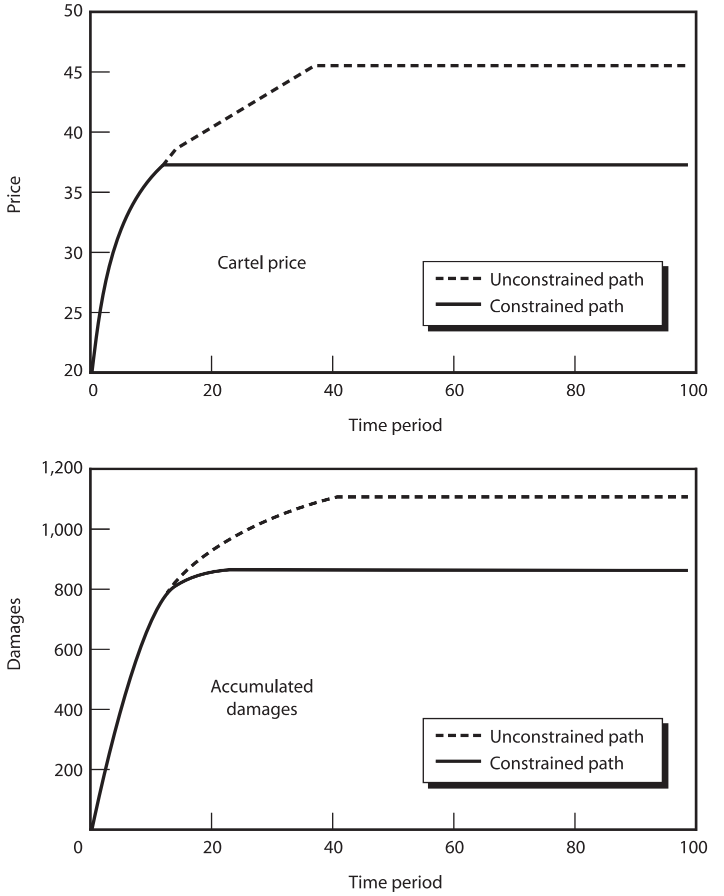


FIGURE 4

MARKOV-PERFECT EQUILIBRIUM PRICE PATH (BENCHMARK CASE)

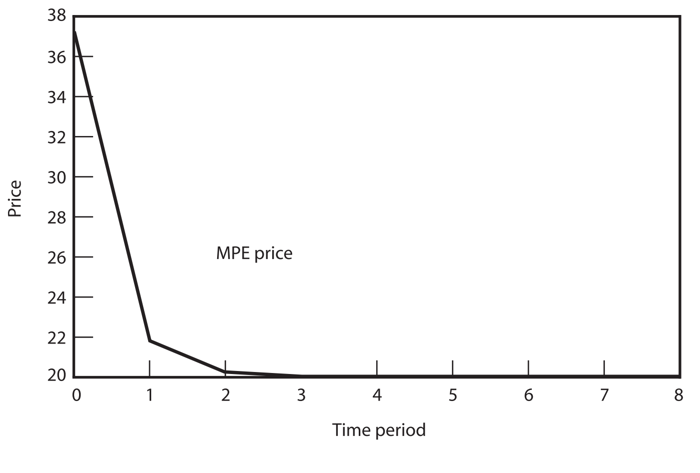
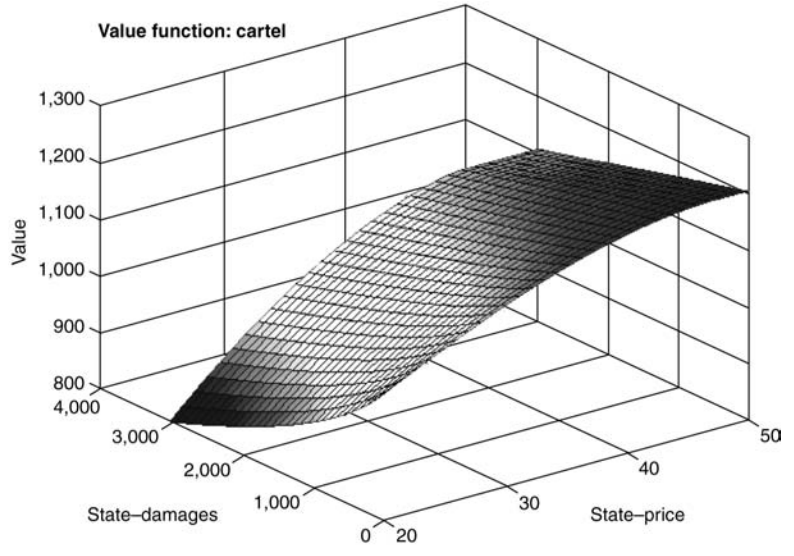
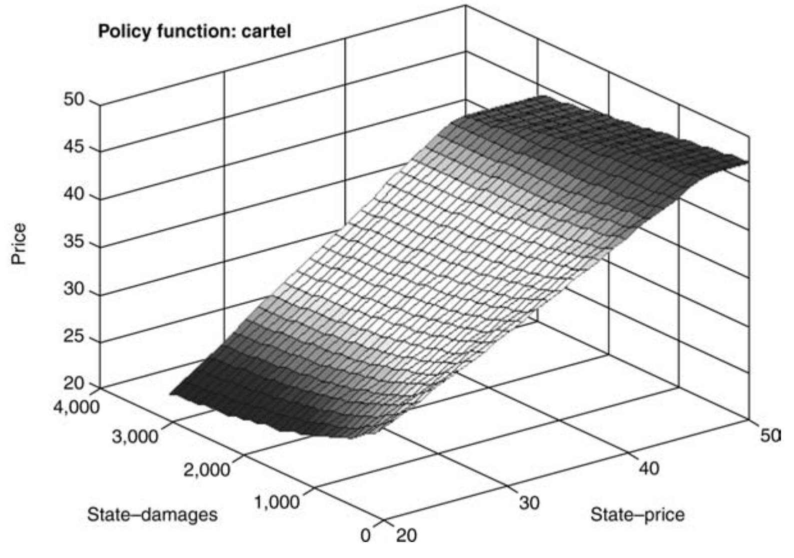


