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Endogenous cartel formation with heterogeneous firms

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In the context of an infinitely repeated capacity-constrained price game, we endogenize the composition of a cartel when firms are heterogeneous in their capacities. When firms are sufficiently patient, there exists a stable cartel involving the largest firms. A firm with sufficiently small capacity is not a member of any stable cartel. When a cartel is not all-inclusive, colluding firms set a price that serves as an umbrella with non-cartel members pricing below it and producing at capacity. Contrary to previous work, our results suggest that the most severe coordinated effects may come from mergers involving moderate-sized firms, rather than the largest or smallest firms.

1. Introduction

■ A common assumption in the theory of collusion is that all firms participate in the cartel. Although clearly there are instances in which cartels are indeed all-inclusive, it is probably more common for a cartel to be lacking some sellers.[1] For example, the global citric acid cartel of the early to mid-1990s comprised Archer Daniels Midland, Cerestar Bioproducts, Haarman & Reimer, Hoffman La Roche, and Jungbunzlauer, who, at the time of cartel formation, encompassed 60% of global production and 67% of EU production. Of particular note is that Chinese suppliers were not part of the citric acid cartel. The exclusion of Chinese suppliers also occurred with cartels in vitamins B1, B2, and C. For vitamin B1, the increase in Chinese supply during the first 3 years of

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[1] The ensuing examples are from Harrington (2006) and are based on information from European Commission decisions over 2000–2004.

collusion resulted in the cartel's global market share declining from 70% to 52%. For vitamin B2, the U.S. producer Coors was also noticeably absent from the cartel. In the 13-year-long European industrial tubes cartel, the cartel controlled about 75%–85% of total production and excluded at least two significant producers. In the Danish district heating pipes cartel, the Swedish firm Powerpipe, which was a sizable competitor, chose not to join the cartel. There are many other examples in which a significant amount of supply was provided by firms who did not participate in the cartel.[2]

The recognition that a cartel need not encompass all firms generates a number of interesting questions. Under what industry conditions can we expect a cartel to be all-inclusive? When a cartel is not all-inclusive and firms are heterogeneous, what are the traits of those firms that join the cartel? How does a merger affect the composition and size of a cartel? Once the composition of the cartel is endogenized, more traditional issues—such as the determinants of the cartel price and properties of cartel price paths—could also be affected.

The objective of this article is to address these and related questions by endogenizing cartel formation in the context of an infinitely repeated price game with homogeneous goods where firms are heterogeneous in their capacities. For a given composition, a cartel is assumed to achieve the best collusive outcome while respecting incentive compatibility constraints (which ensure stability of that outcome) and taking account of the behavior of firms outside the cartel. We then focus on the set of stable cartels where a cartel is stable when all cartel members prefer to be in the cartel and all non-cartel members prefer to be outside the cartel. Although there has been previous work that endogenizes cartel membership, which we briefly review below, our model is the first to endogenize the composition of a cartel in the context of an infinitely repeated game with heterogeneous firms.

In summarizing some of our findings, we show that stable cartels are often not all-inclusive. When a cartel does not encompass all firms, equilibrium has the cartel setting a price which serves as an umbrella for the non-cartel members in that they price just below the cartel price. Non-cartel members produce at capacity, and cartel members produce below capacity. In exploring the incentives associated with joining a cartel, a firm faces a tradeoff. By becoming a member of the cartel, more capacity is brought under the control of the cartel, which leads to a higher cartel price. Hence, a firm benefits from a higher price-cost margin by joining the cartel. The downside is that it is forced to reduce its sales; it goes from producing at capacity to below capacity. A firm finds it optimal not to join the cartel when its capacity is sufficiently low because the effect of its membership on price is trivial but, at the same time, it experiences a nontrivial reduction in its output. Thus, we should not expect a cartel to include very small firms.

Toward understanding general properties of stable cartels, a characterization result is provided. If firms are sufficiently patient, then there always exists a stable cartel comprising the largest firms in the industry. To further make the case that we can expect larger firms to be in cartels, we show that if a firm finds it optimal to join a cartel then any larger firm also finds it optimal, whereas if a firm finds it optimal not to join a cartel then any smaller firm also finds it better to be outside the cartel. Although there can be stable cartels that do not comprise the largest firms, there is much to argue that we should expect cartels to be made up of the largest firms. Some additional results related to mergers are reviewed below after discussing the literature.

Previous research has endogenized cartel membership and found cartels to be less than all-inclusive but, with one recent exception, all that work has been conducted in a static framework. Early pioneering work includes Selten (1973), d'Aspremont et al. (1983), Donsimoni (1985), and Donsimoni, Economides, and Polemarchakis (1986). Although this work has been highly useful, it is subject to the criticism that it presumes price is set to maximize joint cartel profit and thus does not satisfy the incentive compatibility constraints ensuring cartel stability.

[2] Hay and Kelly (1974), for instance, analyze 65 cartel cases in the United States between 1963 and 1972. They report market shares of cartels in 45 cases, and in approximately two thirds of these cases the cartel was not all-inclusive. See also Griffin (1989), who studies 54 well-known international cartels, 53 of which were incomplete.

Within the infinitely repeated game framework, Compte, Jenny, and Rey (2002) and Vasconcelos (2005) also consider collusion when firms are heterogeneous in terms of capital stocks, although they make the standard assumption of an all-inclusive cartel. As in our model, Compte et al. (2002) consider the homogeneous goods capacity-constrained price game when firms have different capacity stocks. They explore the impact of the distribution of capacity on the minimum discount factor for sustaining the joint profit maximum. When the largest firm is not too large, the minimum discount factor depends only on the level of aggregate capacity and not on how it is distributed across firms. Thus, marginal reallocations of capacity across firms has no effect on the ease of collusion. When instead the largest firm is sufficiently large (more specifically, the aggregate capacity of all firms but the largest firm is insufficient to meet market demand at the competitive price), the minimum discount factor for the cartel is determined by the capacity of the largest firm as it has the strongest incentive to cheat. Hence, shifting capacity from the largest firm to the other firms makes collusion easier.

Vasconcelos (2005) considers the homogeneous goods quantity game in which more capital reduces marginal cost for a convex cost function, and investigates, like Compte et al. (2002), the determinants of the minimum discount factor for sustaining the joint profit maximum. The minimum discount factor depends on the capacity of the largest firm but, contrary to Compte et al. (2002), also depends on the capacity of the smallest firm. In fact, it is the smallest firm that has the greatest incentive to deviate from the collusive outcome, whereas it is the largest firm that has the greatest incentive to deviate from the punishment (which is a most severe punishment). Thus, collusion is easier when capital is transferred from the largest firm to the smallest firm; in that sense, less asymmetry is conducive to collusion.

With the assumption of an all-inclusive cartel, Compte et al. (2002) and Vasconcelos (2005) then find that it is the capital stocks of the extremal firms that matter. This is not the case in our model when cartel membership is endogenous and incomplete. To begin, sufficiently small firms are not part of the cartel and thus modest changes in their capacity have no effect. Furthermore, shifts in capacity among firms who would be members of the cartel anyway also do not have an effect.[3] Rather, our analysis suggests that it is the reallocation of capacity among moderate-sized firms that is likely to have the most significant coordinated effects. For example, the merger of two moderate-sized firms could induce them to join the cartel when, in the pre-merger scenario, they would choose to be outside the cartel.

There is a pair of recent articles which do endogenize cartel composition in the context of an infinitely repeated game. Escrihuela-Villar (2008a, 2009) examines an infinitely repeated quantity setting with identical firms and homogeneous goods.[4] Similar to Compte et al. (2002) and Vasconcelos (2005), the collusive outcome is assumed to be the joint profit maximum (given the cartel size). The main result is that firms only have an incentive to form the smallest sustainable cartel. Firms have no incentive to form a larger cartel because outsiders earn more profits than insiders. However, cartel members of the smallest sustainable cartel have no incentive to become a fringe member as leaving the cartel makes collusion unfeasible. As it is assumed there are no capital stocks and firms are identical, our analysis is complementary in that we focus on the traits of those firms that make up stable cartels. Moreover, in our setting, firms might have an incentive to form a larger cartel than is required to sustain some collusion.

In the next section, the model is described. In Section 3, the equilibrium pricing behavior of the non-cartel members is characterized. The results of Section 3 are then embodied in Section 4, where a cartel's pricing problem is modelled and solved. That analysis takes as given a particular composition to the cartel. This composition is endogenized in Section 5, and we explore the characteristics of stable cartels. In doing so, we assume a proportional sharing rule among cartel

[3] This property also holds for Compte et al. (2002) when the aggregate capacity of all firms but one is sufficient to meet demand at the competitive price, which is an assumption we make throughout the article.

[4] Escrihuela-Villar (2008b) derives some implications of mergers when the cartel is partial, but in that paper, cartel size is exogenous.

members. In Appendix A, we provide a justification for it based on fairness. Section 6 investigates the coordinated effects of a merger. In Section 7, we discuss some alternative definitions of cartel stability and alternative allocation rules. Section 8 concludes. All proofs are in Appendix B.

2. Model

■ There are n firms competing in an infinitely repeated capacity-constrained price game with homogeneous products. Capacity stocks are fixed and, at any moment, the entire history is common knowledge so there is perfect monitoring. $\delta \in (0, 1)$ is the common firm discount factor and each firm's payoff is the expected present value of its profit stream.

The market demand function is $D(p)$, which is assumed to be a twice continuously differentiable and decreasing function of price. (It can have a choke price and thus be zero for prices at or above the choke price.) Firms have a common marginal cost $c \geq 0$. Assume $D(c) > 0$ and monopoly profit, $(p - c)D(p)$, is strictly concave. Let p^m denote the monopoly price: $D(p^m) + (p^m - c)D'(p^m) = 0$.

Firms simultaneously make price decisions and then a firm produces to meet its demand up to capacity. Let k_i denote the capacity of firm i and $D_i(p_i, p_{-i})$ denote its demand given its price, p_i, and the vector of other firms' prices, p_{-i}. Two assumptions are made on firm demand, both of which are quite general. In stating these assumptions, let $\Omega(p) \equiv \{j : p_j = p\}$ denote the set of firms that price at p and define $p_{-i}^{\min} \equiv \min\{p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n\}$.

$$\text{A1} \quad \lim_{\eta \to 0^+} D_i\left(p_{-i}^{\min} + \eta, p_{-i}\right) = \max\left\{D\left(p_{-i}^{\min}\right) - \sum_{j \in \Omega(p_{-i}^{\min})} k_j, 0\right\}.$$

$$\text{A2} \quad \text{If } 0 < \sum_{i \in \Omega(p)} D_i(p_i, p_{-i}) < \sum_{i \in \Omega(p)} k_i \text{ then } 0 < D_i(p_i, p_{-i}) < k_i, \ \forall i \in \Omega(p).$$

A1 holds for any well-behaved residual demand function.[5] A2 imposes some symmetry across firms. It says that if the firms that charge a common price have, in sum, excess capacity, then residual demand is allocated so that all firms have excess capacity; and if residual demand is positive, then they all have positive demand.

To simplify the analysis, some plausible though restrictive assumptions are placed on firms' capacities. It is assumed that each firm has insufficient capacity to supply the monopoly demand, and any $n - 1$ firms can meet competitive demand.

$$\text{A3} \quad k_i < D(p^m) \quad \text{and} \quad \sum_{j \neq i} k_j \geq D(c), \ \forall i.$$

The well-known implication of the second part of A3 is that the competitive solution is a stage game Nash equilibrium. The first part has the implication that, when it has the lowest price (subject to the price not exceeding the monopoly price), a firm's demand exceeds its capacity which implies it will produce at capacity. A3 then requires that the largest firm is not too large. This property will significantly simplify the analysis and seems plausible for many markets.