FIGURE 5

VALUE AND POLICY CARTEL FUNCTIONS (NONMONOTONIC CARTEL PRICE PATH)





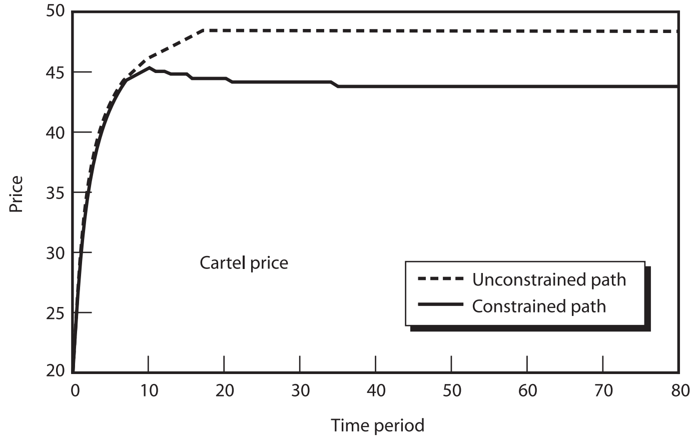
Assumption B1. If $P' \geq P$ and $P' > P^0$, then $\phi(P', P) - \phi((P', \ldots, P^0, \ldots, P'), P)$ is nonincreasing in P.

To interpret Assumption B1, suppose that the lagged price is P and the cartel is to raise it to P'. If an individual firm considers deviating to a price of P^0, $\phi(P', P) - \phi((P', \ldots, P^0, \ldots, P'), P)$ is the associated difference in the probability of detection between colluding and deviating. Assumption B1 says that if the cartel is raising price by a greater amount, then this differential in the probability is greater. Section 2 described a class of probability-of-detection functions whereby Assumption B1 holds. The presumed property for a MPE payoff is stated as Assumption B2.

Assumption B2. $\hat{\pi}/(1-\delta) \geq V_i^{mpe}(\underline{P}) \geq (\hat{\pi}/(1-\delta)) - F, \forall\ \underline{P} \in \Omega^n$.

FIGURE 6

NONMONOTONIC CARTEL PRICE PATH



Theorem 2 shows that when penalties are fixed and only detection is sensitive to the price path, the cartel price path is nondecreasing over time.

Theorem 2. Assume A1–A2, A4–A5, B1–B2, and $\gamma = 0$. If $\{\overline{P}^t\}_{t=1}^{\infty}$ is an OSSPE path, then $\{\overline{P}^t\}_{t=1}^{\infty}$ is nondecreasing over time.

Proof. See the web Appendix.

When the cartel is unconstrained by concerns about stability (that is, the ICCs are not binding), the optimal price path is nondecreasing over time. Since bigger price movements are more likely to trigger suspicions about a cartel having been formed, the cartel gradually raises price so as to balance profit and the probability of detection. Thus, if the price path is decreasing when ICCs bind, it is because incentive compatibility requires it. The issue then is under what circumstances the cartel finds itself charging a price that it can't sustain. In the proof of Theorem 2, it is established that if it is IC to raise price to some level, then it is IC to keep price at that level. Therefore, it is never necessary to reduce price in order to maintain the stability of the cartel, which implies that the price path is nondecreasing over time.

This result, and in particular the role of Assumption B1, can be explained as follows. Recall that the probability of detection is greater when price changes are larger in absolute value. If a cartel keeps price constant, then cheating—with the associated drop in price—cannot make detection any less likely. In contrast, if the cartel is raising price, then cheating—by not raising price as much—can reduce the extent of price fluctuations and thereby make detection less likely. Thus, if a firm found it unprofitable to cheat when the cartel raised price to some level, it isn't then profitable when the cartel is keeping price at that level. In brief, concerns about detection makes cheating less profitable, *ceteris paribus*, when the cartel is keeping prices stable than when it is raising price. It follows that price need never be lowered in order to maintain the stability of the cartel and, therefore, the price path is nondecreasing over time. In conclusion, when the dynamics are solely due to how the price path influences the likelihood of detection, concerns about cartel stability do not alter the qualitative properties of the optimal cartel price path—it is increasing just as when ICCs do not bind.

□ **Pricing dynamics with endogenous penalties.** In proving Theorem 2, it was crucial that penalties were fixed, for if penalties evolve, then the ICCs could change so that it may not be IC to keep price constant. To consider the dynamics emanating from the endogeneity of penalties, suppose detection is independent of prices—being exclusively driven by such factors as internal whistleblowers—and $\gamma > 0$ so that penalties are sensitive to the prices set.

Assumption C1. $\exists\, \phi^0 \in (0, 1)$ such that $\phi(\underline{P}', \underline{P}^0) = \phi^0\ \forall\ \underline{P}', \underline{P}^0 \in \Omega^n$.

It will be useful to explicitly specify the likelihood of detection after the cartel has collapsed. Let $\rho(\tau)$ denote the probability of detection τ periods after the last cartel meeting (which was in the period during which a firm cheated). As specified in Assumption C2, detection is less likely in at least some periods when the cartel is inactive than when it is active.

Assumption C2. $\rho(0) = \phi^0$, $\rho(\tau) \leq \phi^0\ \forall\ \tau$, and $\rho(\tau) < \phi^0$ for some τ.

Because the probability of detection is fixed, the problem simplifies considerably. First, it is straightforward to show that the unique MPE is the infinite repetition of the static Nash equilibrium.[20] Second, the optimal deviation price is that which maximizes current profit, $\psi(P^t)$. Since the probability of detection is fixed and the price at which a firm deviates doesn't influence penalties (where, recall, it is assumed that damages are not assessed when the cartel is not functioning), a deviating firm's price affects only current profit. Using these properties, the cartel's problem can be stated as

$$\max_{\{P^t\}_{t=1}^{\infty} \in \Omega^{\infty}} \sum_{t=1}^{\infty} \delta^{t-1}(1-\phi^0)^{t-1}[\pi(P^t) - \theta^c \gamma x(P^t)] + \kappa^c \left[\frac{\hat{\pi}}{1-\delta} - F\right], \tag{3}$$

subject to

$$\sum_{\tau=t}^{\infty} \delta^{\tau-t}(1-\phi^0)^{\tau-t}[\pi(P^\tau) - \theta^c \gamma x(P^\tau)] + \kappa^c \left[\frac{\hat{\pi}}{1-\delta} - F\right] - \theta^c \beta X^{t-1}$$
$$\geq \overline{\pi}(\psi(P^t); P^t) + \delta \frac{\hat{\pi}}{1-\delta} - \theta^d \beta X^{t-1} - \kappa^d F, \qquad \forall\, t \geq 1,$$

where $\theta^c \equiv \delta\phi^0/[1-\delta\beta(1-\phi^0)]$, $\theta^d \equiv \sum_{\tau=0}^{\infty} \delta(\delta\beta)^\tau \Pi_{h=0}^{\tau-1}(1-\rho(h))\rho(\tau)$, $\kappa^c \equiv [\delta\phi^0/(1-\delta(1-\phi^0))]$, and $\kappa^d \equiv \sum_{\tau=0}^{\infty} \delta^{\tau+1} \prod_{h=0}^{\tau-1}(1-\rho(h))\rho(\tau)$. $\pi(P^t) - \theta^c\gamma x(P^t)$ represents the net income from collusion in period t. A firm receives profit of $\pi(P^t)$ by colluding but incurs a liability in the form of $\theta^c\gamma x(P^t)$, which is the expected present value of damages. This expression is multiplied by $(1-\phi^0)^{t-1}$, which is the probability that the cartel has not yet been detected. Turning to the ICCs, θ^c (θ^d) and κ^c (κ^d) measure the marginal effect of damages and fines, respectively, on the collusive (punishment) payoff. It follows from Assumptions C1 and C2 that $\theta^d < \theta^c$ and $\kappa^d < \kappa^c$, a key implication of which is that if, starting from period t, some price path is IC given $X^{t-1} = X'$, then it is also IC if $X^{t-1} < X'$, as the collusive payoff is decreasing with respect to damages at a faster rate than the deviation payoff.

The next assumption says that the difference between the maximal current profit and the collusive profit is increasing in the collusive price. It will imply that if a price path is IC, then so is a price path that is identical except that the period t price is lower.

Assumption C3. $\overline{\pi}(\psi(P), P) - \pi(P)$ is increasing in $P\ \forall\ P \geq \hat{P}$.

In proving the results of this section, it will be useful to pose the cartel's problem as choosing a level of damages rather than price. As this approach requires that $x(\cdot)$ be one-to-one, Assumption C4 strengthens Assumption A3 by assuming that the damage function is strictly monotonic over the relevant domain.

Assumption C4. $x(\cdot)$ is differentiable and nondecreasing, $x(\hat{P}) = 0$, and x is strictly increasing over $[\hat{P}, P^m)$.

Defining $\xi(x)$ as the price that generates current damage penalties of d, it is implicitly defined by $d = \gamma x(\xi(d))$. ξ is well defined $\forall\ d \in [\gamma x(\hat{P}), \gamma x(P^m)]$.

[20] The reasoning is simple. Since both the probability of detection and penalties are independent of a firm's price, a firm's future payoff is independent of its price. Thus, a MPE must have each firm choose price to maximize its current profit.

Assumption C5. $\pi(\xi(d))$ is concave in $d \forall\, d \in [\gamma x(\hat{P}), \gamma x(P^m)]$.

It can be shown that Assumption C5 holds when demand is weakly concave, marginal cost is constant, damages take the standard form, and the "but for" price weakly exceeds the competitive price.[21] Note that Assumption C4 is also implied by these conditions.

The next result shows that damages are nondecreasing over time.

Lemma 1. Assume A1–A2 and C1–C5. If $\{\overline{X}^t\}_{t=1}^{\infty}$ is consistent with an OSSPE, then $\{\overline{X}^t\}_{t=1}^{\infty}$ is nondecreasing.

Proof. See the web Appendix.

Assumption C6 imposes quasi-concavity of net income: profit less the expected present value of damages. Sufficient conditions for Assumption C6 are strict concavity of the profit and damage functions.

Assumption C6. $\exists\, P^+ \in (\hat{P}, P^m]$ such that $\pi'(P) - \theta^c \gamma x'(P) \gtreqless 0$ as $P \lesseqgtr P^+ \;\forall\, P \in [\hat{P}, P^m]$.

Theorem 3 shows that though the cartel raises price in the first period, it (weakly) decreases price thereafter. Recall that it is assumed the probability of detection is fixed but penalties are sensitive to the price path.[22]

Theorem 3. Assume A1–A2 and C1–C6. If $\{\overline{P}^t\}_{t=1}^{\infty}$ is an OSSPE price path, then $\overline{P}^1 > P^0$ and it is nonincreasing $\forall\, t \geq 1$.

Proof. See the web Appendix.

The logic behind the proof and the result is as follows. As the probability of detection is independent of lagged prices, all dynamics come from the evolution of damages. Since detection is more likely when the cartel is active, the collusive payoff is more sensitive than the deviation payoff to damages. Given that damages grow on the cartel price path (Lemma 1), the collusive payoff is then declining at a faster rate over time than is the deviation payoff. This tightens ICCs, and to ensure they are satisfied, the cartel needs to lower price (by Assumption C3). Note that though price is falling over time, its decline is sufficiently moderate so that damages rise.

It is easy to argue that when ICCs are binding, the price path is strictly decreasing in some periods. Suppose, contrary to the claim, that the price path never decreases. Then, by Theorem 3, it is constant starting with period 1, and let P' be this constant price. With a constant price, X^t is strictly increasing and converging to $\gamma x(P')/(1-\beta)$. Suppose the ICC at t' is binding so that the collusive payoff equals the payoff to cheating. Given, by supposition, that the cartel price is P' in both t' and $t'+1$ periods, the ICC at $t'+1$ is identical to that at t' except that inherited damages are higher at $t'+1$. As the collusive payoff declines faster with respect to damages than the payoff to cheating, if the two payoffs are equal at t', then, since $X^{t'-1} < X^{t'}$, the collusive payoff is strictly less than the payoff to cheating at $t'+1$, which violates incentive compatibility. This contradiction means that the original supposition that the price path is constant is false. Combined with Theorem 3, the price path is then decreasing in some periods.