Although A3 implies $n \geq 3$, this implication is not restrictive because the focus of our research is on endogenizing membership in a cartel and the analysis only becomes relevant when there are at least three firms. To further explore the implications of A3, suppose demand is linear:

[5] As noted by Davidson and Deneckere (1986), if firm i prices at $p_{-i}^{\min} + \eta$ and all firms other than those in $\Omega(p_{-i}^{\min})$ price above firm i—which holds as $\eta \to 0$—then firm i's demand curve is bounded as follows:

$$D\left(p_{-i}^{\min} + \eta\right) - \sum_{j \in \Omega\left(p_{-i}^{\min}\right)} k_j \leq D_i\left(p_{-i}^{\min} + \eta, \underline{p}_{-i}\right) \leq D\left(p_{-i}^{\min}\right) - \sum_{j \in \Omega\left(p_{-i}^{\min}\right)} k_j.$$

Letting $\eta \to 0$, these bounds imply

$$D_i\left(p_{-i}^{\min}, \underline{p}_{-i}\right) = D\left(p_{-i}^{\min}\right) - \sum_{j \in \Omega\left(p_{-i}^{\min}\right)} k_j.$$

$D(p) = a - bp$ where $a - bc > 0$. Because $p^m = \frac{a+bc}{2b}$ then $k_i < D(p^m)$ means $k_i < \frac{a-bc}{2}$. Hence, A3 requires

$$\sum_{j \neq i} k_j \geq a - bc \quad \text{and} \quad k_i < \frac{a - bc}{2} \; \forall i.$$

Summing up the latter condition across all firms but i and combining with the first condition, we have

$$\frac{(n-1)(a-bc)}{2} > \sum_{j \neq i} k_j \geq a - bc.$$

A necessary condition for A3 to be satisfied with linear demand is then

$$\frac{(n-1)(a-bc)}{2} > a - bc \Leftrightarrow n > 3.$$

Hence, if demand is linear, then a necessary condition for A3 to be true is that there are at least four firms. It is clear that A3 comes with some loss of generality, but is likely to hold for many industries.

Finally, for technical reasons, the set of feasible prices is assumed to be countable with an increment of $\varepsilon > 0$. A firm then chooses its price from the set $\{0, \varepsilon, \ldots, c - \varepsilon, c, c + \varepsilon, \ldots\}$. Results will be derived for when ε is small.

3. Static Nash equilibrium for non-cartel members

■ In sustaining collusion, we will focus on equilibrium strategy profiles with several properties. First, it results in a stationary collusive outcome. Second, any deviation from the collusive price by a cartel member results in infinite reversion to a static Nash equilibrium. Because $\sum_{j \neq i} k_j \geq D(c) > 0 \; \forall i$, the static game has two symmetric Nash equilibria: one has all firms price at cost, and the other has all firms price at $c + \varepsilon$. As results will be characterized when ε is small, there is not a substantive difference between these equilibria. Also note that, as $\varepsilon \to 0$, the punishment results in the lowest continuation equilibrium payoff. Third, past behavior by non-cartel members has no effect on cartel members' current behavior.

In this section, we characterize the pricing behavior of non-cartel members in the presence of a single cartel. Because a non-cartel member's price does not affect its continuation payoff, in equilibrium a non-cartel member necessarily chooses price to maximize current profit. Thus, non-cartel members are assumed to achieve a static Nash equilibrium while taking as given the common price set by cartel members. Implicitly, all firms—including those not in the cartel—are then aware that there is a cartel.

We want to focus on cartels and collusive prices whereby the cartel earns more than it would if it did not collude. This requires that the collusive price exceeds $c + \varepsilon$ and, at the collusive price, the cartel has positive demand. In that case, Lemma 1 shows that each non-colluding firm's equilibrium price puts zero mass on prices equal to or above the price charged by the cartel. Thus, non-cartel members undercut the collusive price. Let Γ denote the set of firms in the cartel. All proofs are in Appendix B.

Lemma 1. Assume cartel Γ prices at $p' > c + \varepsilon$ and non-colluding firms set Nash equilibrium prices. If the cartel earns positive profit then, for sufficiently small ε, each non-colluding firm prices below p' with probability one.

Suppose $p'(> c + \varepsilon)$ is a collusive price that, given the non-colluding firms price optimally, yields positive profit for the cartel. By Lemma 1, if $\sum_{j \notin \Gamma} k_j \geq D(p')$ then cartel demand and profit are zero, which contradicts p' yielding positive cartel profit. Thus, if p' is a collusive price that yields positive profit for the cartel then it must be true that $D(p') > \sum_{j \notin \Gamma} k_j$, so there is residual demand for the cartel. Given that property, Lemma 2 shows that an equilibrium has the non-colluding firms all pricing just below the cartel price.

Lemma 2. If cartel Γ prices at $p' > c + \varepsilon$ and $D(p') > \sum_{j \notin \Gamma} k_j$ then, for sufficiently small ε, the unique Nash equilibrium has non-colluding firm j price at $p' - \varepsilon$ and sell k_j units, $\forall j \notin \Gamma$.

To summarize, necessary conditions for the cartel to earn higher profit than at a static Nash equilibrium are that the cartel price p' exceeds $c + \varepsilon$ and the non-cartel members have insufficient capacity to meet demand at the cartel price, $D(p') > \sum_{j \notin \Gamma} k_j$. When the price grid is sufficiently fine, the equilibrium response of the non-cartel members is to just undercut the collusive price with a price of $p' - \varepsilon$. As this leaves residual demand to the cartel of $D(p') - \sum_{j \notin \Gamma} k_j$, total cartel profit is then

$$(p' - c)\left[D(p') - \sum_{j \notin \Gamma} k_j\right].$$

To derive precise predictions, it is critical to perform an equilibrium selection. In particular, multiplicity of equilibrium outcomes is rampant because of the many ways that cartel demand can be allocated among its members. We will assume that demand is allocated within the cartel proportional to a firm's capacity so that colluding firm i's profit is

$$(p' - c)\left[D(p') - (K - K_\Gamma)\right]\left(\frac{k_i}{K_\Gamma}\right),$$

where $K_\Gamma \equiv \sum_{j \in \Gamma} k_j$ and $K \equiv \sum_{j=1}^{n} k_j$. There are two primary reasons for making this specification. First, this allocation rule has occurred in practice; for example, it was used by the Norwegian cement cartel (Röller and Steen, 2006) and several German cartels during the early 20th century (Bloch, 1932; cited in Scherer, 1980). Also see Vasconcelos (2005), who assumed this allocation rule and cites some other cartels that used it. Second, this sharing rule can be derived by applying the notion of fairness prescribed by Rawls (1971) in *A Theory of Justice*. The derivation is provided in Appendix A, which is best read after reading Section 4. We also explore the robustness of our results to the allocation rule in Section 7.

4. Equilibrium price and cartel size

■ Given a cartel Γ and a cartel price p, cartel member $i \in \Gamma$ earns a current profit of

$$(p - c)\left[D(p) - (K - K_\Gamma)\right]\left(\frac{k_i}{K_\Gamma}\right).$$

The collusive value for firm i is then

$$V_i(p, \Gamma) \equiv \left(\frac{1}{1-\delta}\right)(p - c)\left[D(p) - (K - K_\Gamma)\right]\left(\frac{k_i}{K_\Gamma}\right) = k_i V(p, \Gamma),$$

where

$$V(p, \Gamma) \equiv \left(\frac{1}{1-\delta}\right)(p - c)\left[\frac{D(p) - (K - K_\Gamma)}{K_\Gamma}\right]$$

is the present value of the profit stream per unit of capacity for a cartel member. Note that a cartel member's profit depends only on its capacity and the amount of capacity in the cartel; in particular, it does not matter how the capacity is distributed among the other cartel members or among the non-cartel members.

Recall that the collusive outcome is to be supported by the threat of infinite reversion to a static Nash equilibrium that yields (approximately) zero profit. To derive the incentive compatibility constraint (ICC) for cartel member i, first note that a cartel member that deviates from the collusive price ought to maximize current profit, given the other colluding firms are pricing at p and the non-colluding firms are pricing at $p - \varepsilon$. Consider a cartel member who deviates by pricing at $p - \varepsilon$. If it has excess capacity at that price then, if ε is sufficiently small, it will prefer to price at $p - 2\varepsilon$ rather than $p - \varepsilon$ as doing so leads to a discrete increase in demand. Next note that pricing at $p - 2\varepsilon$ results in it being capacity constrained as $D(p^m) > k_i$ and $p - 2\varepsilon \leq p^m$ imply $D(p - 2\varepsilon) > k_i$. Hence, it will not want to price any lower than $p - 2\varepsilon$. Thus, if the deviant firm is not capacity constrained when pricing at $p - \varepsilon$, it will optimally price at $p - 2\varepsilon$ and earn a profit of $(p - 2\varepsilon - c)k_i$. If instead it is capacity constrained at $p - \varepsilon$ then that price is clearly optimal and its profit from cheating is $(p - \varepsilon - c)k_i$. As we are making ε small, the profit from cheating is approximately $(p - c)k_i$.

The ICC for firm $i \in \Gamma$ is then

$$k_i V(p, \Gamma) \geq (p - c)k_i \quad \text{or} \quad V(p, \Gamma) \geq (p - c).$$

The convenient property to note is that all cartel members have the same ICC, even though they may have different capacities. Furthermore, as cartel member i's collusive value is $k_i V(p, \Gamma)$, all cartel members agree that they want price chosen to maximize $V(p, \Gamma)$. These properties are an implication of assuming that a firm's share of cartel supply equals its share of cartel capacity.

Given a particular cartel (as defined by Γ), the cartel's problem is to choose a price that maximizes the representative firm's value while satisfying the ICC. Because the optimal cartel price depends on the members of the cartel only through the capacity that the cartel controls, K_Γ, we will reflect the dependence of the optimal cartel price on K_Γ:

$$p^*(K_\Gamma) = \arg\max V(p, \Gamma) \quad \text{subject to} \quad V(p, K_\Gamma) - (p - c) \geq 0.$$

Note that $p^*(K_\Gamma)$ maximizes aggregate cartel profits subject to the incentive compatibility constraints. Using the particular expressions, we have

$$p^*(K_\Gamma) = \arg\max \left(\frac{1}{1-\delta}\right)(p - c)\left(\frac{D(p) - (K - K_\Gamma)}{K_\Gamma}\right)$$

subject to

$$D(p) \geq K - \delta K_\Gamma.$$

Define $\widehat{p}(K_\Gamma)$ as the maximum price that satisfies the ICC:

$$\widehat{p}(K_\Gamma) \equiv D^{-1}(K - \delta K_\Gamma).$$

Because the objective is strictly concave,

$$\frac{\partial^2 V}{\partial p^2} = \frac{2D'(p) + (p - c)D''(p)}{(1-\delta)K_\Gamma} < 0,$$

the first-order condition suffices to define the unconstrained solution. Define $p^o(\Gamma)$ as the optimal price in that case:

$$D(p^o) - (K - K_\Gamma) + (p^o - c)D'(p^o) = 0.$$

Because $V(p, \Gamma)$ is strictly concave in p then

$$p^*(K_\Gamma) = \min\{p^o(K_\Gamma), \widehat{p}(K_\Gamma)\}.$$

Finally, define $V^*(\Gamma)$ as the equilibrium collusive value for cartel Γ:

$$V^*(K_\Gamma) \equiv V(p^*(K_\Gamma), \Gamma) = \left(\frac{1}{1-\delta}\right)[p^*(K_\Gamma) - c]\left[\frac{D(p^*(K_\Gamma)) - (K - K_\Gamma)}{K_\Gamma}\right].$$

In exploring the effect of cartel capacity on price, first note that if $D(c) \leq K - \delta K_\Gamma$ then there does not exist $p > c$ such that $D(p) \geq K - \delta K_\Gamma$; that is, there is no price exceeding cost which satisfies the ICC. Hence, if $\delta \leq \frac{K - D(c)}{K_\Gamma}$ then $p^*(K_\Gamma) = c$. If $D(c) > K - \delta K_\Gamma$ then there exists $p > c$ such that $D(p) \geq K - \delta K_\Gamma$, from which we conclude: if $\delta > \frac{K - D(c)}{K_\Gamma}$ then $p^*(K_\Gamma) > c$.

Theorem 3 shows that when the cartel is more inclusive—as reflected in more capacity being controlled by the cartel—then the cartel price is higher. Because non-cartel members price just below the cartel price, all firms' prices are higher when the cartel is more encompassing. Note that industry capacity K is kept fixed, so more cartel capacity means that the cartel controls a larger share of fixed industry capacity.