□ **Discussion and comparative dynamics.** Let us now pull together the various pieces of this section. If the ICCs are not binding, the cartel price path rises over time. When ICCs do bind, the central issue is whether the cartel will, at some point, be forced to lower price so as to maintain cartel stability. Both the sensitivity of detection to price movements and the sensitivity of penalties to price levels are pertinent to this issue. Focusing on the former dynamic, Theorem 2 showed that raising price made cartel stability easier, so that there is never a need to lower price. More specifically, if a firm did not find it optimal to cheat when other firms were raising price,

[21] The proof is available on request.

[22] It is worth noting that when the ICCs do not bind, the cartel raises price in the first period and keeps it fixed thereafter when the probability of detection is fixed.

then it is not optimal for them to cheat when other firms are keeping price constant. Thus, higher prices are easier to sustain as lagged price is higher. In contrast, the evolution of penalties can have the opposite effect—collusion may be more difficult as firms collude longer. As penalties grow, cartel members become increasingly concerned with the prospects of detection. If detection is less likely when collusion stops, there is an added incentive for a firm to cheat. With rising penalties as firms collude longer, the cartel must lower price so as to counterbalance this increased desire to deviate. The extent to which rising penalties make the cartel less stable then depends on whether cheating—with the ensuing collapse of the cartel—makes detection less likely. If the probability of detection is sufficiently insensitive to the price decline that would ensue in the post-cartel periods, then a firm reduces the probability of paying antitrust penalties by cheating and causing the cartel to dissolve. In that case, this dynamic forces price down. But if instead a post-cartel price war is likely to trigger detection, rising penalties serve to stabilize the cartel. Firms increasingly prefer to maintain relatively stable cartel prices than to risk detection by inducing a price war. Thus, when detection is sensitive to price declines, these two dynamics reinforce themselves to result in a rising price path.

Given this discussion, the numerical price paths originally derived are easy to understand. When the probability of detection is sufficiently sensitive to price increases, the cartel will gradually raise price for reasons that are clear. If, in addition, detection is sufficiently sensitive to price decreases, then collusion will become easier over time, which allows further price hikes; so the price path is always increasing. Collusion is becoming easier because penalties are growing—so avoiding detection is increasingly important—and the best way to avoid detection is to maintain moderately rising or stable prices rather than experience a sharp price war. When instead detection is fairly insensitive to price decreases, then the price path, after initially rising, will eventually fall. The growing penalties make cheating increasingly attractive—as it brings collusion to an end and reduces the chances of having to pay these penalties—and the cartel must lower price as a result. Thus, the second dynamic eventually comes to dominate the pricing dynamics.

Comparative dynamics are performed and reported in Figures 7 and 8. The benchmark case is explored under various discount factors, $\delta \in \{.3, .4, .6, .9\}$, and number of firms, $n \in \{2, 4, 6, 8\}$. In the absence of antitrust policy, the standard result is that more-patient firms result in higher cartel prices. The result here is different. As δ is raised, the price path initially shifts down, though in the long run prices are higher. This reflects two countervailing effects of δ. First, there is the standard effect that more-patient firms are less inclined to cheat and this loosens up ICCs and allows for a higher collusive price. This causes the cartel to price higher in the long run. Second, a cartel that raises price faster earns higher current profit but lowers its future payoff because

FIGURE 7

EFFECT OF THE DISCOUNT FACTOR ON THE CARTEL PRICE PATH

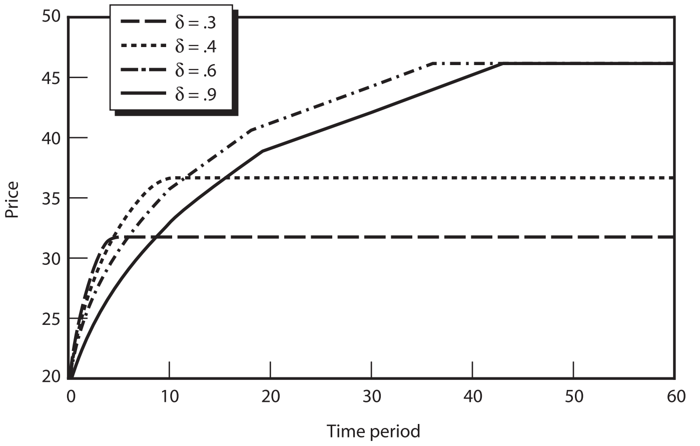
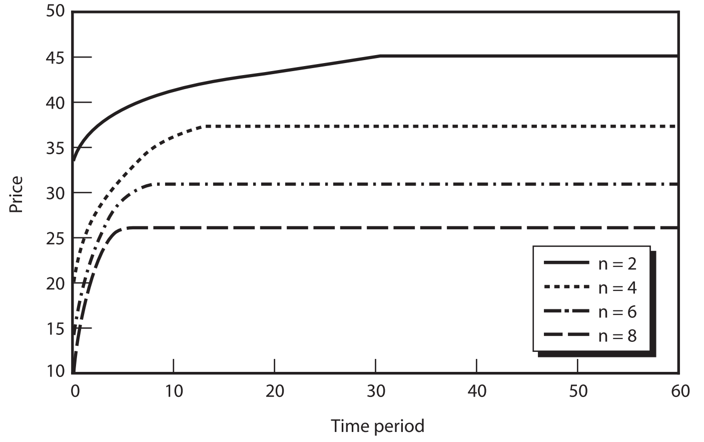


FIGURE 8

EFFECT OF MARKET STRUCTURE ON THE CARTEL PRICE PATH



detection is more likely and damages are larger. Thus, a cartel made up of more-patient firms will raise price slower.

Turning to the impact of market structure, increasing the number of firms has the usual effect of lowering the price that a cartel charges. (Note that the initial price for the cartel, $\hat{P}$, changes with n.) What is interesting, however, is that having more firms results in a shorter transition path. For example, compare a duopoly with the case of four firms. The duopoly raises price from 33 to 45 and takes more than 30 periods to enact this 12-unit price hike. A cartel with four firms raises price from 20 to 37, and this 17-unit price increase is achieved in only 13 periods. Given that a cartel with more firms is starting at a lower price, the increase in profit from a given price hike is greater, which makes the cartel more willing to run the risk of detection. The prediction is then made that a cartel with more firms will raise price more and raise it faster.

5. Possible perverse effects of antitrust laws

■ Having identified some properties of cartel pricing dynamics, the next step is to explore the impact of antitrust laws on the level of cartel prices. Of course, the primary goal of antitrust laws is to deter cartel formation altogether. However, in the event that cartel formation is not deterred, one hopes that antitrust laws will induce the cartel to price lower to reduce the risk of detection and penalties in the event of detection. Furthermore, if the cartel price path is shifted down, then clearly these laws reduce the profitability of forming a cartel—the cartel is induced to price lower, and there is the possibility of penalties—and thus makes it less likely a cartel will form. If, however, antitrust laws induce the cartel to price higher, then it is problematic as to whether these laws are even desirable. Such a possible perverse effect has been noted by Cyrenne (1999). However, for reasons articulated at the end of this section, his model is less than compelling. I then revisit this issue here.

To address the impact of antitrust laws on the cartel price path, the first task is to define the benchmark collusive price in the absence of antitrust laws. If detection considerations are removed, then the model becomes a classical repeated game. In that the unique MPE for that game is infinite repetition of the static Nash equilibrium and given that we use MPE for the punishment in the game with antitrust laws, it is appropriate for the benchmark price to be the highest price supportable by a grim trigger strategy, which I denote $\widetilde{P}$.

Assumption A6. $\widetilde{P}$ exists and is unique, where if

$$\pi(P)/(1-\delta) \geq \overline{\pi}(\psi(P), P) + \delta\left(\frac{\hat{\pi}}{1-\delta}\right) \qquad \forall\, P \in [\hat{P}, P^m],$$

then $\widetilde{P} = P^m$ and otherwise $\widetilde{P} \in [\hat{P}, P^m)$ and is defined by

$$\pi(P)/(1-\delta) \gtreqless \overline{\pi}(\psi(P), P) + \delta(\hat{\pi}/(1-\delta)) \text{ as } P \lesseqgtr \widetilde{P}, \qquad \forall\, P \in [\hat{P}, P^m].$$

It is not difficult to identify assumptions whereby antitrust laws result in lower prices in all periods. Assuming that the probability of detection is fixed will suffice.[23] It is more interesting to consider when antitrust laws can have the perverse effect of raising the prices that the cartel sets. To make for a clean result, let us consider the extreme case when detection depends only on price movements. This is captured by assuming that the baseline probability of detection, which is that associated with the price vector not changing, is zero.

Assumption D1. $\phi : \Omega^{2n} \rightarrow \Re_+$ is continuously differentiable.

Assumption D2. $\phi(P, P) = 0\ \forall P \in \Omega$.

Assumption D3. If $P'' \geq P'$ and $P'' \geq P^0$, then

$$\phi(P'', P') + \phi((P'', \ldots, P^0, \ldots, P''), P'') \geq \phi((P'', \ldots, P^0, \ldots, P''), P').$$

I believe results are robust to minor variations in Assumption D2, and this will be discussed later. Though there is no obviously natural interpretation of Assumption D3, recall from Section 2 that it holds for a general class of probability-of-detection functions.[24]

By Assumption A5, a MPE exists. The following additional property is imposed which holds, for example, when the Bertrand price game is the stage game.

Assumption D4. $V_i^{mpe}(\underline{P}, X)$ is nonincreasing in X and if $\underline{P} \neq (\hat{P}, \ldots, \hat{P})$ and $X > 0$, then

$$\hat{\pi}/(1-\delta) > V_i^{mpe}(\underline{P}, X) \geq \frac{\hat{\pi}}{1-\delta} - \beta X - F.$$

While Assumptions D1–D3 do not imply that the probability of detection is ever positive, such is implicit in Assumption D4. Define $\overline{\Lambda}(P)$ to be the maximal payoff from deviating when the cartel is in a steady state of charging a price of P. This means that P was charged both last period and this period and damages are at their steady-state level of $\gamma x(P)/(1-\beta)$.

$$\begin{aligned}\overline{\Lambda}(P) \equiv \max_{P_i} \overline{\pi}(P_i, P) &+ \delta\phi((P, \ldots, P_i, \ldots, P), P)\left[\frac{\hat{\pi}}{1-\delta} - \beta\left(\frac{\gamma x(P)}{1-\beta}\right) - F\right] \\ &+ \delta[1 - \phi((P, \ldots, P_i, \ldots, P), P)]V_i^{mpe}\left((P, \ldots, P_i, \ldots, P), \frac{\beta\gamma x(P)}{1-\beta}\right).\end{aligned}$$

Note that Assumptions A1–A5 imply that $\overline{\Lambda}(P)$ is defined. In Assumption D5, P^* is defined to be the highest steady-state price path that is IC. By Assumption D2, the steady-state collusive payoff is $\pi(P)/(1-\delta)$.

Assumption D5. P^* exists and is unique where, if

$$\frac{\pi(P)}{1-\delta} \geq \overline{\Lambda}(P) \qquad \forall\, P \in [\hat{P}, P^m],$$

then $P^* = P^m$ and, otherwise, $P^* \in [\hat{P}, P^m)$ and is defined by

[23] A proof is available on request.