Theorem 3. If $K_{\Gamma'} > K_{\Gamma''}$ then: (i) $p^*(K_{\Gamma'}) \geq p^*(K_{\Gamma''})$; and (ii) if $p^*(K_{\Gamma'}) > c$ then $p^*(K_{\Gamma'}) > p^*(K_{\Gamma''})$.

5. Cartel formation

■ Because a cartel Γ is able to support a price in excess of the competitive price if and only if $D(c) > K - \delta K_{\Gamma}$, this section will focus on δ and Γ such that $D(c) > K - \delta K_{\Gamma}$ or, equivalently, $\delta > \frac{K-D(c)}{K_{\Gamma}}$. This condition is assured of holding when δ is sufficiently close to one and Γ is sufficiently inclusive (as, by assumption, $K > D(c)$ and thus $\frac{K-D(c)}{K} < 1$).

In exploring the endogeneity of participation in a cartel, there are two issues to consider: whether a firm wants to be a member of a cartel and whether the existing members of a cartel would want a firm to join. Let us begin by showing that any existing cartel desires to be more inclusive; thus, membership in the cartel is always "open."

Given cartel Γ, recall that if $i \in \Gamma$ then firm i's profit is $k_i V^*(K_{\Gamma})$. Thus, an existing cartel member would want to see firm $j \notin \Gamma$ join the cartel as long as $V^*(K_{\Gamma+\{j\}}) > V^*(K_{\Gamma})$. As the next result shows, a cartel member's profit always increases with a more encompassing cartel.

Theorem 4. Assume $\delta > \frac{K-D(c)}{K_{\Gamma'}}$. If $\Gamma' \subset \Gamma''$ then $V^*(K_{\Gamma''}) > V^*(K_{\Gamma'})$.

Corollary 5. Cartel profit (and cartel profit per unit of cartel capacity) is maximized when the cartel is all-inclusive.

The more problematic issue is whether a firm would want to join the cartel. If failure to join means there would be no collusion, then joining is clearly optimal because positive profit as a cartel member is better than the zero profit earned under competition. But suppose failure to join meant that the remaining cartel members could and would effectively collude. In that case, the next result shows that larger firms are more inclined to join a cartel.

Theorem 6. Assume $\delta > \frac{K-D(c)}{K_{\Gamma}}$ and consider $i, j \notin \Gamma$. If $k_j > k_i$ then: (i) if firm i finds it optimal to join cartel Γ then so does firm j; and (ii) if firm j does not find it optimal to join cartel Γ then neither does firm i.

There are two forces at work behind Theorem 6. By Theorem 3, the cartel price is increasing in the capacity controlled by the cartel, which means that the cartel price is higher when a new member brings more capacity under the control of the cartel. Also, in joining a cartel, a firm experiences a drop in its sales as it goes from producing at capacity—as an outsider to the cartel—to producing less than capacity. Because a cartel member's share of cartel output is equal to its share of cartel capacity, the percentage reduction in sales from joining a cartel is less for a firm with more capacity.[6] Both of these effects are in the direction of providing a larger firm with a stronger incentive to join a cartel.

Remark 7. A firm with more capacity is more inclined to join a cartel.

[6] Holding the cartel price fixed at p', the percentage reduction in firm i's output from joining the cartel is

$$\frac{k_i - \left(\frac{k_i}{k_i + K_{\Gamma}}\right)[D(p') - (K - K_{\Gamma} - k_i)]}{k_i}.$$

As the derivative of this expression with respect to k_i is

$$\left(\frac{1}{k_i + K_{\Gamma}}\right)^2 \left[D(p') - K\right] < 0,$$

a firm with larger capacity experiences a smaller percentage reduction in its output from becoming a cartel member.

Not only is the incentive to join a cartel weaker for a smaller firm, we can make a stronger statement by showing that sufficiently small firms will not join a cartel. When a firm's capacity is sufficiently low, joining the cartel has little effect on the cartel price but results in a firm experiencing a decrease in its sales as it goes from producing at capacity to producing below its capacity. Thus, a small firm experiences a trivial rise in its price-cost margin from becoming a cartel member, whereas suffering a nontrivial fall in its sales.

Theorem 8. Assume $\delta > \frac{K-D(c)}{K_\Gamma}$. If k_i is sufficiently small then firm i does not find it optimal to join cartel Γ.

Although larger firms are more inclined than smaller firms to join a cartel, still unresolved is what a cartel looks like. The first step in addressing this issue is to define exactly what it means for a cartel to be stable. Using the definition of d'Aspremont et al. (1983), a cartel is *stable* if all members prefer to be in the cartel (referred to as "internal stability"), and all non-members prefer to be outside the cartel ("external stability"). In formally stating the definition of cartel stability, note that, given a cartel Γ, the (rescaled) profit of firm $i \in \Gamma$ is $(1-\delta)k_i V^*(K_\Gamma)$, and of firm $j \notin \Gamma$ is $[p^*(K_\Gamma) - c]k_j$.

Definition 9. A cartel Γ is stable if: (i) $(1-\delta)k_i V^*(K_\Gamma) > [p^*(K_\Gamma - k_i) - c]k_i$ for all $i \in \Gamma$; and (ii) $[p^*(K_\Gamma) - c]k_i \geq (1-\delta)k_i V^*(K_{\Gamma+\{i\}})$ for all $i \notin \Gamma$.

With this definition, a cartel member is required to *strictly* prefer being part of the cartel. By requiring that profit be strictly higher by joining a cartel, we can rule out innocuous cartels in which the collusive price is the static Nash equilibrium price, so the cartel has no effect on the market. Thus, a stable cartel necessarily involves a price exceeding the non-collusive price.[7] In Section 7, we discuss other possible definitions of stability.

In light of Theorem 8, we know that stable cartels need not be all-inclusive.

Corollary 10. If one of the firms is sufficiently small, then a stable cartel is not all-inclusive.

The next result shows that a cartel is internally stable if and only if the smallest cartel member finds it optimal to be in a cartel, and it is externally stable if and only if the largest non-cartel member prefers to be an outsider. Thus, in assessing whether a cartel is stable, we need only evaluate the incentives of the smallest cartel member and the largest non-cartel member.

Lemma 11. A cartel Γ is stable if and only if: (i) the smallest firm in Γ finds it optimal to be in the cartel; and (ii) the largest firm not in Γ finds it optimal not to be in the cartel.

The next result provides a general existence and characterization of a stable cartel. Enumerate firms so that $k_1 \geq k_2 \geq \cdots \geq k_n$. Theorem 12 shows there exists a collection of the largest firms which forms a stable cartel. In this theorem, $\phi(m)$ is the stationary profit to firm m from joining cartel $\{1, 2, \ldots, m-1\}$ less the stationary profit to firm m, being outside of that cartel:

$$\phi(m) \equiv \left[p^*\left(\sum_{i=1}^{m} k_i\right) - c\right]\left[D\left(p^*\left(\sum_{i=1}^{m} k_i\right)\right) - \left(K - \sum_{i=1}^{m} k_i\right)\right]\left(\frac{k_m}{\sum_{i=1}^{m} k_i}\right)$$
$$- \left[p^*\left(\sum_{i=1}^{m-1} k_i\right) - c\right]k_m.$$

Thus, firm m finds it optimal to be a member of cartel $\{1, 2, \ldots, m\}$ iff $\phi(m) > 0$. Furthermore, by Lemma 11, $\phi(m) > 0$ is necessary and sufficient for the internal stability of cartel $\{1, 2, \ldots, m\}$.

[7] Our theory of cartel formation does presume that all firms are eligible to join a cartel. Given the issues of illegality and incentive compatibility associated with collusion, some firms may not, commonly speaking, "trust" some other firms and, as a result, exclude them from the cartel. For example, Chinese suppliers may have been excluded from the vitamin cartels because of their unfamiliarity to European, Japanese, and Korean firms. However, if the Chinese suppliers would not have wanted to join the cartel anyway, then our analysis is applicable.

Theorem 12. $\{1, 2, \ldots, m^*\}$ is a stable cartel where

$$m^* = \begin{cases} n & \text{if } \phi(n) > 0 \\ m^o & \text{if } \phi(n) \le 0 \end{cases},$$

and m^o is defined by

$$\phi(m^o) > 0 \ge \phi(m^o + 1).$$

If $\delta > \frac{K - D(c)}{K}$ then $m^* \in \{2, \ldots, n\}$ exists.

A stable cartel is then made up of the m^* largest firms, where m^* is such that firm m^* finds it optimal to join firms $1, \ldots, m^* - 1$ as a cartel whereas firm $m^* + 1$ prefers to stay out of the cartel. If firms are sufficiently patient, such a stable cartel exists. There is no presumption, however, that this stable cartel is unique. However, if we assume market demand is linear, $D(p) = a - bp$, then it can be shown that, among the cartels involving the largest firms, there is a unique stable one.[8]

Theorem 13. Assume demand is linear. If there exists m^* such that $\{1, 2, \ldots, m^*\}$ is a stable cartel, then, generically, m^* is unique.

6. Coordinated effects of a merger

■ An important element in the evaluation of a merger is a determination of its coordinated effects, that is, to what extent the merger would make collusion more likely or more effective. This issue has been explored in previous work for the case of all-inclusive cartels. There it has been shown that a reallocation of capacity within an industry—such as through a merger—affects collusion only when it affects the size of the smallest or largest firm. Compte et al. (2002) shows that the minimum discount factor for an all-inclusive cartel to sustain the joint profit maximum is increasing in the capacity of the largest firm. Hence, a merger involving the largest firm makes collusion more difficult.[9] Vasconcelos (2005) finds that collusion is more difficult when the largest firm is larger and the smallest firm is smaller. Hence, a merger involving the smallest firm makes collusion easier, whereas one involving the largest firm makes it more difficult.

In our model, mergers are neutral with respect to an all-inclusive cartel because a necessary and sufficient condition for an all-inclusive cartel to be effective is $\delta > \frac{K - D(c)}{K}$ and thus depends only on aggregate capacity. Furthermore, as price depends only on how much capacity is controlled by the cartel, price is unaffected by a merger as well. A merger can make a difference, however, when cartels are incomplete and the merger affects the composition of the cartel. It is that issue we explore in this section.

To preview our analysis, we begin with some general results showing that mergers among sufficiently large firms and among sufficiently small firms are neutral. Deriving results for the more general class of mergers is difficult, however, due to the lumpiness of a merger. Assuming linear demand, we offer some examples to suggest that it is mergers among moderate-sized firms that are likely to have the biggest impact. To substantiate this claim, numerical simulations are conducted.[10]

Theorem 14 shows that if two or more members of a stable cartel merge then the remaining set of cartel members (including the merged firm) is a stable cartel. Thus, if a cartel comprises the largest firms then a merger among the largest firms has no coordinated effects.[11]

[8] Uniqueness is generic in the sense that it holds when firms have different capacity stocks. When two firms have identical capacities, there can be multiple stable cartels though all involve the same amount of capacity being controlled by the cartel.

[9] Compte et al. (2002) consider the same model as in this article but they do not impose A3. The preceding statement pertains to results derived when A3 does not hold.

[10] Keep in mind that we can only consider mergers that respect assumption A3, which rules out mergers that make the largest firm too large.

[11] If all of the firms in a cartel merge then the degenerate "one-firm cartel" composed of that merged firm is, technically, a stable cartel.

Theorem 14. Consider a cartel Γ and suppose that a set of firms $\omega \subset \Gamma$ merges. Let i_ω denote the merged firm. If Γ is a stable cartel then $\Gamma - \omega + \{i_\omega\}$ is a stable cartel.

If the firms involved in the merger are sufficiently small then again there are no coordinated effects. This is not surprising in light of Theorem 8 showing that a sufficiently small firm will choose not to collude.

Theorem 15. Suppose that a set of firms ω merges. If the firms in ω are sufficiently small then the set of stable cartels is unchanged.

For the remainder of this section, linear demand is assumed, $D(p) = a - bp$, and we focus on the set of stable cartels comprising the largest firms (as characterized in Theorem 12). By Theorem 13, we know that there is a unique cartel in that set. Recall that firms are enumerated according to their capacity: $k_1 \geq k_2 \geq \cdots \geq k_n$. If we further suppose that the discount factor δ is sufficiently close to one so that the ICC for price is not binding for any (profitable) cartel, it can be shown that cartel $\{1, \ldots, m\}$ is stable iff

$$k_m > \sqrt{\left(\sum_{i=1}^{m-1} k_i\right)^2 - (K - (a - bc))^2} \tag{1}$$

and

$$k_{m+1} \leq \sqrt{\left(\sum_{i=1}^{m} k_i\right)^2 - (K - (a - bc))^2}. \tag{2}$$

By (1), we need the smallest firm in the cartel to be sufficiently large; and, by (2), we need the largest firm outside of the cartel to be sufficiently small. By Theorem 13, there is a unique value for m satisfying (1) and (2).