[24] Referring to this class, Assumption D3 does not require that $\widetilde{\phi}$ be weakly convex for price increases; it just requires that it be nonincreasing for price decreases and nondecreasing for price increases.

$$\frac{\pi(P)}{1-\delta} \gtreqqless \overline{\Lambda}(P) \text{ as } P \lesseqqgtr P^*, \qquad \forall\, P \in [\hat{P}, P^m].$$

Furthermore, it is straightforward to show that $P^* \geq \widetilde{P}$, and if $P^m > \widetilde{P}$, then $P^* > \widetilde{P}$. It follows from Assumption D4 that

$$\overline{\pi}(\psi(P), P) + \delta\left(\frac{\widehat{\pi}}{1-\delta}\right) > \overline{\Lambda}(P).$$

It is then true that $\pi(\widetilde{P})/(1-\delta) \geq \overline{\Lambda}(\widetilde{P})$, which implies $P^* \geq \widetilde{P}$. If $\widetilde{P} < P^m$, then

$$\frac{\pi(\widetilde{P})}{1-\delta} = \overline{\pi}(\psi(P), P) + \delta\left(\frac{\hat{\pi}}{1-\delta}\right) > \overline{\Lambda}(\widetilde{P})$$

and therefore $P^* > \widetilde{P}$.

Theorem 4 states that the price path is bounded below P^* and converges to it. If ICCs are binding in the absence of antitrust laws, so that $\widetilde{P} < P^m$, then the introduction of antitrust laws causes the cartel to eventually price higher.

Theorem 4. Assume A1–A6 and D1–D5. If $\{\overline{P}^t\}_{t=1}^{\infty}$ is an OSSPE price path, then $\overline{P}^t \leq P^* \;\forall\, t$ and $\lim_{t\to\infty} \overline{P}^t = P^*$.

Proof. See the web Appendix.

Given the prospects of detection, the cartel will tend to gradually raise price so as to reduce the likelihood of triggering suspicions that a cartel has been formed. This could cause the cartel price path to initially lie below $\widetilde{P}$, which is the cartel price in the absence of antitrust laws. Theorem 4 establishes that eventually the cartel will price in excess of $\widetilde{P}$ *because* detection may occur and antitrust laws result in the levying of penalties. For example, suppose the MPE is infinite repetition of the static Nash equilibrium. The post-deviation period is then characterized by firms lowering their prices from some collusive level to $\hat{P}$. This "price war" has associated with it some probability of triggering suspicions that firms may not be competing, leading to an investigation and the levying of costly antitrust penalties. These expected penalties represent an additional cost associated with deviation, which serves to lower the payoff to deviating. Of course, detection can also occur with collusion, which lowers the collusive payoff. However, since $\phi(P, P) = 0$ and the cartel price path eventually settles down, the probability of detection if firms continue colluding is approaching zero, and therefore the collusive payoff is approaching that value which occurs without antitrust laws. In the long run, antitrust laws then cause a loosening of ICCs, which allows the cartel to support prices in excess of $\widetilde{P}$.[25]

As just argued, the assumption that $\phi(P, P) = 0$ means that antitrust penalties have no impact on the collusive payoff in the long run because the probability of detection is converging to zero. However, they do have an impact on the payoff from deviating, since deviation results in price discretely falling, which means a positive probability of detection. If instead $\phi(P, P) > 0$, then the presence of an antitrust authority depresses both the collusive payoff and the payoff from deviating, so its effect on ICCs in the long run is ambiguous. Still, by continuity, Theorem 4 would seem to hold as long as a deviation-induced price war were more likely to generate detection than the stable prices associated with continued collusion. The more general idea is that once parties engage in a conspiracy, detection is often more likely if they discontinue it—resulting in an abrupt change in behavior that might trigger suspicions—than if they continue with the charade. This perverse effect of antitrust policy on cartel pricing may then be quite general.[26]

---

[25] Let me now comment on why I cannot *a priori* dismiss the possibility that an OSSPE path could entail prices in excess of P^m. By pricing above P^m, the cartel may make deviation less profitable, as it could cause the MPE price path to involve bigger price decreases and thus be more likely to induce detection.

[26] For very different reasons, McCutcheon (1997), Fershtman and Pakes (2000), and Athey and Bagwell (2001) identify some perverse effects of antitrust law with respect to price fixing.

In returning to Cyrenne (1999), he modifies the imperfect monitoring model of Green and Porter (1984) by assuming that the transition into a punishment phase entails an additional fixed cost that is interpreted as an antitrust fine. He shows that average price is increasing in the size of the fine. While intriguing, this result is predicated on a less than compelling specification of the detection process. As part of the standard Green-Porter mechanism, the cartel specifies a trigger price such that reversion to the static Nash equilibrium occurs when price falls below it. It is the process of price falling below the trigger price that brings forth cartel detection; no other element of the price series influences detection. If P' is the trigger price, then the probability of detection equals one if firms are colluding in t and $P^t < P'$ and is zero otherwise. This has odd properties. For example, a small change in price can trigger detection—if price goes from being above P' to below P'—while a large change in price (up or down) can avoid detection as long as price remains above P'. Though Cyrenne motivates this specification by the notion that large price movements induce detection, his specification does not appear to capture that idea very well. Nevertheless, his general insight that antitrust penalties can, in some circumstances, make cheating less attractive and thereby support higher collusive prices is on target.

6. Concluding remarks

■ This article has enriched the classical repeated-game model of collusion by taking account of how the manner in which a cartel prices may affect its detection and, in that event, the levying of penalties. Due to the complex way in which detection and penalties influence the conditions for the internal stability of the cartel, there is an array of implications. First, the introduction of antitrust laws can lower the prices set by the cartel but can also allow them to charge higher prices by loosening the incentive-compatibility constraints associated with collusion. Second, while the optimal cartel price path is increasing when incentive-compatibility constraints are not binding, when they do bind, the properties of the path depend on whether those constraints are loosening or tightening over time. When penalties are fixed, collusion becomes easier over time, and this results in the price path being increasing. When penalties are endogenous but the probability of detection is fixed, collusion becomes more difficult over time as penalties accumulate. As a result, the cartel price path is decreasing over time, after initially being raised right after cartel formation. Combining these two dynamics, numerical analysis identifies two possible paths: (i) the cartel gradually raises price and it converges to a steady state, and (ii) the cartel gradually raises price but, after some point, lowers price and it converges to a steady state. While there are many actual cases in which the cartel initially engages in a gradual price rise, it is a much more subtle issue whether price converges on a steady state and whether it declines in doing so. The degree to which those results are consistent with the evidence will have to await careful empirical analysis.

This is a rich area for further investigation. The focus of this article has been on detection through the change in a common price, being motivated by the potentially suspicious nature of price increases. Another source of suspicion is parallel behavior by firms. One can also explore how cartel stability and detection are affected by corporate leniency programs that allow the first cartel member to report to avoid government penalties (though not damages). The most challenging direction is to model the role of buyers so as to endogenize the detection process. The current article has focused its energy on deriving the pricing implications of a simple and exogenous detection technology. It is important, however, to make progress in modelling detection. If the objective is to understand how cartels price in the current environment, I do not recommend modelling detection from an equilibrium perspective in which buyers or the antitrust authority are players in a game trying to detect cartels. Because antitrust authorities do not engage in detection, assuming they do is a nonstarter. While buyers have indeed detected cartels, an equilibrium approach requires that buyers have a good idea of how a cartel prices. Such an assumption is highly problematic. In my opinion, the challenge is to model how buyers become suspicious about a cartel having formed without assuming they know exactly how a cartel prices. Research is currently in progress along those lines.

Appendix

■ **Numerical method.** For numerical analysis, the state space is Δ^*, which is a discretized version of $\Delta \equiv [\hat{P}, P^m] \times [0, \gamma x(P^m)/(1-\beta)]$. For all of the numerical results reported here, it is assumed that Δ^* is 30×30 and thus has 900 states. The numerical method involves two stages: (i) solving for a MPE for the post-deviation game and (ii) solving the cartel's problem. As U.S. law has a statute of limitations with respect to antitrust violations, I assume that penalties can only be levied if detection occurs within T periods of the cartel's dissolution. This allows a MPE to be solved through backward induction. Let $W^{\tau}(f', X')$ denote a firm's MPE payoff in the τth period after a deviation given a lagged average transaction price of f' and damages of X'. As $W^{T+1}(f, X) = \hat{\pi}/(1-\delta)$, the post-deviation period T symmetric equilibrium price is defined by

$$\begin{aligned}\widetilde{P}^T \in \arg\max_{P_i} \overline{\pi}\left(P_i, \widetilde{P}^T\right) + \delta\phi\left(f\left(\widetilde{P}^T, \ldots, P_i, \ldots, \widetilde{P}^T\right), f'\right)\left[\frac{\hat{\pi}}{1-\delta} - \beta X' - F\right] \\ + \delta\left[1 - \phi\left(f\left(\widetilde{P}^T, \ldots, P_i, \ldots, \widetilde{P}^T\right), f'\right)\right]\frac{\hat{\pi}}{1-\delta}.\end{aligned}$$

Using the first-order condition, it is possible to derive a closed-form solution for $\widetilde{P}^T$ with which one can derive $W^T(f', X')$. This is done for each $(f', X') \in \Delta^*$. Using a Chebychev polynomial to interpolate, the evaluation of $W^T(f', X')$ is extended to Δ. Interpolation involves 20 basis functions and an equal number of interpolation nodes. The $T - 1^{st}$ post-deviation equilibrium price is defined by:

$$\begin{aligned}\widetilde{P}^{T-1} \in \arg\max_{P_i} \overline{\pi}\left(P_i, \widetilde{P}^{T-1}\right) + \delta\phi\left(f\left(\widetilde{P}^{T-1}, \ldots, P_i, \ldots, \widetilde{P}^{T-1}\right), f'\right)\left[\frac{\hat{\pi}}{1-\delta} - \beta X' - F\right] \\ + \delta\left[1 - \phi\left(f\left(\widetilde{P}^{T-1}, \ldots, P_i, \ldots, \widetilde{P}^{T-1}\right), f'\right)\right] W^T\left(f\left(\widetilde{P}^{T-1}, \ldots, P_i, \ldots, \widetilde{P}^{T-1}\right), \beta X'\right).\end{aligned}$$

As the first-order condition does not have a closed-form solution, I solve it using the bisection method starting with bounds of $\hat{P}$ and f' (as one can show that the MPE price cannot be higher than f'). Note that this method requires not only a good approximation of the post-deviation value function but also its derivative. To make sure the approximation is a good one, I compare the solution with that derived using exhaustive search of the price space (which does not rely on approximating the derivative), for several parameter configurations. The two solutions are very close. Solving for the symmetric equilibrium price for all states in Δ^*, interpolation is used again to derive $W^{T-1}(f', X') \; \forall \; (f', X') \in \Delta$. Iterating this process ultimately leads to the derivation of $W^1(f', X')$, which is exactly $V^{mpe}(f', X')$.

Given the MPE payoff function, the remaining problem is a single-agent constrained dynamic programming problem:

$$\begin{aligned}V\left(P^{t-1}, X^{t-1}\right) = \max_{P \in \Omega^*} \pi(P) + \delta\phi\left(P, P^{t-1}\right)\left[\frac{\hat{\pi}}{1-\delta} - \beta X^{t-1} - \gamma x(P) - F\right] \\ + \delta\left[1 - \phi\left(P, P^{t-1}\right)\right] V\left(P, \beta X^{t-1} + \gamma x(P)\right) \qquad \text{(B1)}\end{aligned}$$

subject to

$$\begin{aligned}\pi(P) + \delta\phi\left(P, P^{t-1}\right)\left[\frac{\hat{\pi}}{1-\delta} - \beta X^{t-1} - \gamma x(P) - F\right] + \delta\left[1 - \phi\left(P, P^{t-1}\right)\right] V\left(P, \beta X^{t-1} + \gamma x(P)\right) \\ \geq \max_{P_i \in \Omega^*} \overline{\pi}(P_i, P) + \delta\phi\left(f(P, \ldots, P_i, \ldots, P), P^{t-1}\right)\left[\frac{\hat{\pi}}{1-\delta} - \beta X^{t-1} - F\right] \\ + \delta\left[1 - \phi\left(f(P, \ldots, P_i, \ldots, P), P^{t-1}\right)\right] V^{mpe}\left(f(P, \ldots, P_i, \ldots, P), \beta X^{t-1}\right). \qquad \text{(B2)}\end{aligned}$$