Suppose the pre-merger cartel is Γ. Focusing on a two-firm merger between firms i and j, there are three cases to consider. Let the merged firm be denoted i/j.

(i) $i, j \in \Gamma$ so that the merger involves two of the larger firms in that, prior to the merger, they both would have been in the cartel. By Theorem 14, this merger has no effect.

(ii) $i \in \Gamma, j \notin \Gamma$ so that the merger involves one of the larger firms (as, prior to the merger, it would have been in the cartel) and one of the smaller firms (as, prior to the merger, it would not have been in the cartel). We want to show that cartel capacity expands by at most the capacity of the smaller firm, k_j.

   (a) Let firm h be the largest firm, other than j, which is not in Γ. As the cartel involves the largest firms, for cartel capacity to go up by more than k_j in response to the merger, firm h must find it optimal to join a cartel with capacity $K_\Gamma + k_j$:

   $$k_h > \sqrt{(K_\Gamma + k_j)^2 - (K - (a - bc))^2}.$$

   Because, prior to the merger, firm h did not want to join,

   $$k_h \leq \sqrt{K_\Gamma^2 - (K - (a - bc))^2}.$$

   Combining these two inequalities, we have

   $$\sqrt{K_\Gamma^2 - (K - (a - bc))^2} \geq k_h > \sqrt{(K_\Gamma + k_j)^2 - (K - (a - bc))^2},$$

   which gives us a contradiction. Hence, the post-merger cartel must be a subset of $\Gamma - \{i\} + \{i/j\}$, which means post-merger capacity has an upper bound of $K_\Gamma + k_j$.

   (b) Let us show by way of example that the change in cartel capacity from the merger can be less than k_j and, in fact, can even be negative. That is, a merger between a large and small firm can reduce the amount of capacity in the cartel and thus lower price. Assume $D(p) = 1 - p$ and $c = 0$ for this and the next example. The static Nash equilibrium supply is then 1 and monopoly supply is $\frac{1}{2}$. Note that A3 is satisfied iff $\sum_{i=2}^{n} k_i \geq 1$ and $\frac{1}{2} > k_1$. Assume there are eight firms with the following capacity distribution:

$$k_1 = \frac{4}{10}, \quad k_2 = k_3 = k_4 = k_5 = k_6 = \frac{2}{10}, \quad k_7 = k_8 = \frac{1}{10}.$$

In the pre-merger situation, a stable cartel involves firm 1 and any pair of firms from $\{2, 3, 4, 5, 6\}$ though, to ease the discussion, let us suppose the stable cartel is $\Gamma = \{1, 2, 3\}$.[12] If there is a merger between firms 2 and 7 then the merged firm is the second largest firm, with capacity of $\frac{3}{10}$. The post-merger stable cartel then surely includes firms 1 and 2/7 and, in fact, firm 3 no longer desires to be part of the cartel (nor does any other firm). The addition of the merged firm has increased cartel capacity to the point that firm 3 prefers to be outside of the cartel. Because firm 3's capacity exceeds that of firm 7, total cartel capacity declines, which means that the merger has *reduced* the equilibrium price.

(iii) $i, j \notin \Gamma$ so that the merger involves two of the smaller firms (as, prior to the merger, they would not have been in the cartel). Although the merger may have no impact, it could also raise cartel capacity by the sum of the capacities of the two firms involved in the merger.

(a) If $k_{m+1} \geq k_i + k_j$ then the merger has no impact, as it leaves unaffected the largest firm not in the pre-merger cartel. Thus, if the two firms involved in the merger are sufficiently small, they will not join the cartel even after the merger.

(b) To show that a merger could induce firms to join the cartel, suppose there are eight pre-merger firms with capacities of

$$k_1 = k_2 = \frac{4}{10}, \quad k_3 = k_4 = k_5 = \frac{2}{10}, \quad k_6 = k_7 = k_8 = \frac{1}{10}.$$

The pre-merger cartel is composed of firms 1 and 2. If firms 3 and 4 merge, then the post-merger cartel can be shown to be made up of firms 1, 2, and the merged firm 3/4. A merger between firms 3 and 4 expands their capacity sufficiently that the merged firm wants to join firms 1 and 2 in the cartel when, prior to the merger, both firms 3 and 4 preferred not to be in the cartel. The post-merger cartel controls 50% more capacity.

The previous analysis suggests that coordinated effects are likely to be greatest for a merger involving two moderate-sized firms. To more fully explore this claim, simulations are performed when $D(p) = 1 - p$ and $c = 0$. Contrary to the preceding examples, it is not assumed that δ is sufficiently high that the ICC on price is not binding. A single simulation involves the following five steps.

(i) Fix the number of firms, n.

(ii) Randomly select a vector of capacities $(k_1, \ldots, k_n)$ according to a uniform distribution over $(0, \frac{1}{2})^n$. Requiring that each firm's capacity is less than $\frac{1}{2}$ ensures that the first part of A3 is satisfied. However, we do allow the merged firm's capacity to exceed $\frac{1}{2}$.[13] Next check that $\sum_{h \neq i,j} k_h \geq 1, \ \forall i, j$. If that condition holds then the second part of A3 is satisfied both in the pre-merger and any post-merger situation, in which case go to step (iii). If

[12] As mentioned prior to Theorem 13, uniqueness is generic. We can easily have a unique stable cartel by assuming firms 2 and 3 each have capacity of .20001.

[13] Let us show that this possibility does not alter the characterization of the solution. With a collusive price of p and a cartel Γ, the ICCs are

$$\left(\frac{1}{1-\delta}\right)(p-c)\left[D(p) - (K - K_\Gamma)\right]\left(\frac{k_i}{K_\Gamma}\right) \geq (p-c)\min\{k_i, D(p)\}, i \in \Gamma.$$

If A3 holds then $k_i \leq D(p^m)$, which implies $k_i < D(p) \, \forall p < p^m$ in which case the collection of these ICCs is replaced with this common one:

$$\left(\frac{1}{1-\delta}\right)(p-c)\left[D(p) - (K - K_\Gamma)\right]\left(\frac{k_i}{K_\Gamma}\right) \geq (p-c)k_i \Leftrightarrow D(p) \geq K - \delta K_\Gamma.$$

TABLE 1 Average Price Change Due to a Merger, $n = 5$

Capacity Rank of Merger Partners		
Firm A	Firm B	Average Price Change
4	5	0.0297
3	5	0.0185
1	5	0.0114
2	5	0.0114
3	4	0.0099
1	4	0.0047
2	4	0.0047
1	2	0
1	3	0
2	3	0

instead $\sum_{h \neq i,j} k_h < 1$ for some i, j then redraw the vector of capacities until the condition is satisfied.

(iii) Given $(k_1, \ldots, k_n)$, randomly select the discount factor δ according to a uniform distribution over $(\frac{K-(a-bc)}{K}, 1)$, where $K \equiv \sum_{i=1}^{n} k_i$. By drawing δ from this interval, some collusion is sustainable. (Otherwise, a merger has no effect.)

(iv) Given $(k_1, \ldots, k_n)$ and δ, derive the unique stable cartel involving the largest firms. Record the pre-merger price.

(v) Consider every possible two-firm merger and, for each of them, derive the new stable cartel and post-merger price. Record the change in price due to the merger as well as the rank of the firms (in terms of capacity) involved in the merger.

This procedure is repeated 100,000 times and we report the price change from a merger (averaged over those 100,000 simulations) between the two largest firms, the largest and second largest firms, and so forth down to the two smallest firms. With n firms, there are $1 + 2 + \cdots + (n-1)$ possible mergers. This procedure is performed for $n \in \{5, 6, 7, 8, 9, 10\}$.

Table 1 reports results for the case of five firms, and we have ordered mergers in terms of the size of their price effects. The biggest price effect occurs when the two smallest firms merge; the average price increase is 0.0297. The next biggest price effect occurs when the median firm and smallest firm merge. A merger among any of the three largest firms has no price effect. Roughly speaking, larger price effects tend to occur when smaller firms are involved in the merger. However, as the number of non-large and non-small firms is limited when there are only five firms, more robust findings occur when we allow for more firms.

Now suppose a cartel has a merged firm whose capacity exceeds $D(p)$. The merger firm's ICC is

$$\left(\frac{1}{1-\delta}\right)(p-c)\left[D(p)-(K-K_\Gamma)\right]\left(\frac{k_i}{K_\Gamma}\right) \geq (p-c)D(p) \Leftrightarrow$$

$$\left(\frac{1}{1-\delta}\right)\left[D(p)-(K-K_\Gamma)\right]\left(\frac{1}{K_\Gamma}\right) \geq \frac{D(p)}{k_i}, \tag{3}$$

while for the other cartel members the ICC is

$$\left(\frac{1}{1-\delta}\right)(p-c)\left[D(p)-(K-K_\Gamma)\right]\left(\frac{1}{K_\Gamma}\right) \geq 1. \tag{4}$$

Because $D(p)/k_i < 1$, the binding ICC is (4). Thus, if one cartel member has capacity exceeding $D(p^m)$, the cartel problem is still characterized as

$$\max_p \left(\frac{1}{1-\delta}\right)(p-c)\left(\frac{D(p)-(K-K_\Gamma)}{K_\Gamma}\right)$$

subject to

$$D(p) \geq K - \delta K_\Gamma.$$

TABLE 2 Average Price Change Due to a Merger, $n \in \{6, 7, 8, 9, 10\}$

	Categorization of Firms[a]			Merger Type[b]					
n	Large (L)	Medium (M)	Small (S)	L/L	L/M	L/S	M/M	M/S	S/S
6	1,2	3,4	5,6	0	0.0036	0.0161	0.0153	**0.0255**	0.0240
7	1,2,3	4,5	6,7	0	0.0125	0.0139	**0.0258**	0.0202	0.0138
	1,2	3,4,5	6,7	0	0.0067	0.0131	**0.0201**	0.0186	0.0138
	1,2	3,4	5,6,7	0	0.0034	0.0132	0.0144	**0.0193**	0.0184
8	1,2,3	4,5,6	7,8	0	0.0110	0.0099	**0.0212**	0.0149	0.0087
	1,2,3	4,5	6,7,8	0	0.0096	0.0112	**0.0200**	0.0173	0.0125
	1,2	3,4,5	6,7,8	0	0.0054	0.0109	0.0151	**0.0155**	0.0125
9	1,2,3	4,5,6	7,8,9	0	0.0081	0.0090	**0.0169**	0.0136	0.0094
10	1,2,3,4	5,6,7	8,9,10	0.0011	0.0085	0.0081	**0.0163**	0.0118	0.0074
	1,2,3	4,5,6,7	8,9,10	0	0.0063	0.0077	**0.0136**	0.0111	0.0074
	1,2,3	4,5,6	7,8,9,10	0	0.0063	0.0083	0.0117	**0.0120**	0.0097

[a] "Categorization of Firms" allocates a firm to being large, medium, or small based on its rank in terms of capacity.
[b] "Merger Type" refers to the size—large, medium, or small—of the firms participating in the merger.

When $n > 5$, there are many types of merger so we organize the data by partitioning firms into three categories: large, medium, and small. With six or nine firms, the appropriate categorization is clear; when $n = 6$ (9), the firms with the two (three) largest capacities are labeled *large*, the firms with the two (three) smallest capacities are labeled *small*, and the remaining firms are labeled *medium*. When the number of firms is not divisible by three, we consider the three partitions that are closest to having $n/3$ in each category. For example, when $n = 7$, one partition has firms ranked 1st, 2nd, and 3rd (in terms of capacity) being large, those ranked 4th and 5th being medium, and those ranked 6th and 7th being small; a second partition has firms ranked 1st and 2nd being large, those ranked 3rd, 4th, and 5th being medium, and those ranked 6th and 7th being small; and a third partition has firms ranked 1st and 2nd being large, those ranked 3rd and 4th being medium, and those ranked 5th, 6th, and 7th being small.

Table 2 reports the price effects from various mergers. Let us first analyze the case of six and nine firms and then identify some general findings. When there are six firms, a merger between a medium and small firm has the biggest impact as price increases by 0.0255. A merger between two small firms (that is, the two smallest firms) is almost as significant with a price increase of 0.024. A merger involving a large firm has a trivial impact when it is with a large or medium firm. When there are nine firms, a merger between two medium firms has the biggest price effect, and a merger between a medium and small firm has the next largest impact.

Looking across all of the cases in Table 2, there are at least two general properties. First, the merger with the biggest price effect involves a medium firm and either another medium firm or a small firm. Second, a merger between two large firms has the smallest impact and, with two exceptions, a merger between a large and a medium firm has the next lowest impact.[14] Thus, contrary to previous research based on all-inclusive cartels, we do not find it is mergers involving the smallest or largest firms that have the biggest coordinated effects. Rather, it is mergers involving more moderate-sized firms that have the biggest impact on price. Intuitively, a merger between two moderate-sized firms may significantly expand the amount of capacity controlled by the cartel by inducing the merged firm to become a cartel member. In concluding, it is important to emphasize that we do not consider mergers that are too large. Although we allow a merged firm to have capacity exceeding monopoly supply, the remaining capacity of the non-merged firms must exceed competitive supply and this limits how large mergers can be.

[14] One exception occurs when there are 10 firms and the partition has the four largest firms classified as large, and the other occurs when there are 8 firms and the two smallest firms are classified as small.

7. Discussion

■ There are two key assumptions underlying how we have solved for stable cartels. First is the proportional allocation rule specifying that a firm's share of cartel supply equals its share of cartel capacity. Second is the definition of cartel stability from d'Aspremont et al. (1983) whereby a cartel is stable if a cartel member cannot earn higher profit by leaving the cartel and a firm outside of the cartel cannot earn higher profit by joining the cartel. In this section, we discuss these specifications.

□ **Alternative definitions of cartel stability.** As the literature on coalition formation is vast with definitions, our discussion will focus on closely related definitions and how our results may be robust to these modifications. With our definition of cartel stability, a cartel is stable when none of the firms has an incentive to change its membership decision while taking into account the price adjustments of all rivals to such a change. The literature on coalition formation provides at least two alternative definitions of stability. First, a cartel is stable when none of the firms—by changing its membership decision—can induce a stable cartel that makes the deviating firm better off. By contrast, under our definition, if, say, a cartel member chooses to leave the cartel, its exit leaves the cartel unstable. However, after such an exit, other firms may have an incentive to alter their membership decision, and this could ultimately yield a new stable cartel (or the original one). An alternative notion of cartel stability then presumes firms are *farsighted* and thereby requires that each firm compares its profit under the current stable cartel with the stable cartel that would be induced if it changed its membership decision. Second, our definition of cartel stability does not allow firms to deviate by forming a nonbinding coalition. That is, even though firms individually have no incentive to deviate, firms could find a *joint deviation* profitable. An alternative definition of cartel stability then additionally requires that there exists no deviating coalition that is mutually beneficial. Note that both alternative concepts of cartel stability can be viewed as a refinement in the sense that they (weakly) reduce the set of stable cartels.

The possibility that firms are farsighted has been explored in Diamantoudi (2005) when firms are identical. The focus there is on cartel size where a cartel size is unstable when a firm can induce a larger or smaller stable cartel in which it earns higher profit. Applying this notion of stability to our setting is problematic in that nonexistence may occur. Consider a cartel comprising the largest firms and suppose that the smallest insider and the largest outsider are approximately of equal size. This has two key implications: first, the cartel is still stable if these two firms change places; second, the outsider earns more profits than the insider as both receive the same price and the outsider sells more. According to our definition of stability, the smallest insider has no incentive to deviate. Suppose, however, that the smallest insider did exit the cartel. In the new situation, the largest outsider now has an incentive to join the cartel, which again is stable according to our definition. In turn, however, the new insider may defect, which provides an incentive for the largest outsider to join again. Thus, when firms are farsighted then cycling may occur and, as a result, there is no stable cartel.[15]

The idea that firms might deviate by forming mutually beneficial nonbinding agreements is explored in Bernheim, Peleg, and Whinston (1987) and Bernheim and Whinston (1987), where the concept of coalition-proof Nash equilibrium is introduced. They argue that the Nash best response (as implied by our definition of stability) is necessary, but may not be sufficient. For example, a stable cartel that is Pareto dominated is not coalition proof. We can show, however, that a stable cartel (according to our definition) with the highest total capacity is a coalition proof Nash equilibrium. Within the set of stable cartels comprising the largest firms, the candidate for being coalition proof is then the largest one, that is, the highest m such that $\{1, \ldots, m\}$ is a stable cartel. Whereas that cartel could well have the largest capacity among all stable cartels—and

[15] This also occurs when firms are identical, which is why Diamantoudi (2005) focuses only on cartel *size* and is unconcerned with the identities of the firms in the cartel.

thereby be coalition proof—we do not believe that is universally true. In conclusion, although there is no reason to think that the largest stable cartel comprising the largest firms is not coalition proof, a proof of it eludes us.

□ **Alternative allocation rules.** The proportional rule for allocating cartel supply may leave room for Pareto improvements.[16] For a cartel to be stable under the proportional rule, an outsider of the cartel cannot find it profitable to join the cartel when its share of cartel supply would equal its share of cartel capacity. It is possible, however, that there is another allocation that would induce the firm to join the cartel—by giving it a bigger share than what the proportional rule dictates—and still allow existing cartel members to earn higher profit. Thus, restricting the cartel to using a proportional rule may leave Pareto improvements unexploited. We have two points to argue against taking that approach, and two points to directly support the proportional rule.

First, endogenizing the allocation—as opposed to setting a fixed rule such as the proportional rule—would make the model intractable because allocations would be defined by a recursive system. To see the complexities introduced, consider a cartel with firms 1 and 2. Now suppose one can find a price and allocation among firms 1, 2, and 3 which is incentive compatible, firm 3 earns higher profit than as an outsider (when the cartel has only firms 1 and 2), and firms 1 and 2 earn higher profit than when the cartel has only firms 1 and 2. Given a cartel with those three firms, we can similarly ask whether there is a price and allocation which allow firm 4 to join the cartel. Note that the characteristics of the four-firm cartel allocation will depend on the allocation previously derived to get firm 3 to join the cartel because the four-firm allocation must give firm 3 enough so that it does not prefer the cartel with only firms 1, 2, and 3. Thus, there is a recursivity to the allocations which tremendously reduces the tractability of the model and would make it extremely difficult to derive useful results.

Second, we have worked with some examples and it is true that there are allocations which can induce a firm to join a cartel which is stable under the proportional rule. However, these examples can have a problematic feature: the allocation may require quotas to be *inversely* related to capacity. Consider the second example in Section 6 (after firms 3 and 4 merged):

$$k_1 = k_2 = k_3 = \frac{4}{10}, \quad k_4 = \frac{2}{10}, \quad k_5 = k_6 = k_7 = k_8 = \frac{1}{10}.$$

Under the proportional rule, a cartel among firms 1, 2, and 3 is stable. One can show that firm 4 can be induced to join the cartel—and firms 1, 2, and 3 are made better off—if firm 4's share of cartel supply is at least 41% and firms 1, 2, and 3 each receive a share of at least 17%. For example, firms 1, 2, and 3 each receiving 19% and firm 4 receiving 43% will work. Note that firm 4 produces more than twice as much as firm 1 (or 2 or 3) even though it has only half as much capacity. This property seems unreasonable and is surely counterfactual. To upset a cartel stable under the proportional rule may then require unreasonable allocations.

Let us next turn to arguing for the proportional rule. We have already pointed out that the proportional rule has been used by actual cartels and that it satisfies Rawls's notion of fairness. But is there a reason why existing cartel members would not deviate from the proportional rule even when doing so could expand the cartel and increase its profits? We think there is. As soon as the cartel raises the possibility of a nonproportional allocation for the purpose of inducing another firm to join, what is to prevent existing small members from wanting to renegotiate their allocation? The practical challenges faced by firms in coming to an agreement over an allocation are not to be taken lightly. In one sample of international cartels, almost 50% collapsed because of lack of agreement on the allocation of profits (Eckbo, 1976). Likewise, Levenstein and Suslow (2004) state: "Bargaining problems were much more likely to undermine collusion than was secret cheating. About one quarter of the cartel episodes ended because of bargaining problems. Bargaining issues affected virtually every industry studied." If there is a

[16] We thank a referee for making this insightful observation.

simple acceptable allocation rule that allows for collusion, cartel members may be hesitant to entertain modifications to it out of fear that it will lead to negotiations that may prove difficult to resolve and could ultimately result in the collapse of collusion.

Finally, many of our results seem robust to the allocation rule. First, the result that the cartel sets an umbrella price (with the non-colluding firms undercutting it) is not dependent on the allocation rule. Second, consider the identified tradeoff associated with joining a cartel: it results in a higher price but the firm is required to reduce its supply relative to when it was outside of the cartel. That tradeoff surely holds for a wide class of allocation rules. A third finding is that a larger firm has a stronger incentive to join a cartel. With the proportional rule, this result is because price rises more with its addition (as more capacity comes under the control of the cartel) and its quantity does not fall as much as for a smaller firm because higher capacity means a higher share of cartel supply. The first condition is largely independent of the allocation rule, though the second condition is not. However, as long as the larger firm's supply does not decline too much with it joining a cartel (compared to the decrease in supply for a smaller firm), the first effect will dominate and again the larger firm will have a stronger incentive to join a cartel. A fourth result is that the coordinated effects of a merger are largest for moderate-sized firms. This finding is driven by the third result, which is that larger firms are more inclined to join a cartel. Thus, a merger which combines two non-cartel members may result in a firm large enough that it finds it is profitable to join the cartel. Again, that result is likely to hold for other allocation rules. In sum, the intuition driving many of our results do not appear dependent on the use of the proportional rule, whereas the proportional rule has many benefits in terms of tractability and descriptive realism.

8. Concluding remarks

■ How encompassing is a cartel? What are the characteristics of the firms which choose to collude? In spite of the large body of theoretical work on collusion, there is very little research that addresses these questions within the infinitely repeated game framework. The objective of our research was to shed some light on these questions. We find that cartels are often incomplete. When a cartel is incomplete, colluding firms set a price that serves as an umbrella, with non-cartel members pricing below it and producing at capacity whereas cartel members restrict supply below capacity. Sufficiently small firms will not be part of a cartel and, more generally, a larger firm is more inclined to join a cartel. Cartel membership is driven by the confluence of two forces. First, the cartel price is increasing in the capacity controlled by the cartel, which means that the cartel (and market) price is higher when a new member brings more capacity under the control of the cartel. Thus, a firm with more capacity raises the cartel price more by joining. Second, in becoming a cartel member, a firm experiences a drop in its sales as it goes from producing at capacity—as an outsider to the cartel—to producing less than capacity. Because a cartel member's share of cartel output is assumed to equal its share of cartel capacity, the percentage reduction in sales from joining a cartel is less for a firm with more capacity. Both of these effects serve to make a larger firm more inclined to join a cartel.

One of the potentially more significant policy contributions of the article concerns the coordinated effects of a merger. Previous theory focused attention on mergers that affect the size of the largest or smallest firm, whereas our analysis suggests that it is mergers involving firms that lie between those extremes that may have the largest coordinated effects. A merger between two moderate-sized firms may significantly expand the size and profitability of a potential cartel by inducing the merged firm to be a cartel member. From the perspective of an antitrust or competition authority, concerns about coordinated effects may be most severe for these mergers involving firms which are not small, but not large either.

Appendix A

Derivation of the proportional allocation rule is described.

What is a reasonable way for a cartel to allocate demand among its members? To address this question, we begin by defining an allocation rule. For any capacity vector $(k_1, \ldots, k_n)$ and cartel Γ, an allocation rule ϕ prescribes an allocation of cartel demand, $D(p) - (K - K_\Gamma)$, given a cartel price p. Assume allocation rules are anonymous in that they do not depend on a firm's identity, only its capacity.