Ω^* is a discretized version of Ω and contains 100 equidistant prices from $[\hat{P}, P^m]$. (B1)–(B2) is solved through function iteration with a discretized state space of 30×30. The value function is approximated by a linear spline with 30 basis functions and an equal number of interpolation nodes.[27] An initial value function is specified, for which the above constrained optimization problem was solved.[28] This produces new values for each state in Δ^*. Interpolation using a linear spline then produces a new value function defined on Δ. This process is iterated until convergence is achieved, where the criterion is the norm of the difference of the coefficient vectors between iterations is less than 5×10^{-10}. For purposes of comparison, the same process is run on the unconstrained dynamic programming problem as defined by (B1).

[27] As this numerical method does not require approximation of the derivative of the value function, I use the linear spline rather than the Chebychev polynomial.

[28] The initial coefficients for the linear spline are set at 10,000, resulting in the initial value function well exceeding the present value of the unconstrained joint profit maximum. Thus, convergence occurs from above. This is important, since if the initial value function is set too low, it could converge to $V^{mpe}(\cdot)$ or there may not exist any price that satisfies (A28). Note that this operator on the value function is not assured of being a contraction mapping.

References

Athey, S. and Bagwell, K. "Optimal Collusion with Private Information." *RAND Journal of Economics*, Vol. 32 (2001), pp. 428–465.

Baker, J.B. "Private Information and the Deterrent Effect of Antitrust Damage Remedies." *Journal of Law, Economics and Organization*, Vol. 4 (1988), pp. 385–408.

Besanko, D. and Spulber, D.F. "Antitrust Enforcement Under Asymmetric Information," *Economic Journal*, Vol. 99 (1989), pp. 408–425.

——— and ———. "Are Treble Damages Neutral? Sequential Equilibrium and Private Antitrust Enforcement." *American Economic Review*, Vol. 80 (1990), pp. 870–887.

Block, M.K., Nold, F.C., and Sidak, J.G. "The Deterrent Effect of Antitrust Enforcement." *Journal of Political Economy*, Vol. 89 (1981), pp. 429–445.

Christie, W.G. and Schultz, P.H. "Why Do NASDAQ Market Makers Avoid Odd-Eighth Quotes?" *Journal of Finance*, Vol. 49 (1994), pp. 1813–1840.

Connor, J.M. *Global Price Fixing: Our Customers are the Enemy*. Boston: Kluwer Academic, 2001.

Cyrenne, P. "On Antitrust Enforcement and the Deterrence of Collusive Behaviour." *Review of Industrial Organization*, Vol. 14 (1999), pp. 257–272.

Fersthman, C. and Pakes, A. "A Dynamic Oligopoly with Collusion and Price Wars." *RAND Journal of Economics*, Vol. 31 (2000), pp. 207–236.

Fudenberg, D. and Tirole, J. *Game Theory*. Cambridge, Mass.: MIT Press, 1991.

Green, E.J. and Porter, R.H. "Noncooperative Collusion Under Imperfect Price Information." *Econometrica*, Vol. 52 (1984), pp. 87–100.

Harrington, J.E., Jr. "Collusion and Predation Under (Almost) Free Entry." *International Journal of Industrial Organization*, Vol. 7 (1989), pp. 381–401.

———. "Some Implications of Antitrust Laws for Cartel Pricing." *Economics Letters*, Vol. 79 (2003), pp. 377–383.

———. "Optimal Cartel Pricing in the Presence of an Antitrust Authority." *International Economic Review*, forthcoming.

Hay, G.A. and Kelly, D. "An Empirical Survey of Price Fixing Conspiracies." *Journal of Law and Economics*, Vol. 17 (1974), pp. 13–38.

LaCasse, C. "Bid Rigging and the Threat of Government Prosecution." *RAND Journal of Economics*, Vol. 26 (1995), pp. 398–417.

Levenstein, M. and Suslow, V. "Private International Cartels and Their Effect on Developing Countries." Unpublished background paper for the World Bank *World Development Report 2001*, January 9, 2001.

McAnney, J.W. "The Justice Department's Crusade Against Price-Fixing: Initiative or Reaction?" *Antitrust Bulletin*, Fall 1991, pp. 521–542.

McCutcheon, B. "Do Meetings in Smoke-Filled Rooms Facilitate Collusion?" *Journal of Political Economy*, Vol. 105 (1997), pp. 330–350.

Motta, M. and Polo, M. "Leniency Programs and Cartel Prosecution." *International Journal of Industrial Organization*, Vol. 21 (2003), pp. 347–379.

Rey, P. "Towards a Theory of Competition Policy." In M. Dewatripont, L.P. Hansen, and S.J. Turnovsky, eds., *Advances in Economics and Econometrics: Theory and Applications*, Eight World Congress. New York: Cambridge University Press, 2003.

Salant, S.W. "Treble Damage Awards in Private Lawsuits for Price Fixing." *Journal of Political Economy*, Vol. 95 (1987), pp. 1326–1336.

Schinkel, M.P. and Tuinstra, J. "Imperfect Antitrust Enforcement." Mimeo, Universiteit Maastricht,s November 2002.

Souam, S. "Optimal Antitrust Policy Under Different Regimes of Fines." *International Journal of Industrial Organization*, Vol. 19 (2001), pp. 1–26.

Spagnolo, G. "Optimal Leniency Programs." Fondazione Eni Enrico Mattei, Note di Lavoro no. 42/2000.

Spiller, P.T. "Treble Damages and Optimal Suing Time." *Research in Law and Economics*, Vol. 9 (1986), pp. 45–56.

Vives, X. *Oligopoly Pricing: Old Ideas and New Tools*, Cambridge, Mass.: MIT Press, 1999.