Given $(k_1, \ldots, k_n, \Gamma)$ and a cartel price p, let the resulting allocation be

$$\{\phi_i(p; k_1, \ldots, k_n, \Gamma)\}_{i\in\Gamma},$$

where ϕ_i is the quantity of firm i. For an allocation to be implementable when $p > c$, it must satisfy the incentive compatibility constraints:

$$\left(\frac{1}{1-\delta}\right)(p-c)\phi_i \geq (p-c)k_i, \quad \forall i \in \Gamma$$

or

$$\phi_i \geq (1-\delta)\, k_i, \quad \forall i \in \Gamma, \tag{A1}$$

where

$$\sum_{i\in\Gamma} \phi_i = D(p) - (K - K_\Gamma). \tag{A2}$$

For allocation rule ϕ, $\Omega(k_1, \ldots, k_n, \Gamma; \phi)$ denotes the set of incentive compatible prices given $(k_1, \ldots, k_n, \Gamma)$:

$$\Omega\,(k_1, \ldots, k_n, \Gamma; \phi) \equiv \{p > c : \text{ (A1) and (A2) are satisfied}\}\,.$$

As any allocation rule is required to distribute demand among cartel members so that ICCs are satisfied, it now becomes a point of bargaining as to what is a reasonable allocation. Here, we apply the concept of fairness as articulated by Rawls (1971). His perspective is that it is difficult for people to agree as to what is fair when their place in society is already determined. For example, those who are poor find progressive taxation fair, whereas those who are wealthy find it unfair. To extract from those biases, Rawls proposes a hypothetical situation whereby people go behind the "veil of ignorance" and consider fairness as though their place in society had not yet been determined, what he refers to as the "original state."

Our approach to deriving a fair allocation rule is to apply this Rawlsian logic. Given some allocation of capacities, it is clear that a firm with high capacity will find it fair that it receives more, whereas a firm with low capacity may disagree. Let us then consider the thought experiment in which a firm does not know its capacity. More specifically, given a capacity vector $(k_1, \ldots, k_n)$ and a cartel Γ, a firm knows that it is a member of the cartel and that the cartel has capacities $\{k_i\}_{i\in\Gamma}$, but does not know which of the elements in that set is its capacity. In this original state, we want to show that firms would agree to the proportional sharing rule, denoted $\widetilde{\phi}$:

$$\widetilde{\phi}_i\,(p; k_1, \ldots, k_n, \Gamma) \equiv \left(\frac{k_i}{K_\Gamma}\right)[D(p) - (K - K_\Gamma)]\,.$$

Because allocation rules are anonymous, then, for any price, all cartel members have the same expected share of cartel profit. Therefore, all firms rank allocation rules according to the amount of total cartel profit generated. We then say that allocation rule ϕ' is preferred to ϕ'' when ϕ' produces at least as high total cartel profit as ϕ'' for all $(k_1, \ldots, k_n, \Gamma)$ and strictly higher profit for some $(k_1, \ldots, k_n, \Gamma)$. Let $p^*(k_1, \ldots, k_n, \Gamma; \phi)$ be the profit-maximizing price for the cartel given the allocation rule, ϕ. Noting that the ICCs must be satisfied, it is defined by

$$p^*\,(k_1, \ldots, k_n, \Gamma; \phi) = \arg \max_{p\in\Omega(k_1,\ldots,k_n,\Gamma;\phi)} (p-c)\,[D(p) - (K - K_\Gamma)]\,.$$

Let

$$p^o\,(k_1, \ldots, k_n, \Gamma) = \arg\max_p (p-c)\,[D(p) - (K - K_\Gamma)]$$

be the unconstrained optimal price for the cartel. Given the concavity of the objective function, the cartel's profit is strictly increasing in price for all $p < p^o(k_1, \ldots, k_n, \Gamma; \phi)$. Given the relationship between price and profit, it follows that ϕ' is preferred to ϕ'' iff

$$p^*(k_1, \ldots, k_n, \Gamma; \phi') \geq p^*(k_1, \ldots, k_n, \Gamma; \phi''), \quad \forall\,(k_1, \ldots, k_n, \Gamma)$$

and

$$p^*(k_1, \ldots, k_n, \Gamma; \phi') > p^*(k_1, \ldots, k_n, \Gamma; \phi''), \quad \text{for some} \quad (k_1, \ldots, k_n, \Gamma).$$

Let us show that the proportional sharing rule $\widetilde{\phi}$ is preferred to all other allocation rules. If $(k_1, \ldots, k_n, \Gamma)$ is such that

$$p^*(k_1, \ldots, k_n, \Gamma; \widetilde{\phi}) = p^o(k_1, \ldots, k_n, \Gamma; \phi),$$

then clearly there is no other allocation rule that yields higher profit than the proportional sharing rule.

Now consider $(k_1, \ldots, k_n, \Gamma)$ such that

$$p^*(k_1, \ldots, k_n, \Gamma; \widetilde{\phi}) < p^o(k_1, \ldots, k_n, \Gamma; \phi). \tag{A3}$$

(It is straightforward to show that (A3) holds for some $(k_1, \ldots, k_n, \Gamma)$.) (A3) implies that one or more ICCs are binding. In fact, with $\widetilde{\phi}$, either none or all ICCs are binding. The ICC of firm i is

$$\widetilde{\phi}_i \geq (1-\delta)\, k_i \text{ or } \left(\frac{k_i}{K_\Gamma}\right)[D(p) - (K - K_\Gamma)] \geq (1-\delta)\, k_i,$$

which is equivalent to

$$\left(\frac{1}{K_\Gamma}\right)[D(p) - (K - K_\Gamma)] \geq 1 - \delta.$$

As this ICC is the same for all firms then, if (A3) holds, all firms' ICCs are binding:

$$q_i = (1-\delta)k_i, \quad \forall i \in \Gamma,$$

where

$$q_i = \left(\frac{k_i}{K_\Gamma}\right)[D(p^*(k_1, \ldots, k_n, \Gamma; \widetilde{\phi})) - (K - K_\Gamma)], \quad \forall i \in \Gamma,$$

$$\sum_{i \in \Gamma} q_i = D(p^*(k_1, \ldots, k_n, \Gamma; \widetilde{\phi})) - (K - K_\Gamma).$$

Continuing to assume that (A3) holds, consider a different allocation rule, $\bar{\phi}$. Let us show that

$$p^*(k_1, \ldots, k_n, \Gamma; \widetilde{\phi}) > p^*(k_1, \ldots, k_n, \Gamma; \bar{\phi}).$$

Consider $p = p^*(k_1, \ldots, k_n, \Gamma; \widetilde{\phi})$. If $\bar{\phi}$ differs from $\widetilde{\phi}$ then it allocates total cartel supply $D(p^*(k_1, \ldots, k_n, \Gamma; \widetilde{\phi})) - (K - K_\Gamma)$ differently, which means that at least one cartel member—say firm j—must have lower quantity than what $\widetilde{\phi}$ prescribes. But then $\bar{\phi}_j < (1-\delta)k_j$, in which case firm j's ICC is violated. If $p > p^*(k_1, \ldots, k_n, \Gamma; \widetilde{\phi})$, then total cartel supply is less than $D(p^*(k_1, \ldots, k_n, \Gamma; \widetilde{\phi})) - (K - K_\Gamma)$, in which case, again, there must be some cartel member whose allocated supply is lower than with $\widetilde{\phi}$ and thus its ICC is violated. We conclude that the proportional sharing rule $\widetilde{\phi}$ is preferred to any other allocation rule.

In essence, the proportional sharing rule does the best job of satisfying the ICCs in that, when the unconstrained optimal price cannot be supported, it allocates cartel supply in such a manner so as to loosen up the ICCs as much as possible. Consider another allocation rule which leaves some ICCs slack. In that situation, the cartel could set a higher price and—although there is less cartel supply to go around—the ICCs could still be satisfied by reducing the allocation to those firms with slack ICCs. A proportional sharing rule sets price as high as is possible by allocating cartel demand so that all ICCs bind and thus price cannot be any higher without violating the ICCs. As, behind the veil of ignorance, all firms prefer an allocation rule that produces a higher price, fairness implies that they use the proportional sharing rule.

Appendix B

Proofs

Proof of Lemma 1. Given the cartel earns positive profit, first note that each non-colluding firm earns positive profit because a non-colluding firm can always match the collusive price and it will have positive demand (by A2) and positive profit (as the collusive price must be above cost).

Across non-colluding firms, let p'' be the highest upper bound to the support of their mixed strategies. Suppose $p'' > p'$ (in which case Lemma 1 is not true); p'' may or may not be a mass point. Initially, assume only one non-colluding firm has an upper bound of p''. Because $\sum_{j \neq i} k_j \geq D(c)$ by A3 then $\sum_{j \neq i} k_j > D(p'')$, which implies that this firm has zero demand at a price of p'' which implies zero profit and thus an expected equilibrium profit of zero. This contradicts a non-colluding firm earning positive profit. Now assume there are two or more firms with an upper bound of p''; again it is the highest upper bound across non-colluding firms and $p'' > p'$. If those firms put zero mass on p'' then the preceding argument works to deliver a contradiction. If one or more firms put positive mass on p'' then consider a firm with an upper bound of p'' and for which there are other firms putting positive mass at p''. When ε is sufficiently small, its expected profit is higher by pricing at $p'' - \varepsilon$ than at p'' because it experiences a discrete increase in demand with a trivial fall in the price-cost margin. This does presume that it is not capacity constrained, which is indeed true by A2, and that $\sum_{j=1}^n k_j > D(p'')$. This contradicts having p'' in the support. Hence, the upper bound to all non-colluding firms' supports cannot exceed p'.

Next we want to show that p' is not an upper bound either. Suppose it is. Because a non-colluding firm's expected profit is positive, its expected profit (and demand) must be positive at a price of p' if p' is in its support. In addition, as the cartel is putting unit mass at p', a non-colluding firm experiences a discrete increase in its expected demand by pricing at $p' - \varepsilon$. If ε is sufficiently small, $p' - \varepsilon$ is a profitable deviation unless a firm is capacity constrained. But as $\sum_{j=1}^{n} k_j > D(p')$, A2 implies this firm has excess capacity.

Proof of Lemma 2. By Lemma 1, equilibrium has all non-colluding firms price below p'. Because $D(p') > \sum_{j \notin \Gamma} k_j$, this means that all non-colluding firms are capacity constrained at the equilibrium price vector. Given that firm *j*'s profit is then $(p_j - c)k_j \forall p_j < p'$, its optimal price is $p' - \varepsilon$. The unique Nash equilibrium then has all non-colluding firms pricing at $p' - \varepsilon$ and producing at capacity.

Proof of Theorem 3. If $\delta \leq \frac{K-D(c)}{K_{\Gamma'}}$ then $p^*(K_{\Gamma'}) = c = p^*(K_{\Gamma''})$ and thus (i) is true and (ii) is vacuously true. If $\frac{K-D(c)}{K_{\Gamma'}} < \delta \leq \frac{K-D(c)}{K_{\Gamma''}}$ then $p^*(K_{\Gamma'}) > c = p^*(K_{\Gamma''})$ and thus (i) and (ii) are true. Finally, suppose $\frac{K-D(c)}{K_{\Gamma''}} < \delta$ so that $p^*(K_{\Gamma'}),\ p^*(K_{\Gamma''}) > c$. As (ii) applies, we then need to prove $p^*(K_{\Gamma'}) > p^*(K_{\Gamma''})$.

If the ICC is not binding then $p^*(K_\Gamma)$ is defined by

$$D(p^*(K_\Gamma)) - (K - K_\Gamma) + (p^*(K_\Gamma) - c)D'(p^*(K_\Gamma)) = 0.$$

Take the total derivative with respect to K_Γ,

$$D'(p^*(K_\Gamma))p^{*\prime}(K_\Gamma) + 1 + p^{*\prime}(K_\Gamma)D'(p^*(K_\Gamma)) + (p^*(K_\Gamma) - c)D''(p^*(K_\Gamma))p^{*\prime}(K_\Gamma) = 0$$

$$p^{*\prime}(K_\Gamma) = -\frac{1}{2D'(p^*(K_\Gamma)) + (p^*(K_\Gamma) - c)D''(p^*(K_\Gamma))} > 0,$$

which follows from the second-order condition holding. If instead the ICC is binding, then $p^*(K_\Gamma)$ is defined by

$$p^*(K_\Gamma) = D^{-1}(K - \delta K_\Gamma).$$

Take the total derivative with respect to K_Γ,

$$p^{*\prime}(K_\Gamma) = -\delta D^{-1\prime}(K - \delta K_\Gamma) > 0.$$

Because $p^*(K_\Gamma)$ is continuous and increasing in K_Γ then, when $\frac{K-D(c)}{K_{\Gamma''}} < \delta$, it follows that if $K_{\Gamma'} > K_{\Gamma''}$ then $p^*(K_{\Gamma'}) > p^*(K_{\Gamma''})$.

Proof of Theorem 4. Given the collection of firms Γ is colluding, the cartel's problem is

$$(V^*(K_\Gamma) \equiv) \max_p V(p, \Gamma) = \max_p \left(\frac{1}{1-\delta}\right)(p - c)\left(\frac{D(p) - K + K_\Gamma}{K_\Gamma}\right)$$

subject to

$$D(p) - K + \delta K_\Gamma \geq 0.$$

Suppose the cartel expands from Γ' to Γ'', which means cartel capacity rises from $K_{\Gamma'}$ to $K_{\Gamma''}$. First note that if K_Γ is increased, the cartel's objective increases for any price,

$$\frac{\partial V(p, \Gamma)}{\partial K_\Gamma} = \left(\frac{1}{1-\delta}\right)(p - c)\left(\frac{K - D(p)}{K_\Gamma^2}\right) > 0.$$

Next note that increasing K_Γ loosens the ICC:

$$\frac{\partial(D(p) - K + \delta K_\Gamma)}{\partial K_\Gamma} = \delta > 0.$$

Because $D(c) > K - \delta K_{\Gamma'}$, the set of prices such that the ICC is satisfied is nonempty for Γ' and, by the preceding analysis, is strictly larger for Γ''. Because the set of feasible (that is, incentive compatible) prices is larger with Γ'' and the objective is higher for any price with Γ'', it follows that $V^*(K_{\Gamma''}) > V^*(K_{\Gamma'})$.

Proof of Theorem 6. Consider a cartel Γ and a firm that is not a member of Γ. If its capacity is k, it prefers to join cartel Γ iff

$$[p^*(K_\Gamma + k) - c][D(p^*(K_\Gamma + k)) - (K - K_\Gamma - k)]\left(\frac{k}{K_\Gamma + k}\right) > [p^*(K_\Gamma) - c]k,$$

where the LHS expression is the stationary profit from joining the cartel and the RHS is the stationary profit from remaining outside the cartel. This condition can be rearranged to

$$[p^*(K_\Gamma + k) - c]\left(\frac{D(p^*(K_\Gamma + k)) - (K - K_\Gamma - k)}{K_\Gamma + k}\right) - [p^*(K_\Gamma) - c] > 0. \tag{B1}$$

If the expression in (B1) is increasing in k then $k_j > k_i$ implies that if (B1) holds for firm i then it holds for firm j, and if (B1) does not hold for firm j then it does not hold for firm i.

To establish that the expression in (B1) is increasing in k, consider $k'' > k'$ and let us show

$$\begin{aligned}&\left[p^*(K_\Gamma + k'') - c\right]\left(\frac{D(p^*(K_\Gamma + k'')) - (K - K_\Gamma - k'')}{K_\Gamma + k''}\right)\\ \geq\ &\left[p^*(K_\Gamma + k') - c\right]\left(\frac{D(p^*(K_\Gamma + k')) - (K - K_\Gamma - k'')}{K_\Gamma + k''}\right)\\ >\ &\left[p^*(K_\Gamma + k') - c\right]\left(\frac{D(p^*(K_\Gamma + k')) - (K - K_\Gamma - k')}{K_\Gamma + k'}\right).\end{aligned} \tag{B2}$$

In proving the weak inequality in (B2), first note that the two expressions differ only in that the LHS has a price of $p^*(K_\Gamma + k'')$ and the RHS has a price of $p^*(K_\Gamma + k')$. Recall that

$$\begin{aligned}p^*(K_\Gamma + k'') &= \arg\max (p - c)\left(\frac{D(p) - (K - K_\Gamma - k'')}{K_\Gamma + k''}\right)\\ \text{subject to } p &\leq D^{-1}(K - \delta(K_\Gamma + k'')).\end{aligned} \tag{B3}$$

As the cartel price is nondecreasing in cartel capacity (Theorem 3), $p^*(K_\Gamma + k'') \geq p^*(K_\Gamma + k')$ and, therefore, $p^*(K_\Gamma + k')$ also satisfies the constraint in (B3). Because $p^*(K_\Gamma + k'')$ maximizes this objective, the resulting value must be at least as great as that from a price of $p^*(K_\Gamma + k')$. Hence, the weak inequality in (B2) is true. The strong inequality in (B2) follows because $\frac{D(p^*(K_\Gamma+k))-(K-K_\Gamma-k)}{K_\Gamma+k}$ is increasing in k. Therefore, the expression in (B1) is increasing in k.

Proof of Theorem 8. Evaluate (B1) as a firm's capacity becomes really small:

$$\lim_{k\to 0}[p^*(K_\Gamma + k) - c]\left(\frac{D(p^*(K_\Gamma + k')) - (K - K_\Gamma - k)}{K_\Gamma + k}\right) - [p^*(K_\Gamma) - c] = -[p^*(K_\Gamma) - c]\left(\frac{K - D(p^*(K_\Gamma))}{K_\Gamma}\right) < 0.$$

Proof of Lemma 11. The "only if" part is obvious, so let us consider the "if" part. Part (ii) follows immediately from Theorem 6. All that remains is to show that if the smallest firm in Γ finds it optimal to be in the cartel then all other firms in Γ find it optimal as well. Consider a firm in Γ with capacity k. It finds it optimal to be a member of the cartel iff

$$\psi(K_\Gamma, k) \equiv [p^*(K_\Gamma) - c][D(p^*(K_\Gamma)) - (K - K_\Gamma)]\left(\frac{k}{K_\Gamma}\right) - [p^*(K_\Gamma - k) - c]k > 0.$$

Dividing through by k, we have

$$\psi(K_\Gamma, k) \equiv [p^*(K_\Gamma) - c][D(p^*(K_\Gamma)) - (K - K_\Gamma)]\left(\frac{1}{K_\Gamma}\right) - p^*(K_\Gamma - k) + c > 0. \tag{B4}$$

Suppose this condition holds for the smallest firm in Γ. Because the cartel price is nondecreasing in cartel capacity, $p^*(K_\Gamma - k)$ is nonincreasing in k. Thus, (B4) holds for larger values of k.

Proof of Theorem 12. If $\phi(n) > 0$ then the smallest firm of cartel $\{1, 2, \ldots, n\}$ prefers to be in the cartel and there is no largest firm outside of the cartel. By Lemma 11, $\{1, 2, \ldots, n\}$ is then a stable cartel. Suppose instead $\phi(n) \leq 0$ and m^o exists. By the definition of m^o, firm m^o prefers to be in cartel $\{1, 2, \ldots, m^o\}$ and firm $m^o + 1$ prefers to be outside the cartel. By Lemma 11, $\{1, 2, \ldots, m^o\}$ is a stable cartel. We conclude that $\{1, 2, \ldots, m^*\}$ is a stable cartel.

To prove existence, if $\phi(n) > 0$ then $m^* = n$ and thus m^* exists. Now suppose $\phi(n) \leq 0$. Because $\delta > \frac{K-D(c)}{K}$ implies $p^*(\sum_{i=1}^n k_i) > c$ then $\phi(n) \leq 0$ implies $p^*(\sum_{i=1}^{n-1} k_i) > c$. Given that $p^*(k_1) = c$ (that is, a "cartel" composed only of firm 1 leads to the static Nash equilibrium price by A3) and $p^*(\sum_{i=1}^{n-1} k_i) > c$, there exists $m' \in \{2, \ldots, n-1\}$ such that

$$p^*\left(\sum_{i=1}^{m'} k_i\right) > c = p^*\left(\sum_{i=1}^{m} k_i\right) \quad \text{for all} \quad m < m'.$$

In other words, $\{1, 2, \ldots, m'\}$ is the least inclusive collection of the largest firms that is able to sustain collusion. Hence,

$$\phi(m') = \left[p^*\left(\sum_{i=1}^{m'} k_i\right) - c\right]\left[D\left(p^*\left(\sum_{i=1}^{m'} k_i\right)\right) - \left(K - \sum_{i=1}^{m'} k_i\right)\right]\left(\frac{k_{m'}}{\sum_{i=1}^{m'} k_i}\right) > 0.$$

Because $\phi(m') > 0 \geq \phi(n)$, there exists $m^o \in \{m', \ldots, n-1\}$ such that $\phi(m^o) > 0 \geq \phi(m^o + 1)$. Thus, m^* exists.

Proof of Theorem 13. We want to prove that there exists a unique stable cartel under the assumption that cartels consist of the largest firms, as characterized by Theorem 12. Suppose there exists a stable cartel comprising the largest firms (Theorem 12 provides sufficient conditions for existence). Let $\Gamma(m) \equiv \{1, \ldots, m\}$ denote a cartel comprising the m

largest firms. Denote the smallest stable cartel to be $\{1, \ldots, m'\}$ or $\Gamma(m')$. To prove uniqueness, we will show that $\Gamma(m)$ is unstable for all $m > m'$.

Because $\Gamma(m')$ is stable, firm $m'+1$ prefers not to be in the cartel:

$$\left[p^*\left(\sum_{i=1}^{m'} k_i\right) - c\right] k_{m'+1} \geq \left[p^*\left(\sum_{i=1}^{m'+1} k_i\right) - c\right]\left[D\left(p^*\left(\sum_{i=1}^{m'+1} k_i\right)\right) - \left(K - \sum_{i=1}^{m'+1} k_i\right)\right]\left(\frac{k_{m'+1}}{\sum_{i=1}^{m'+1} k_i}\right). \tag{B5}$$

Our method of proof will be to show that (B5) implies

$$\left[p^*\left(\sum_{i=1}^{m} k_i\right) - c\right] k_{m+1} \geq \left[p^*\left(\sum_{i=1}^{m+1} k_i\right) - c\right]\left[D\left(p^*\left(\sum_{i=1}^{m+1} k_i\right)\right) - \left(K - \sum_{i=1}^{m+1} k_i\right)\right]\left(\frac{k_{m+1}}{\sum_{i=1}^{m+1} k_i}\right), \quad \forall m \geq m'. \tag{B6}$$

(B6) implies, for all $m > m'$, that $\Gamma(m)$ is not stable because firm m does not want to be part of the cartel.

In working with (B6), the particular form it takes depends on whether or not the ICC for price is binding. If it is not binding for cartel Γ then the cartel price is

$$p^*(K_\Gamma) = \frac{a + bc - K + K_\Gamma}{2b}, \tag{B7}$$

and if it is binding then

$$p^*(K_\Gamma) = \frac{a - K + \delta K_\Gamma}{b}. \tag{B8}$$

As a preliminary result, let us show that if the ICC is not binding for cartel Γ then it is not binding for cartel Γ' where $K_{\Gamma'} > K_\Gamma$. Suppose that property did not hold, so that the ICC is not binding for Γ and is binding for Γ'. Using (B7) and (B8), that it is not binding for Γ means

$$\frac{a - K + \delta K_\Gamma}{b} \geq \frac{a + bc - K + K_\Gamma}{2b} \Leftrightarrow (2\delta - 1) K_\Gamma \geq K - (a - bc),$$

and that it is binding for Γ' means

$$\frac{a + bc - K + K_{\Gamma'}}{2b} \geq \frac{a - K + \delta K_{\Gamma'}}{b} \Leftrightarrow K - (a - bc) \geq (2\delta - 1) K_{\Gamma'}.$$

Combining these two inequalities, we have

$$(2\delta - 1) K_\Gamma \geq K - (a - bc) \geq (2\delta - 1) K_{\Gamma'}.$$

Because $K - (a - bc) > 0$, the LHS inequality implies $\delta > 1/2$. Thus, a necessary condition for this inequality to be true is $K_\Gamma \geq K_{\Gamma'}$, which is a contradiction. Thus, if the ICC is not binding for some cartel then it is not binding for any larger cartel ("larger" meaning it has more capacity).

From the preceding result, there are then three possible cases with respect to cartel Γ: (i) the ICC is not binding for cartel Γ and all larger cartels; (ii) the ICC is binding for cartel Γ and all larger cartels; and (iii) the ICC is binding for cartel Γ and there is some larger cartel Γ' for which it is not binding. Note that case (iii) implies the ICC is binding for all cartels smaller than Γ' and is not binding for all cartels larger than Γ'. In showing that (B5) implies (B6), we will consider each of these three cases for cartel $\Gamma(m')$. Recall that $\Gamma(m')$ is the smallest stable cartel comprising the largest firms.

Let us begin with case (i) so that the ICC is not binding for $\Gamma(m)$, for all $m \geq m'$. Substituting (B7) into (B6) and performing some simplifications, the expression in (B6) can be shown to take the form

$$k_{m+1} \leq \sqrt{\left(\sum_{i=1}^{m} k_i\right)^2 - (K - (a - bc))^2}. \tag{B9}$$

Because

$$k_{m+1} \leq k_{m'+1} \leq \sqrt{\left(\sum_{j=1}^{m'} k_j\right)^2 - (K - (a - bc))^2} < \sqrt{\left(\sum_{j=1}^{m} k_j\right)^2 - (K - (a - bc))^2}, \ \forall m > m', \tag{B10}$$

then (B5) implies (B6).

Next consider case (ii) so that the ICC is binding for $\Gamma(m)$, for all $m \geq m'$. Substituting (B8) into (B6) and performing some simplifications, the expression in (B6) can be shown to take the form

$$k_{m+1} \leq \frac{(a-bc) - K + \delta\left(\sum_{j=1}^{m} k_j\right)}{1-\delta}. \tag{B11}$$

Because

$$k_{m+1} \leq k_{m'+1} \leq \frac{(a-bc) - K + \delta\left(\sum_{j=1}^{m'} k_j\right)}{1-\delta} < \frac{(a-bc) - K + \delta\left(\sum_{j=1}^{m} k_j\right)}{1-\delta}, \quad \forall m > m', \tag{B12}$$

(B5) then implies (B6).

Case (iii) has the ICC binding for cartel $\Gamma(m')$ and there exists $m'' \in \{m', \ldots, n-1\}$ whereby it is binding for $\Gamma(m)$ iff $m \leq m''$. Note that if $\Gamma(m''+1)$ is unstable then, by the analysis associated with case (i), $\Gamma(m)$ is unstable for all $m > m''+1$. We also know that $\Gamma(m'+1)$ is unstable. Case (iii) is divided into two subcases: (a) $m'' = m'$; and (b) $m'' > m'$.

For case (iii)(a), we have $m'' = m'$. We need only show that $\Gamma(m'+2)$ is unstable when the ICC is not binding. Using (B9), $\Gamma(m'+2)$ is unstable iff

$$k_{m'+2} \leq \sqrt{\left(\sum_{j=1}^{m'+1} k_j\right)^2 - (K-(a-bc))^2}.$$

Rearranging, we have

$$k^2_{m'+2} - k^2_{m'+1} \leq \left(\sum_{j=1}^{m'} k_j\right)^2 + 2k_{m'+1}\left(\sum_{j=1}^{m'} k_j\right) - (K-(a-bc))^2. \tag{B13}$$

The LHS of (B13) is weakly negative because $k_{m'+1} \geq k_{m'+2}$. It therefore suffices to show that the RHS of (B13) is nonnegative:

$$\left(\sum_{j=1}^{m'} k_j\right)^2 + 2k_{m'+1}\left(\sum_{j=1}^{m'} k_j\right) - (K-(a-bc))^2 \geq 0 \Leftrightarrow \sqrt{\left(\sum_{j=1}^{m'} k_j\right)^2 + 2k_{m'+1}\left(\sum_{j=1}^{m'} k_j\right)} \geq K-(a-bc). \tag{B14}$$

(B14) holds because the stability of $\Gamma(m')$ implies $\sum_{j=1}^{m'} k_j > K - (a-bc)$.

For case (iii)(b), we have $m'' > m'$; and recall that the ICC is binding iff $m \leq m''$. We need to show that $\Gamma(m)$ is unstable for all $m \in \{m'+2, \ldots, m''+1\}$. By the analysis associated with case (ii), (B6) is satisfied for all $m \in \{m'+1, \ldots, m''-1\}$ and, therefore, $\Gamma(m)$ is unstable for all $m \in \{m'+1, \ldots, m''-1\}$. Thus, we just need to show that $\Gamma(m)$ is unstable for $m \in \{m'', m''+1\}$.

In considering $\Gamma(m'')$, first note that the internal stability condition of firm m'' for cartel $\Gamma(m'')$ is the converse of the external stability condition of firm m'' for cartel $\Gamma(m''-1)$; the former condition requires that firm m'' strictly prefers to join $\Gamma(m''-1)$, whereas the latter condition requires that firm m'' weakly prefers not to join $\Gamma(m''-1)$. As we know, the latter condition holds then that the internal stability condition for firm m'' for $\Gamma(m'')$ is violated. Hence, $\Gamma(m'')$ is unstable.

This leaves the case of cartel $\Gamma(m''+1)$. First observe that

$$\frac{K - D(c) + \sum_{j=1}^{m''} k_j}{2\sum_{j=1}^{m''} k_j} \geq \delta \geq \frac{K-(a-bc) + \sum_{j=1}^{m''+1} k_j}{2\sum_{j=1}^{m''+1} k_j}, \tag{B15}$$

as the ICC is binding for $\Gamma(m'')$ (which is the LHS inequality) and the ICC is not binding for $\Gamma(m''+1)$ (which is the RHS inequality). We will show that firm $m''+1$ has no incentive to join $\Gamma(m'')$, which is the case iff

$$\left[p^*\left(\sum_{j=1}^{m''+1} k_j\right) - c\right]\left[\frac{a - bp^*\left(\sum_{j=1}^{m''+1} k_j\right) - K + \sum_{j=1}^{m''+1} k_j}{\sum_{j=1}^{m''+1} k_j}\right] - \left[p^*\left(\sum_{j=1}^{m''} k_j\right) - c\right] \leq 0,$$

or

$$\left((a-bc)-K+\sum_{j=1}^{m''+1}k_j\right)^2-4\left(\sum_{j=1}^{m''+1}k_j\right)\left((a-bc)-K+\delta\sum_{j=1}^{m''}k_j\right)\leq 0.$$

Rearranging this condition, we have to show

$$\delta\geq\frac{\left((a-bc)-K+\sum_{j=1}^{m''+1}k_j\right)^2+4\sum_{j=1}^{m''+1}k_j\left(K-(a-bc)\right)}{4\sum_{j=1}^{m''+1}k_j\sum_{j=1}^{m''}k_j}. \tag{B16}$$

We know from (B15) that

$$\delta\geq\frac{K-(a-bc)+\sum_{j=1}^{m''+1}k_j}{2\sum_{j=1}^{m''+1}k_j}. \tag{B17}$$

(B16) then holds if the RHS of (B17) exceeds the RHS of (B16):

$$\frac{K-D(c)+\sum_{j=1}^{m''+1}k_j}{2\sum_{j=1}^{m''+1}k_j}\geq\frac{\left((a-bc)-K+K\sum_{j=1}^{m''+1}k_j\right)^2+4\sum_{j=1}^{m''+1}k_j\left(K-D(c)\right)}{4\sum_{j=1}^{m''+1}k_j\sum_{j=1}^{m''}k_j},$$

which is equivalent to

$$\sum_{j=1}^{m''}k_j\geq K-(a-bc)+k_{m''+1}. \tag{B18}$$

Rearranging (B18), firm $m''+1$ prefers not to join $\Gamma(m'')$ if

$$k_{m''+1}\leq(a-bc)-K+\sum_{j=1}^{m''}k_j,$$

which is equivalent to

$$k_{m''+1}\leq(a-bc)-K+\sum_{j=1}^{m''-1}k_j+k_{m''}. \tag{B19}$$

We know that residual demand for cartel $\Gamma(m'')$ is positive,

$$(a-bc)-K+\sum_{j=1}^{m''-1}k_j>0,$$

and that $k_{m''}\geq k_{m''+1}$. Hence, (B19) holds and firm $m''+1$ has no incentive to join cartel $\Gamma(m'')$, which implies that $\Gamma(m''+1)$ is unstable.

Proof of Theorem 14. For cartel Γ, the conditions for its members to find it optimal to be part of the cartel are

$$\left[p^*(K_\Gamma)-c\right]\left[D\left(p^*(K_\Gamma)\right)-(K-K_\Gamma)\right]\left(\frac{k_h}{K_\Gamma}\right)>\left[p^*\left(K_\Gamma-k_h\right)-c\right]k_h,\ \forall h\in\Gamma, \tag{B20}$$

and for non-members to find it optimal not to join are

$$\left[p^*(K_\Gamma)-c\right]k_h\geq\left[p^*\left(K_\Gamma+k_h\right)-c\right]\left[D\left(p^*\left(K_\Gamma+k_h\right)\right)-\left(K-K_\Gamma-k_h\right)\right]\left(\frac{k_h}{K_\Gamma+k_h}\right),\ \forall h\notin\Gamma. \tag{B21}$$

Consider what happens to these conditions when firms $i, j\in\Gamma$ merge to form firm i/j. As K_Γ is unaffected, the conditions in (B21) for the non-cartel members are unchanged. Also, the conditions in (B20) for $h\in\Gamma-\{i,j\}$ are unchanged as well. All that we need to assess is whether the newly merged firm wants to be part of the cartel, which is the case iff

$$\left[p^*(K_\Gamma)-c\right]\left[D(p^*(\Gamma))-(K-K_\Gamma)\right]\left(\frac{k_i+k_j}{K_\Gamma}\right)>\left[p^*\left(K_\Gamma-k_i-k_j\right)-c\right]\left(k_i+k_j\right)$$

or

$$[p^*(K_\Gamma) - c]\left(\frac{D(p^*(\Gamma)) - (K - K_\Gamma)}{K_\Gamma}\right) > p^*(K_\Gamma - k_i - k_j) - c. \tag{B22}$$

That firms *i* and *j* find it individually profitable to be part of the cartel means that

$$[p^*(K_\Gamma) - c]\,[D\,(p^*(K_\Gamma)) - (K - K_\Gamma)]\left(\frac{k_i}{K_\Gamma}\right) > [p^*\,(K_\Gamma - k_i) - c]\,k_i \quad \text{and}$$

$$[p^*(K_\Gamma) - c]\,[D\,(p^*(K_\Gamma)) - (K - K_\Gamma)]\left(\frac{k_j}{K_\Gamma}\right) > [p^*(K_\Gamma - k_j) - c]k_j$$

or

$$[p^*(K_\Gamma) - c]\left(\frac{D(p^*(\Gamma)) - (K - K_\Gamma)}{K_\Gamma}\right) > \max\{p^*(K_\Gamma - k_i),\, p^*(K_\Gamma - k_j)\} - c. \tag{B23}$$

We need to show that if (B23) is true then (B22) is true. This property is immediate as the LHS expressions are identical and

$$\max\{p^*(K_\Gamma - k_i),\, p^*(K_\Gamma - k_j)\} > p^*(K_\Gamma - k_i - k_j)$$

by Theorem 3.

The preceding argument can be used to show that any merger among prospective cartel members is conducive to collusion. For example, suppose we consider a merger among firms $h, i, j \in \Gamma$. If Γ is stable then so is $\Gamma - \{i, j\} + \{i/j\}$ and, if $\Gamma - \{i, j\} + \{i/j\}$ is stable then so is $\Gamma - \{h, i, j\} + \{h/i/j\}$, where firm $h/i/j$ has capacity $k_h + k_i + k_j$.

Proof of Theorem 15. Consider a cartel that was stable prior to the merger. The conditions for internal stability are unaffected by the merger as, given the merger participants are sufficiently small, they were not part of the cartel. For a non-merger participant that was not part of a pre-merger condition, its external stability condition is unaffected. Finally, by Theorem 8, if the merged firm is sufficiently small then it will not be part of any stable cartel.

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